# Molecular Gas Consumption: Theory

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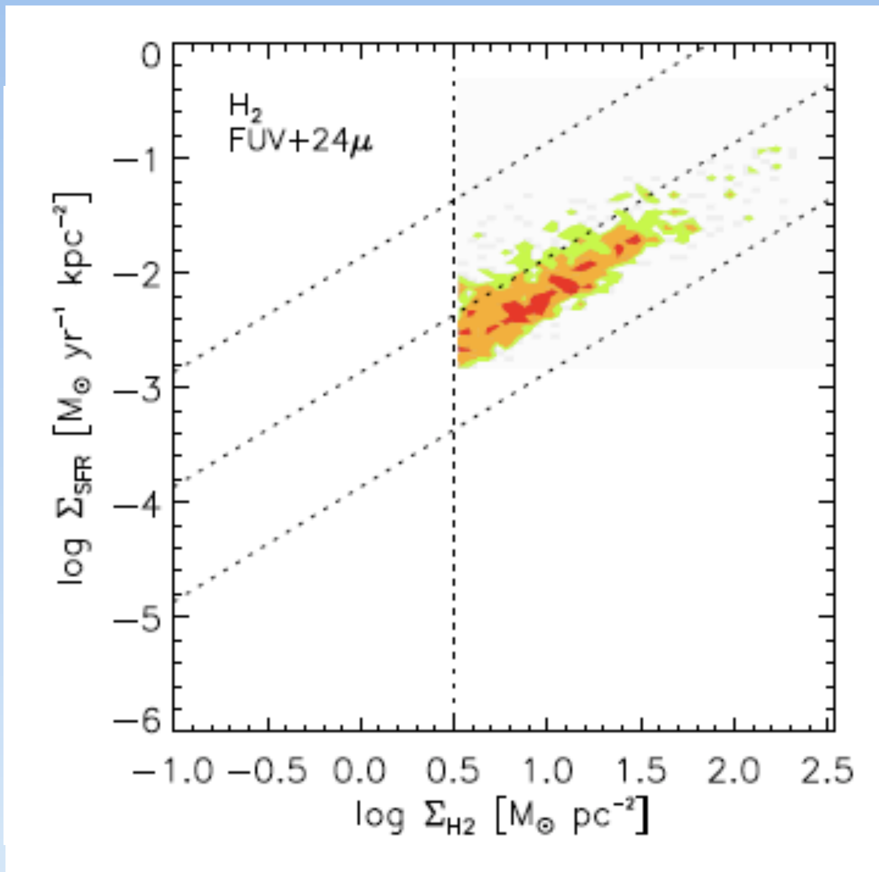
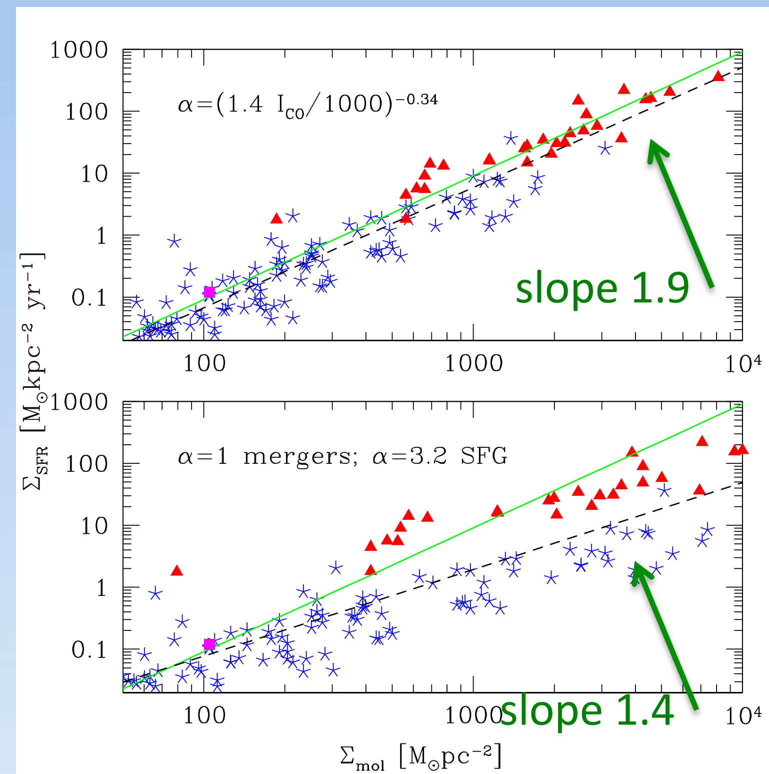
# $\Sigma_{SFR}$ vs. $\Sigma_{H2}$




