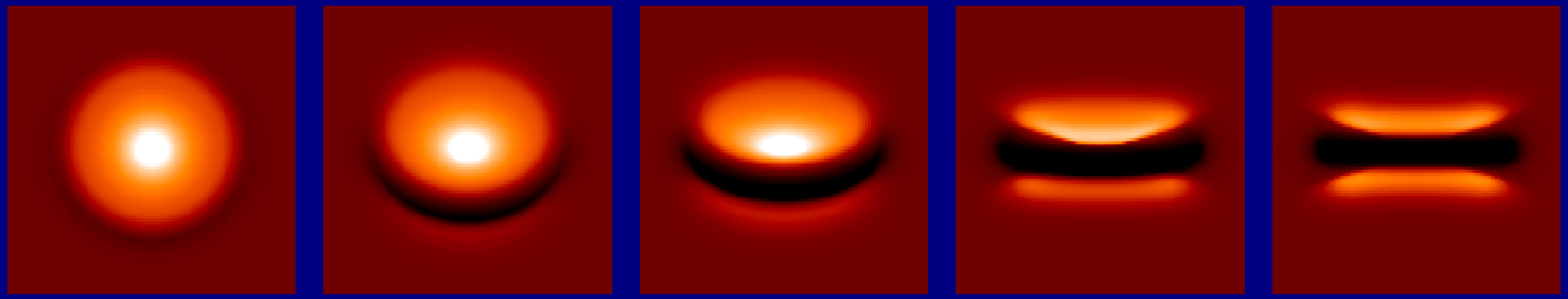
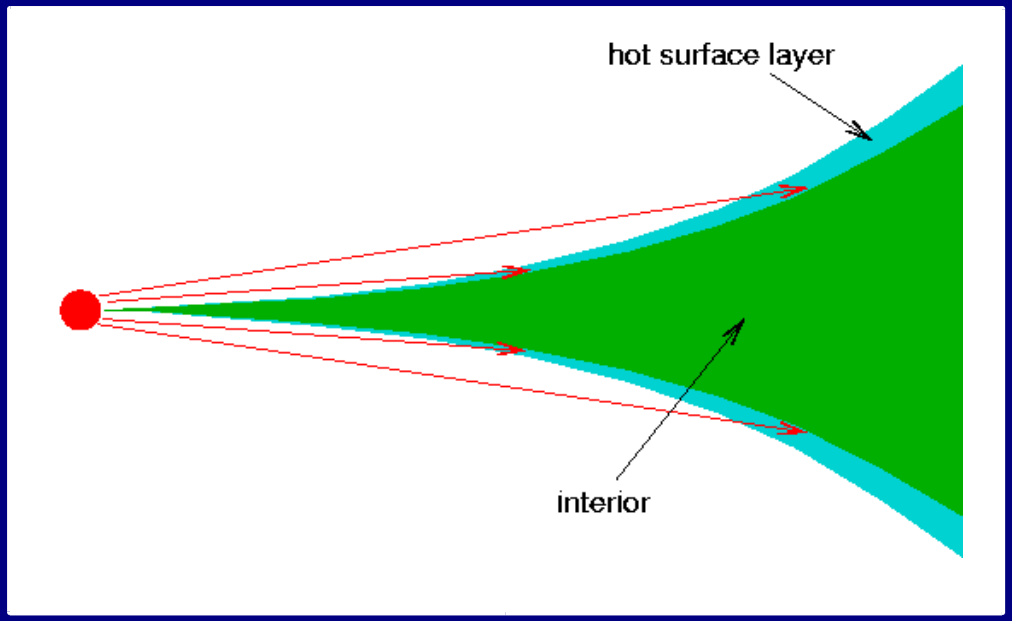
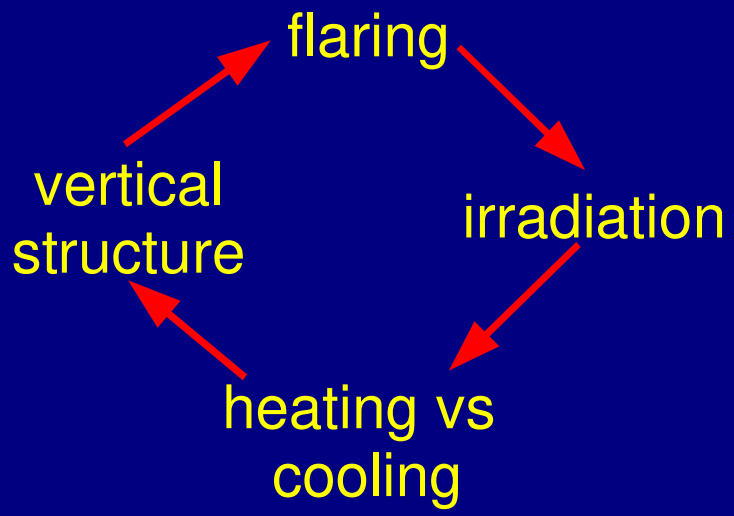
2-D/3-D Models of Protoplanetary Disks



C.P. Dullemond

What determines structure of disk?





Vertical density & temperature structure determined from continuum radiative transfer and hydrostatic equilibrium.

SED and images can be computed from this model using a ray-tracer.

Equations to solve:

Radiative transfer equations:

- Formal transfer eq:

$$\Omega \cdot \nabla I_\nu = \rho \kappa_\nu [B(T) - I_\nu]$$

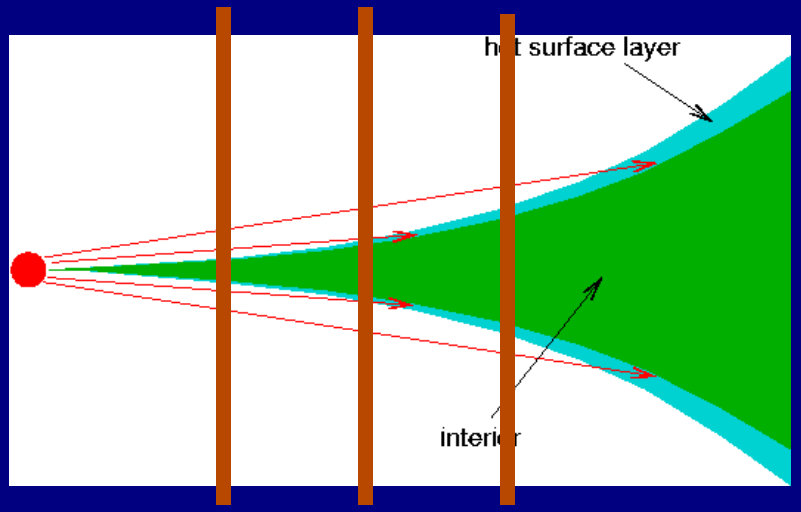
- Solving T:

$$\int \kappa_\nu B_\nu(T)\, d\nu = \frac{1}{4\pi} \int \int \kappa_\nu I_\nu \, d\Omega \, d\nu$$

Hydrostatic equilibrium equation:

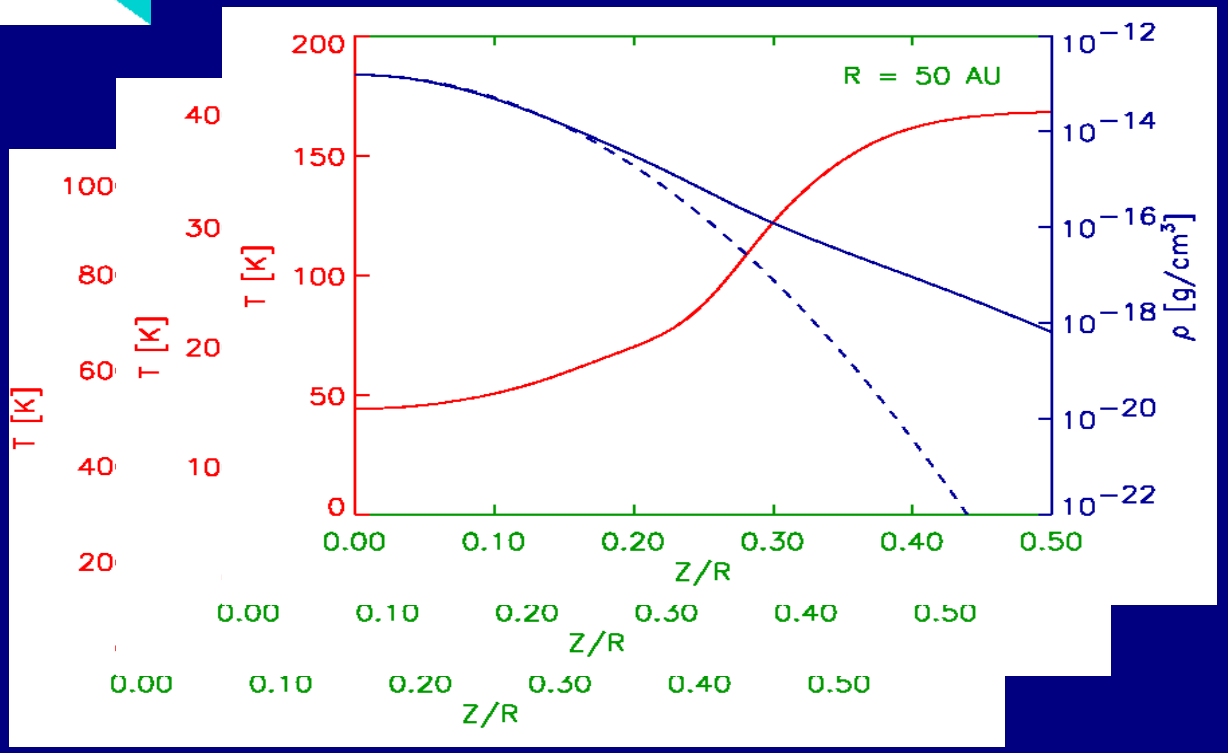
$$\frac{dP}{dz} = -\rho \frac{GM_*}{R^3} z$$

'Old' approach: 1+1D splitting

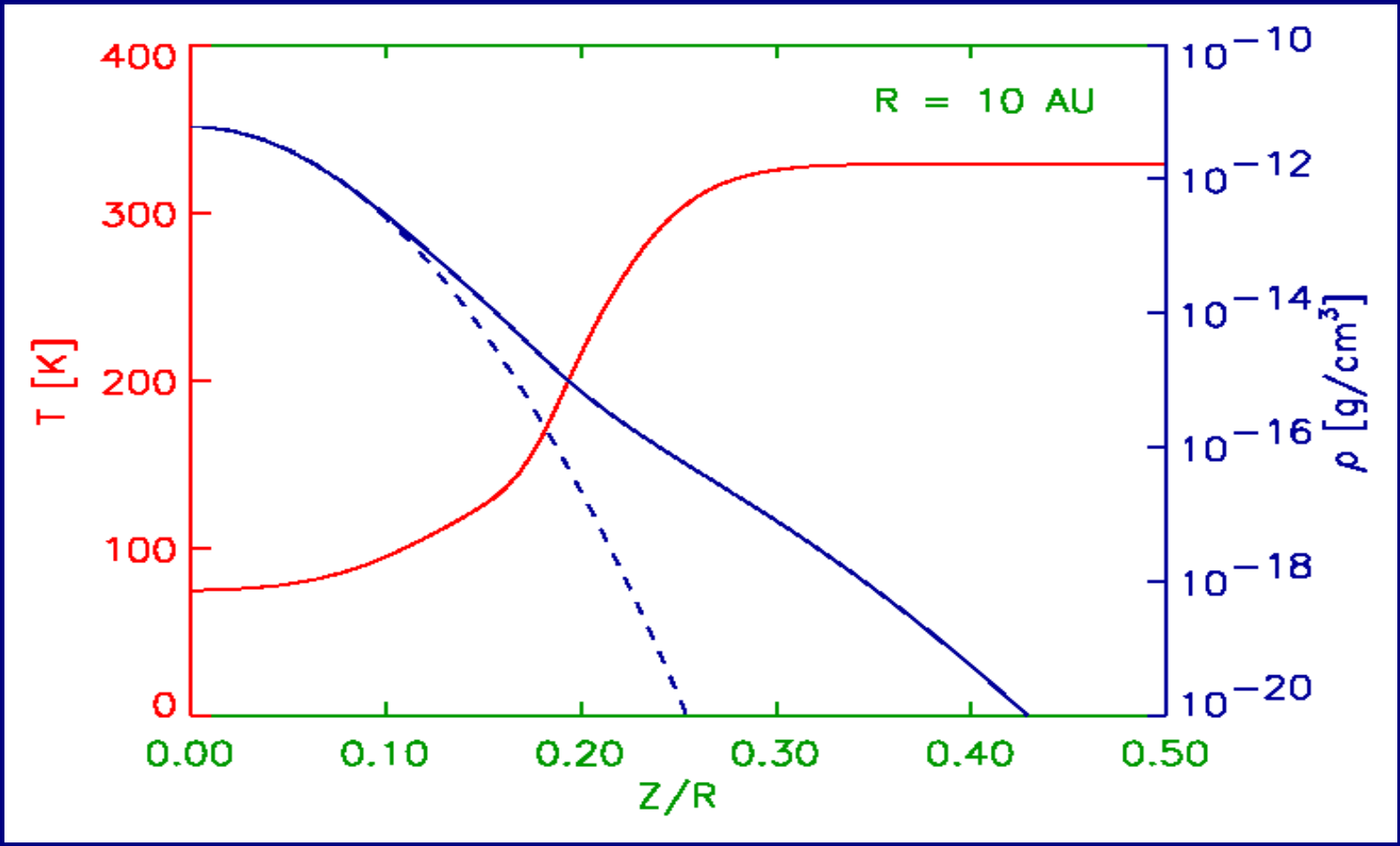


Global disk model...

... consists of vertical slices, each forming a 1D problem. All slices are independent from each other.



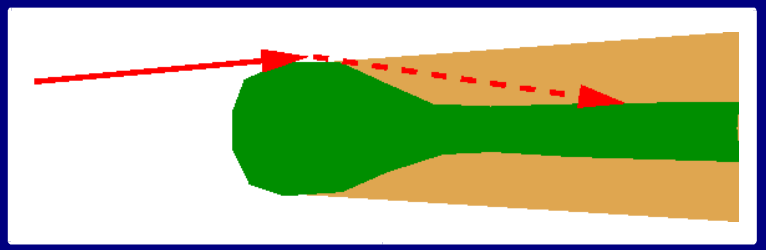
A closer look at one slice:



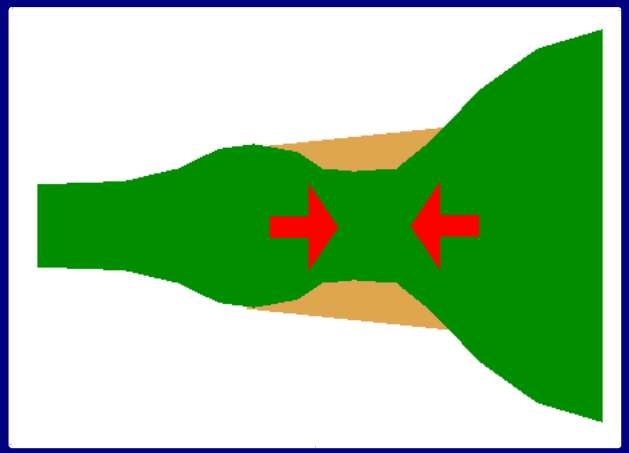
The need for 2-D/3-D models

- Disk models so far:
  - → 1+0D: Two-layer (Chiang & Goldreich; Lachaume et al.)
  - → 1+1D: Vertical structure (D'Alessio et al; Malbet et al.; Bell et al.; etc.)

Can't do:



Indirect irradiation



Radial radiative diffusion

Build-up of 2-D/3-D model package

	Insert:	Obtain:
Ray tracer:	ρ, T →	SED, image(s)
Radiative Transfer:	ρ →	T
Vertical Hydrostatics:	T →	ρ

Radiative Transfer Package

Build-up of 2-D/3-D model package

	Insert:		Obtain:
Ray tracer:	ρ, T	→	SED, image(s)
Radiative Transfer:	ρ	→	T
Vertical Hydrostatics:	T	→	ρ

Full Disk Model Package

Problem:

Disks can have extremely high optical depths, in particular at small radii (inward of few AU).