Schruba et al (2011)

Bigiel et al (2008)




Data from Genzel et al (2010) sample

- SFR linear* in $H_2$ at moderate $\Sigma_{H2} \lesssim 100 M_\odot\, pc^{-2}$ :

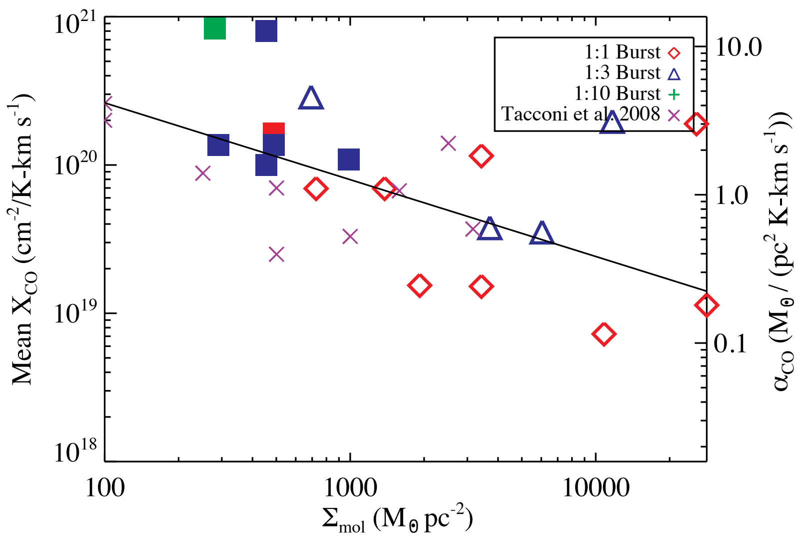
$\Sigma_{SFR} = \Sigma_{H2}/t_{SF}(H_2)$ with $t_{SF}(H_2) = 2\times10^9$ yr

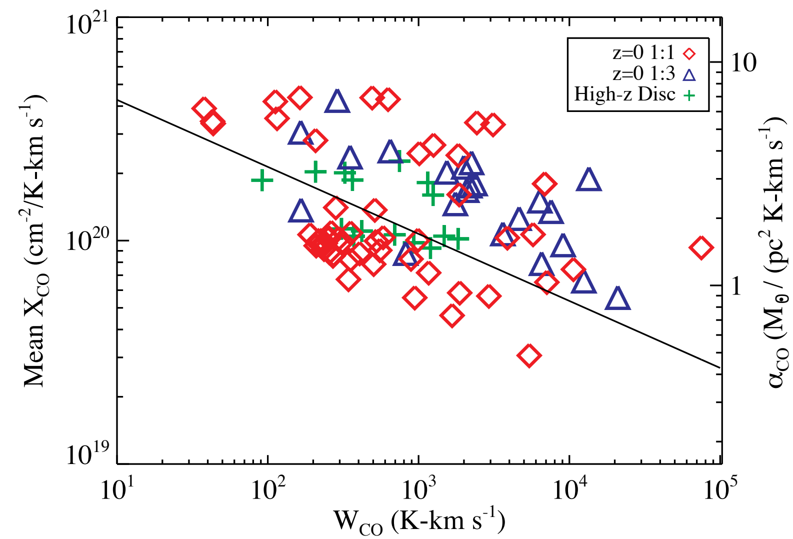
- Steeper slopes for $\Sigma_{SFR}$ vs. $\Sigma_{H2}$ in "starburst" regions, at

$\Sigma_{H2} > \Sigma_{GMC} \sim 100\ M_\odot\ pc^{-2}$

* or slightly sublinear – see Shetty poster

# $X_{CO} = \Sigma_{H2}/W_{CO}$




Narayanan, Krumholz, Ostriker, & Hernquist (2012)