Most radiative transfer methods are known to have difficulties with extreme optical depths.

Two codes available:

- "RADMC"

  A Monte Carlo type code

- "RADICAL"

  A Variable Eddington Tensor based code

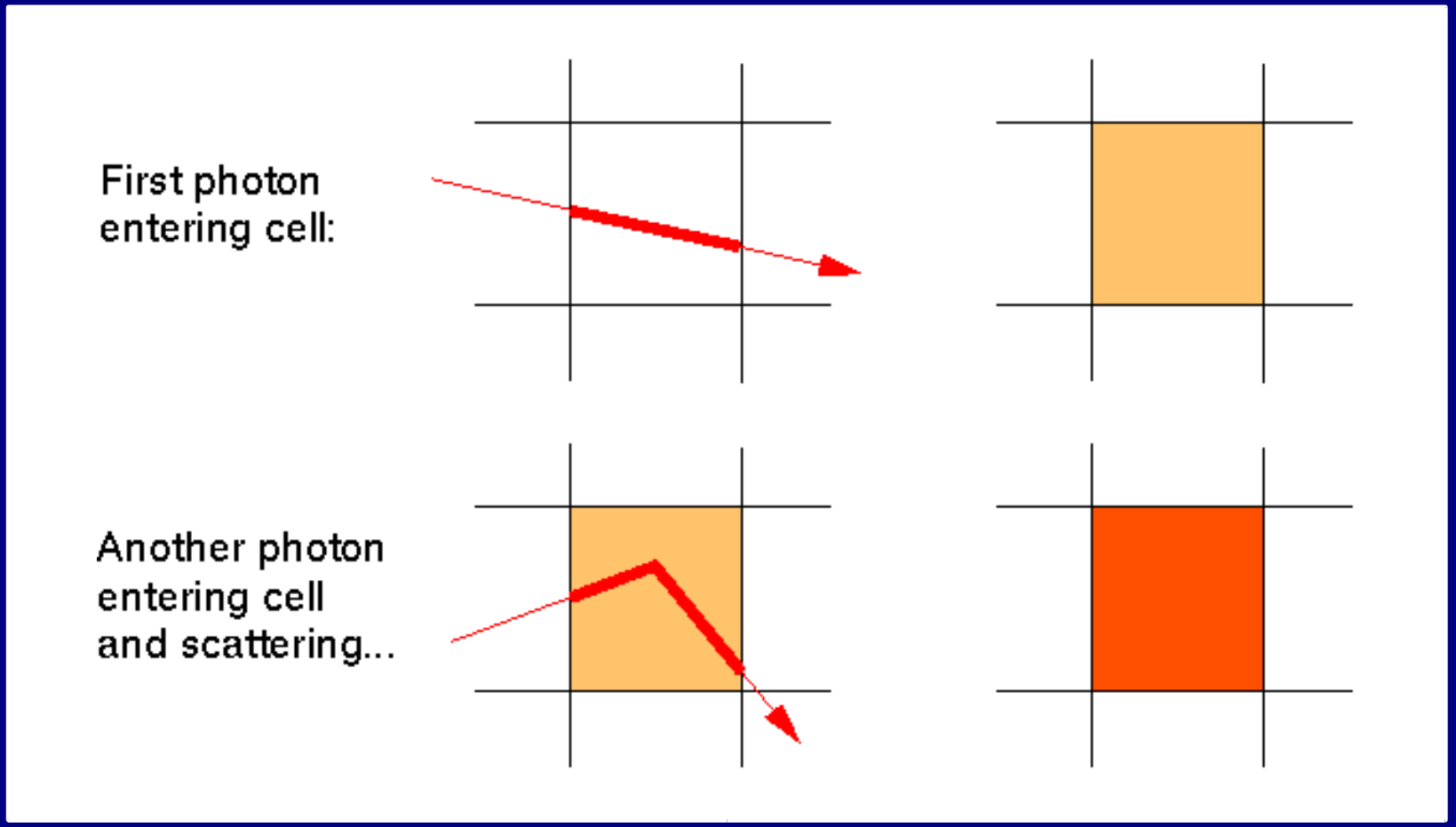
RADMC: Monte Carlo Method

Method: adaption of Bjorkman & Wood (2001)

- Chop stellar radiation field in N photon packages. Each one is launched from the star into arbitrary direction.
- Photons travel through disk and change direction regularly due to:
  - → Scattering events
  - → Absorption events
- In addition to absorption events we drop energy into cells continously

RADMC:
Monte Carlo Method

Method: adaption of Bjorkman & Wood (2001)



RADMC: Monte Carlo Method

Method: adaption of Bjorkman & Wood (2001)

- Advantages:
  - → Very robust and reliable. “Fool proof”.
  - → Intuitive: it is a “simulation”
- Disadvantages:
  - → Numerical noise
  - → At high optical depths (e.g. disk midplane): noise gets catastrophic
  - → High optical depths: long execution times

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

The transfer equation:

$$\mu \frac{dI_{\mu,\nu}}{dz} = \rho \kappa_\nu [B(T) - I_{\mu,\nu}]$$

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

Define the “moments”:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_{\mu,\nu}\, d\mu$$

$$H_\nu = \frac{1}{2}\int_{-1}^{1} I_{\mu,\nu}\, \mu\, d\mu$$

$$K_\nu = \frac{1}{2}\int_{-1}^{1} I_{\mu,\nu}\, \mu^2\, d\mu$$

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

Express transfer equation in terms of moments:

$$\frac{dH_\nu}{dz} = \rho\kappa_\nu(B_\nu(T) - J_\nu)$$
$$\frac{dK_\nu}{dz} = -\rho\kappa_\nu H_\nu$$

Closure problem: 2 equations, 3 unknowns

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

Express K_ν in terms of J_ν times a so-called "Eddington Factor":

$$K_\nu = f_\nu J_\nu$$

Compute f_ν using full 2-D/3-D ray-tracer-like radiative transfer code (we use a "short characteristics method")

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

Iterate the following procedures:

1) Initial guess for the temperature
2) Compute f_ν
3) Solve moment equations
4) Find temperature from moments
5) If not yet converged, then goto 2)

RADICAL:
Variable Eddington Tensor Method

(For simplicity: 1-D equations shown here)

- Advantages:
  - → Convergence in ~ 6 iterations
  - → Can handle extreme optical depths (e.g. $\tau = 10^5$)
  - → Smooth temperature profiles
- Disadvantages:
  - → Can have small, but non-negligible systematic errors in temperature due to gridding effects

Requirements for Radiative Transfer Code

Hydrostatic Equilibrium Solver:

H_p goes as $T^{1/2}$ → Small errors in $\langle T \rangle$ are OK...

Need smooth T at all R,z → Monte Carlo noise BIG trouble!

~~RADMC (Monte Carlo)~~

RADICAL (Edd Tensors)

SEDs and images (ray-tracer):

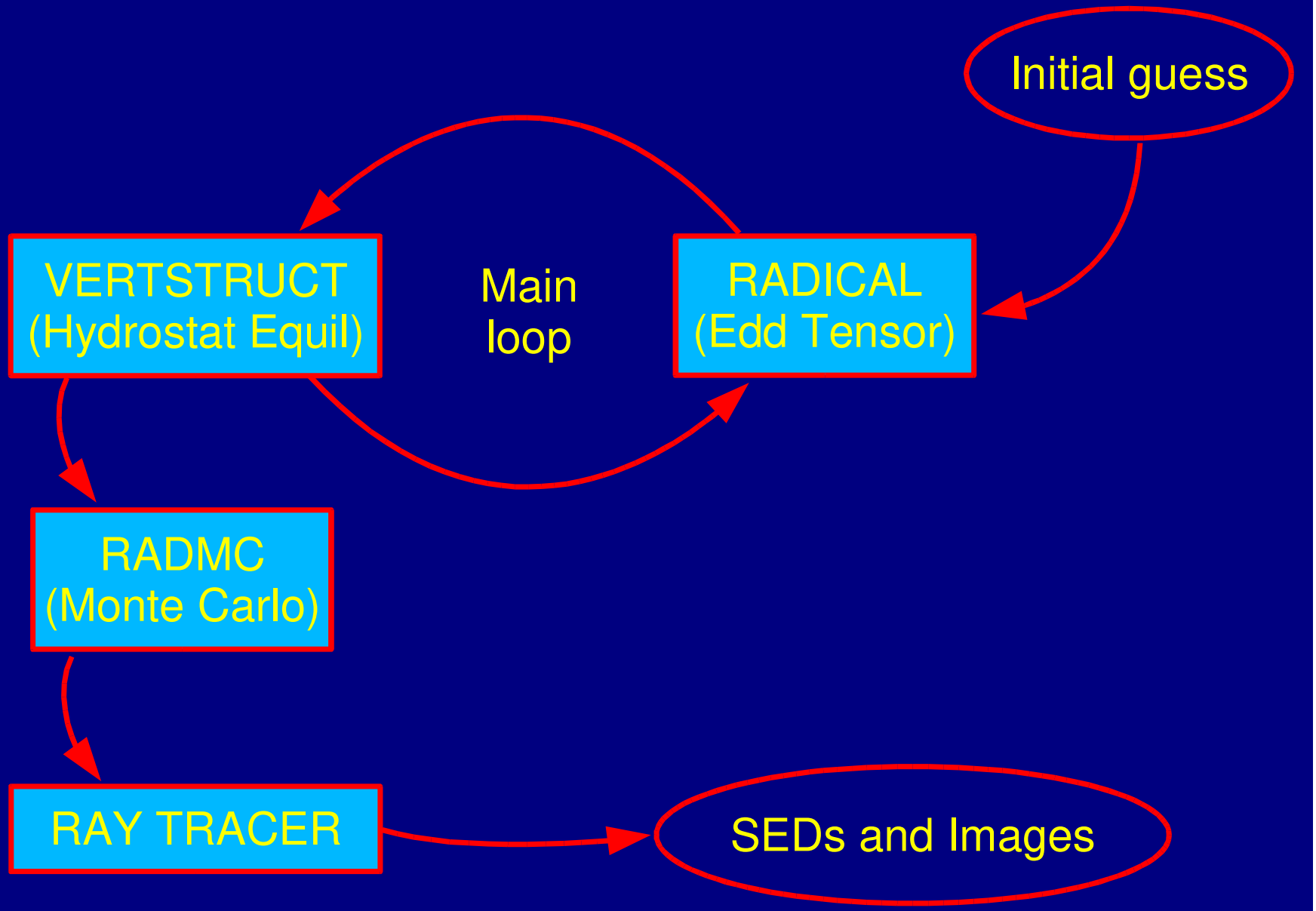
Spectrum goes as T^4 → Require accurate $\langle T \rangle$

Noise in T is washed out → Monte Carlo noise no problem

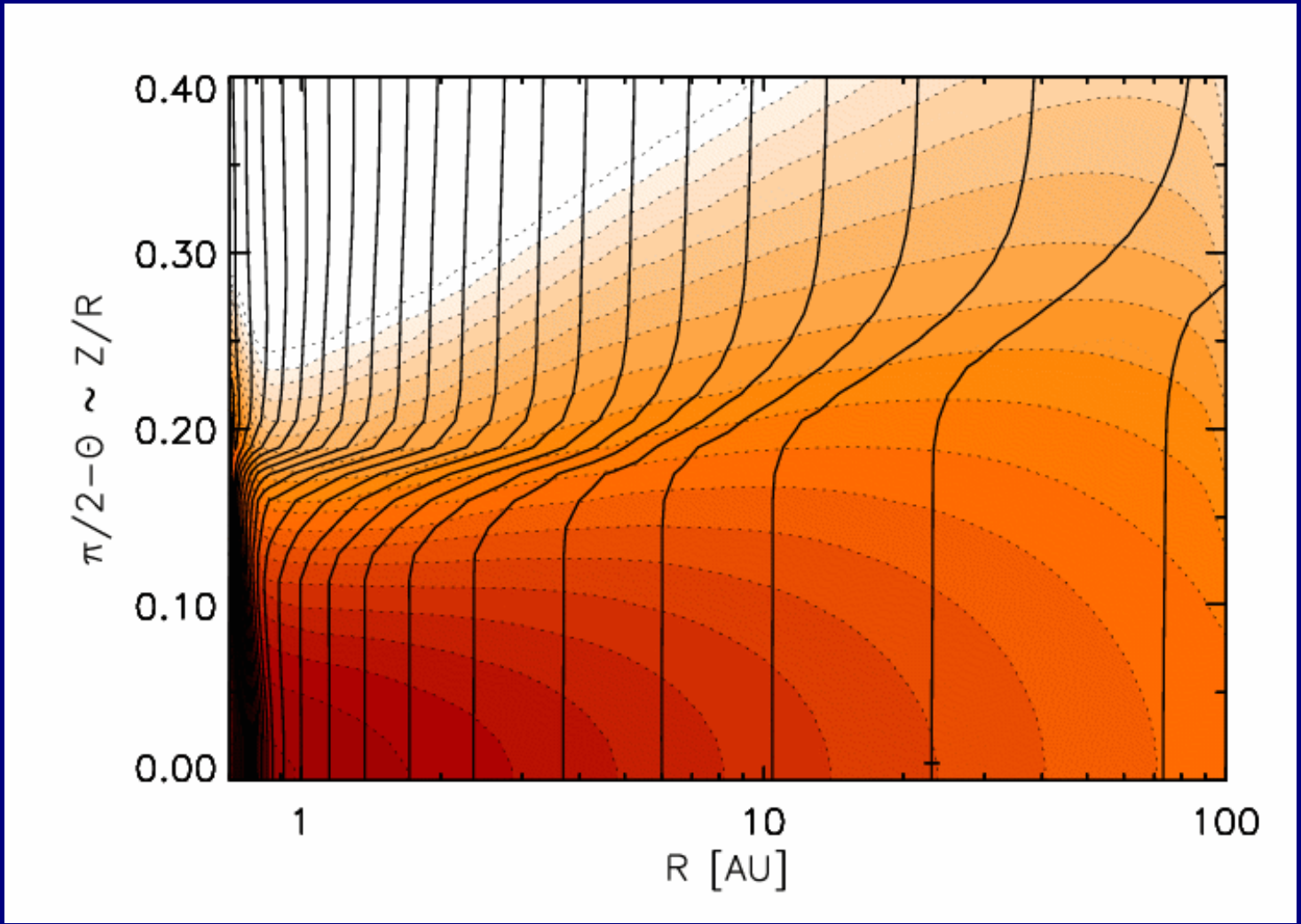
RADMC (Monte Carlo)

~~RADICAL (Edd Tensors)~~

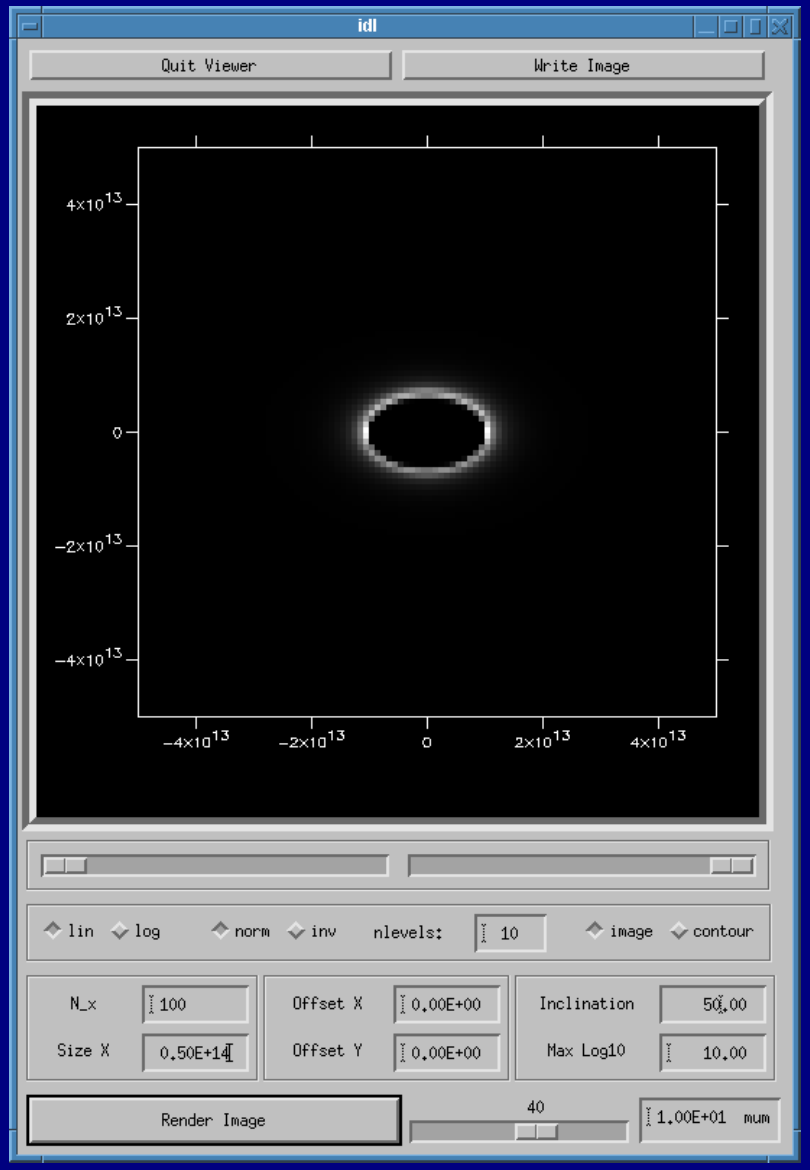
Combined method



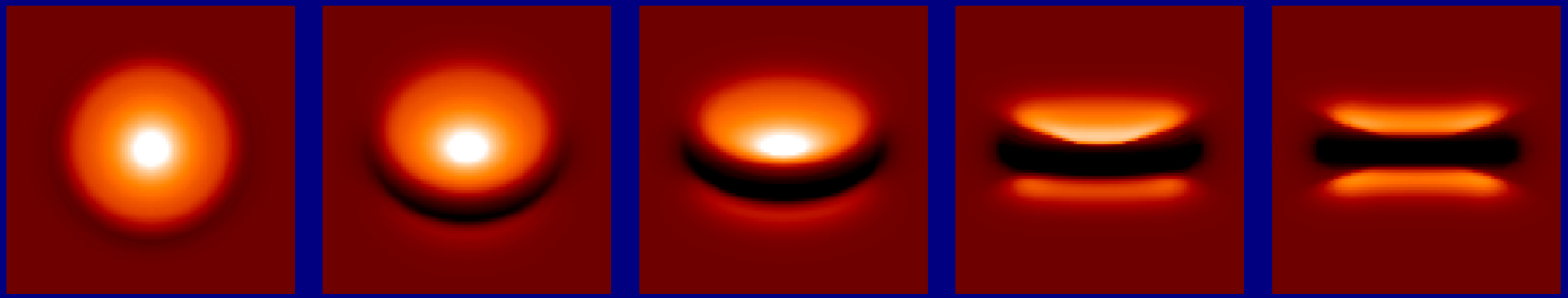
Flared disk in 2-D



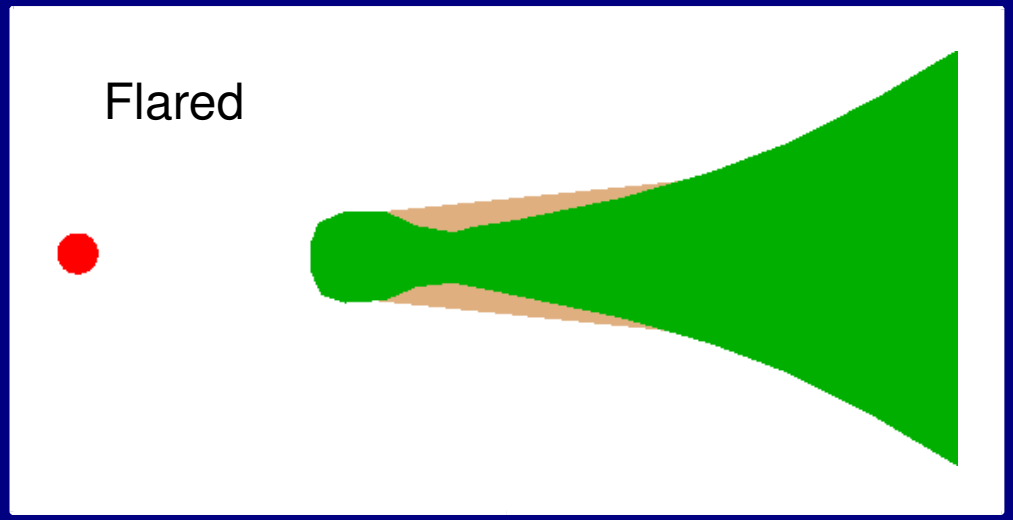
Ray Tracer: an interactive widgit tool

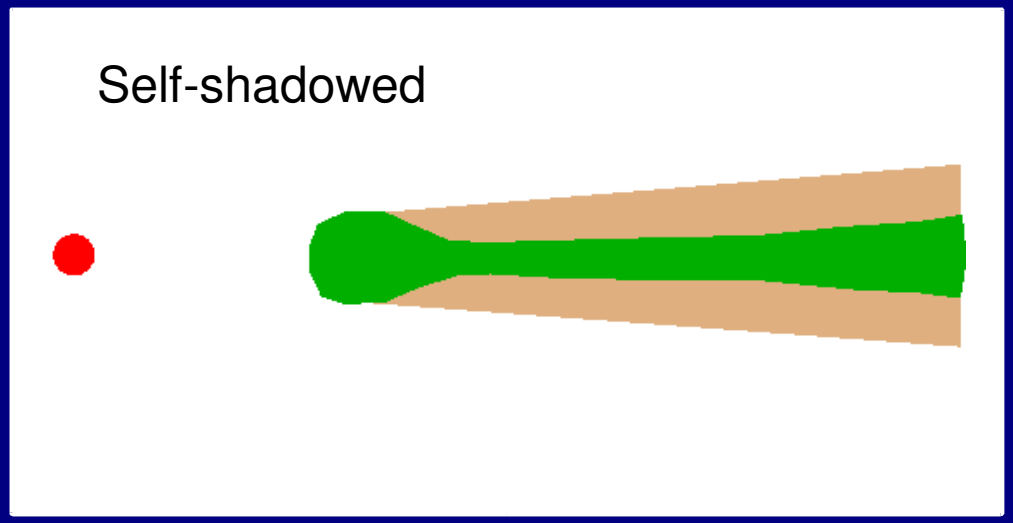


Images: scattered light + background

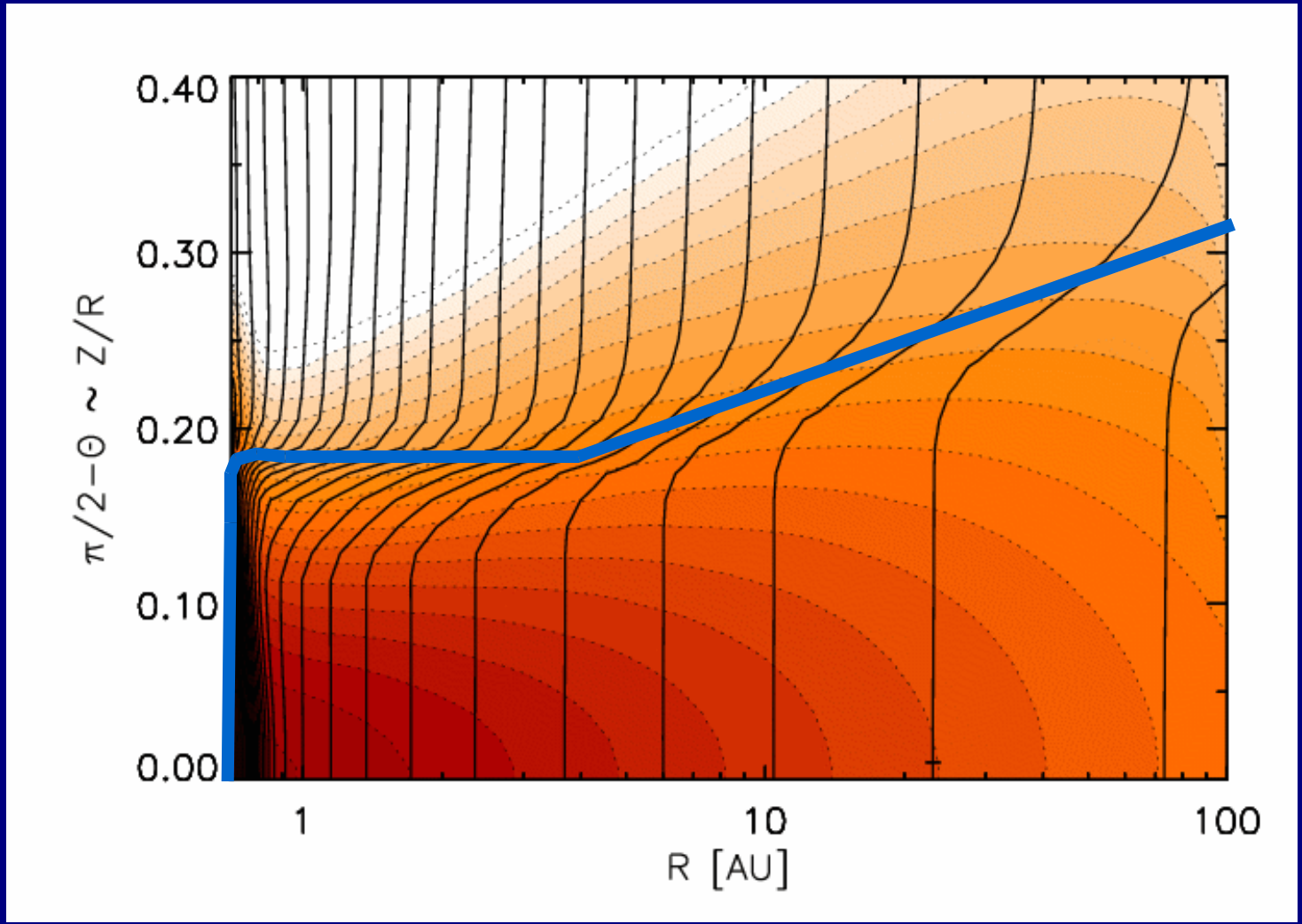


Pictograms of the 2 geometries found

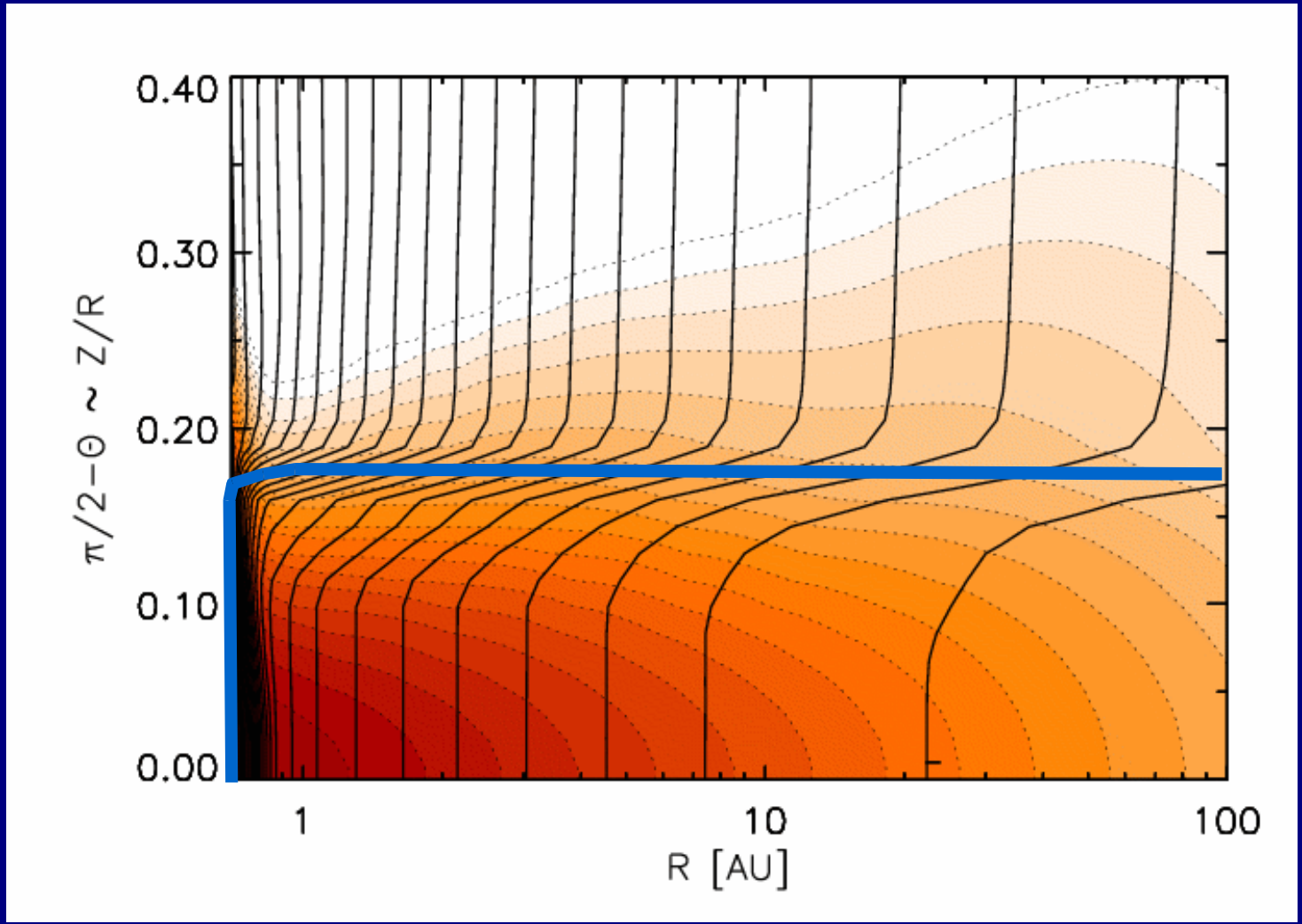




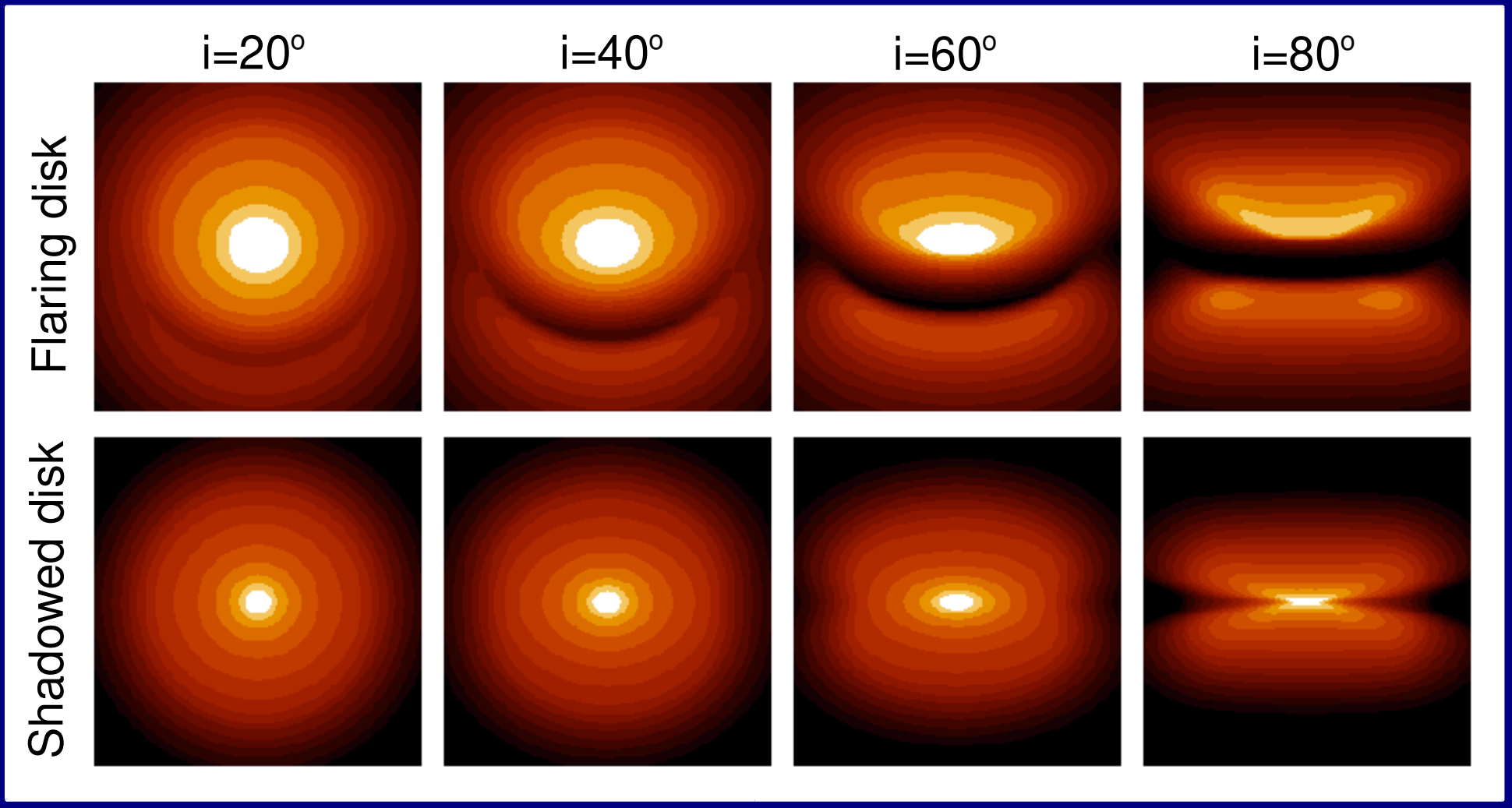
Flared disk in 2-D



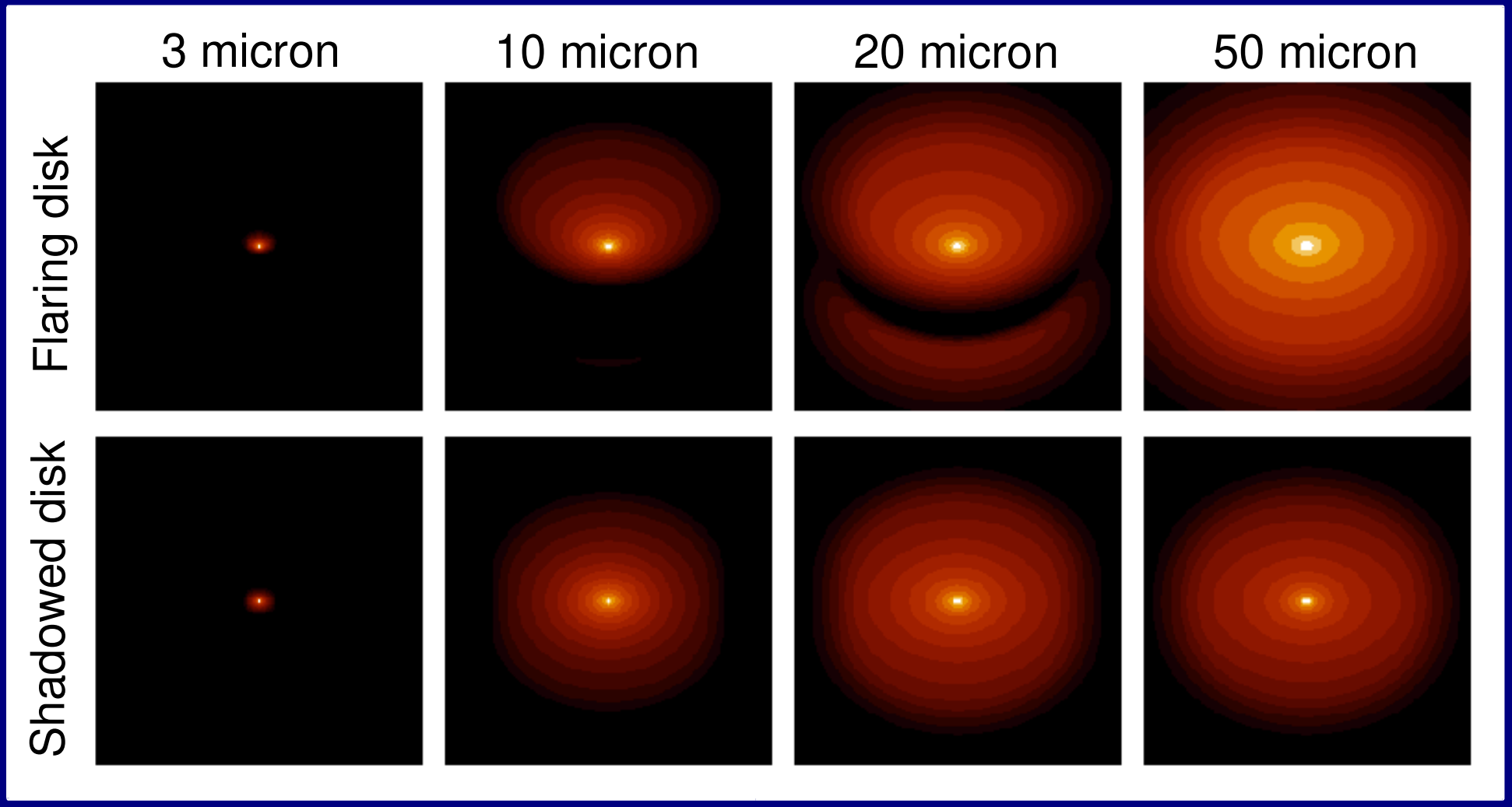
Self-shadowed disk in 2-D



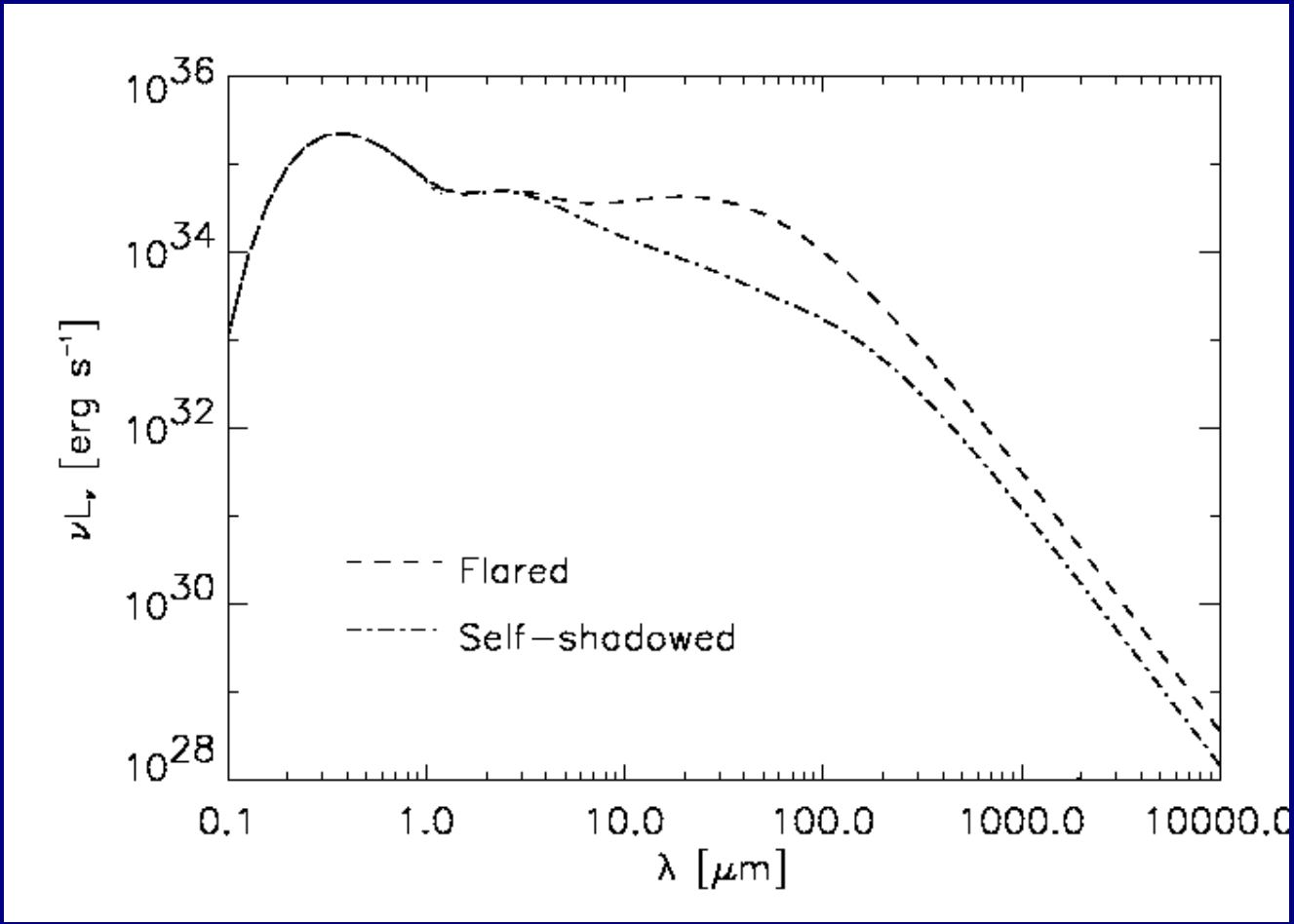
Thermal emission flared/shadowed



Thermal emission flared/shadowed



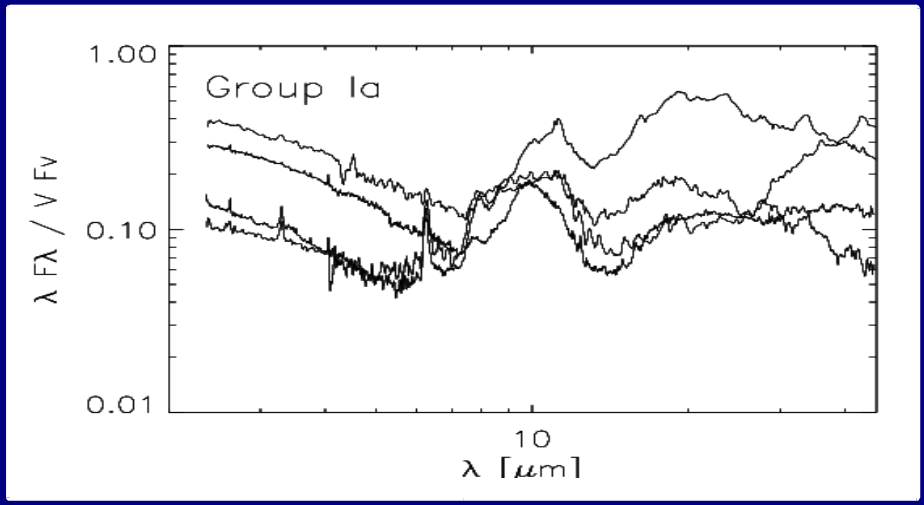
SEDs flaring / self-shadowed disks



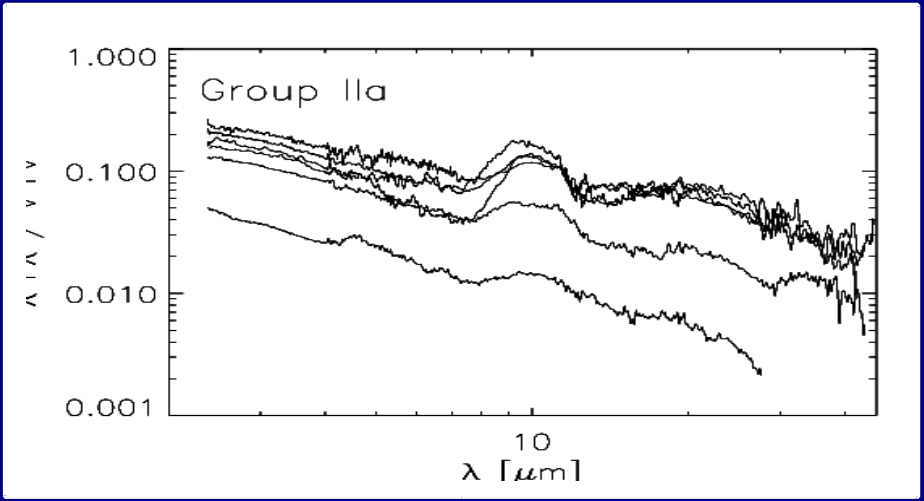
Classifying Herbig Ae/Be stars

(Meeus et al. 2001)

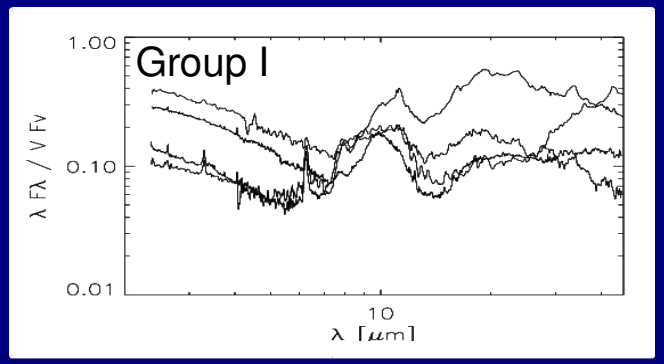
Group I



Group II



Conclusion of this comparison:



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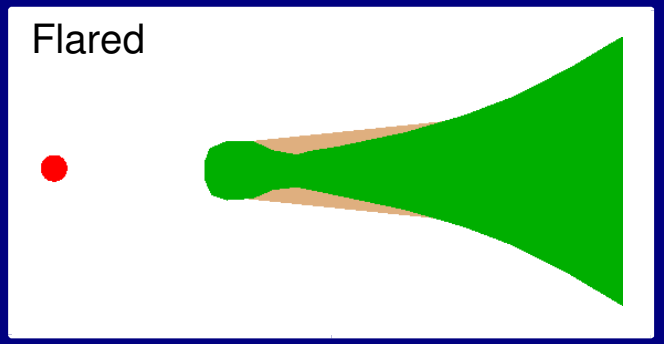

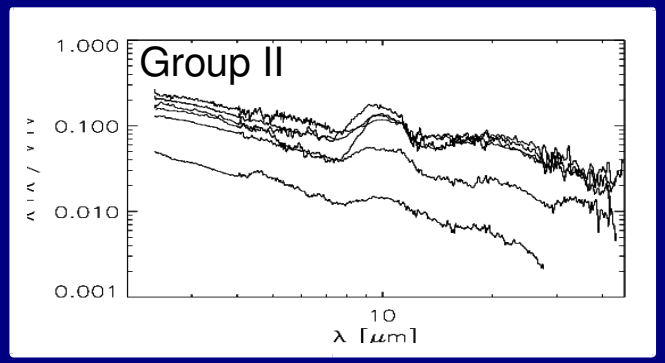

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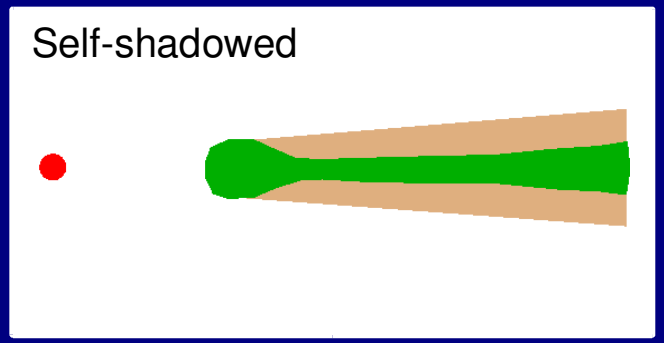

Evolutionary scenario

- Disks start massive with large optical depth
  - → Flaring geometry
  - → Small grains
  - → GROUP I

- Dust settles and coagulates
  - → Self-shadowed geometry
  - → Small and large grains → blackbody mm slope

    (e.g. talk by L. Testi earlier)
  - → GROUP II