$$X_{CO} = 1.3 \times 10^{21} / [Z' \Sigma_{H2}^{0.5}]$$

$$X_{CO} = 6.8 \times 10^{20} / [Z'^{0.65} W_{CO}^{0.32}]$$

# Gas consumption efficiency

- Interpretation of $t_{SF}(H_2)$~ const. at $\Sigma_{H2} \lesssim 100 M_\odot\, pc^{-2}$:
  "isolated" GMCs have ~uniform properties and SFE that is low and ~ independent of local environment
  $t_{SF}(H_2) = 2\times10^9$ yr requires $\varepsilon_{GMC}=0.01$ if $t_{GMC} = 20$ Myr,
  $\varepsilon_{ff} = 0.003$ if $\langle n_H \rangle \sim 50\ cm^{-3}$
- Interpretation of $t_{SF}(H_2)$ decreasing in starbursts:
  where GMCs "overlap," density increases and relevant dynamical timescales are shorter
- Gas consumption timescale $t_{SF,gas} \equiv \Sigma_{gas}/\Sigma_{SFR}$ :
  - ~10% efficiency per orbital time $t_{orb} = 2\pi/\Omega$
  - Lower efficiency over timescales in local ISM layer
    $t_{ff} = (3\pi/32 G\rho_{gas})^{1/2} \sim 0.2\, t_{orb}$ , $t_{ver} = H/v_z \sim 0.05\, t_{orb}$
- Star formation is *inefficient at consuming gas* over timescales relevant to the ISM dynamics either in individual clouds or in overall ISM

# Gravity and turbulence timescales

- Gravitational free-fall time:

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 4.3\mathrm{Myr}\left(\frac{n_H}{100\ \mathrm{cm}^{-3}}\right)^{-1/2}$$

- Dynamical crossing time:

$$t_{dyn} = \frac{L}{v_{\mathrm{turb}}} = 9.8\mathrm{Myr}\frac{L/10\mathrm{pc}}{v_{turb}/\mathrm{km\ s}^{-1}}$$

- Ratio:

$$\frac{t_{\mathrm{ff}}}{t_{\mathrm{dyn}}} = 0.9\alpha_{\mathrm{vir}}^{1/2} \qquad \alpha_{\mathrm{vir}} \equiv \frac{5v_{\mathrm{turb}}^2 R}{3GM}$$

# Gravity and turbulence for equilibrium slab

- Gravitational free-fall time (midplane):

$$t_{\rm ff} = \frac{\sqrt{3}}{4}\frac{v_z}{G\Sigma} = 9.9{\rm Myr}\frac{v_z/10{\rm km\ s^{-1}}}{\Sigma/100M_\odot\ {\rm pc^{-2}}}$$

- Dynamical crossing time:

$$t_{\rm dyn} = \frac{H}{v_z} = \frac{v_z}{\pi G\Sigma} = 7.2{\rm Myr}\frac{v_z/10{\rm km\ s^{-1}}}{\Sigma/100M_\odot\ {\rm pc^{-2}}}$$

- Ratio:

$$\frac{t_{\rm ff}}{t_{\rm dyn}} = 1.4$$

# Gravity and turbulence for externally-confined gas

- Gravitational free-fall time (gas):

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 43{\rm Myr}\left(\frac{n_H}{1\ {\rm cm}^{-3}}\right)^{-1/2}$$

- Dynamical crossing time:

$$t_{\rm dyn} = \frac{1}{(4\pi G\rho_*)^{1/2}} = 13{\rm Myr}\left(\rho_*/0.1M_\odot\ {\rm pc}^{-3}\right)^{-1/2}$$

- Ratio:

$$\frac{t_{\rm ff}}{t_{\rm dyn}} = \left(\frac{3\pi^2\rho_*}{8\rho}\right)^{1/2} = 3.3\left(\frac{\rho_*/0.1M_\odot\ {\rm pc}^{-3}}{n_H/1\ {\rm cm}^{-3}}\right)^{1/2}$$

# Questions

- Why is consumption efficiency low in GMCs?
- Why is the consumption efficiency low in galactic ISM?

# Giant Molecular Clouds

- GMCs are turbulent; internal velocities increase with scale

    Cloud:

    $$v^2 \sim \alpha_{vir} GM/R \sim \alpha_{vir} G(\Sigma R^2)/R \sim \alpha_{vir} G\Sigma R$$

    Interior:

    $$\delta v(s) \sim (\alpha_{vir} G\Sigma)^{1/2} s^{1/2} \equiv c_s \, (s/L_{sonic})^{1/2}$$

- Sonic scale is where $\delta v(L_{sonic}) = c_s \sim 0.2$ km s$^{-1}$

    $L_{sonic} \sim c_s^2/(\alpha_{vir} G\Sigma)$

    $\sim 0.1$ pc for $\alpha_{vir} \sim 1$ and $\Sigma \sim 100\ M_{\odot}\ pc^{-2}$

    Alternatively,

$$L_{sonic} \sim (c_s/v)\, c_s/(\alpha_{vir} G\rho_0)^{1/2} \sim (c_s/v)\, L_{jeans}(\rho_0)/\alpha_{vir}^{1/2}$$

# Turbulence and density structure

- Supersonic turbulence creates density structure
- Successive compressions and rarefactions are independent:

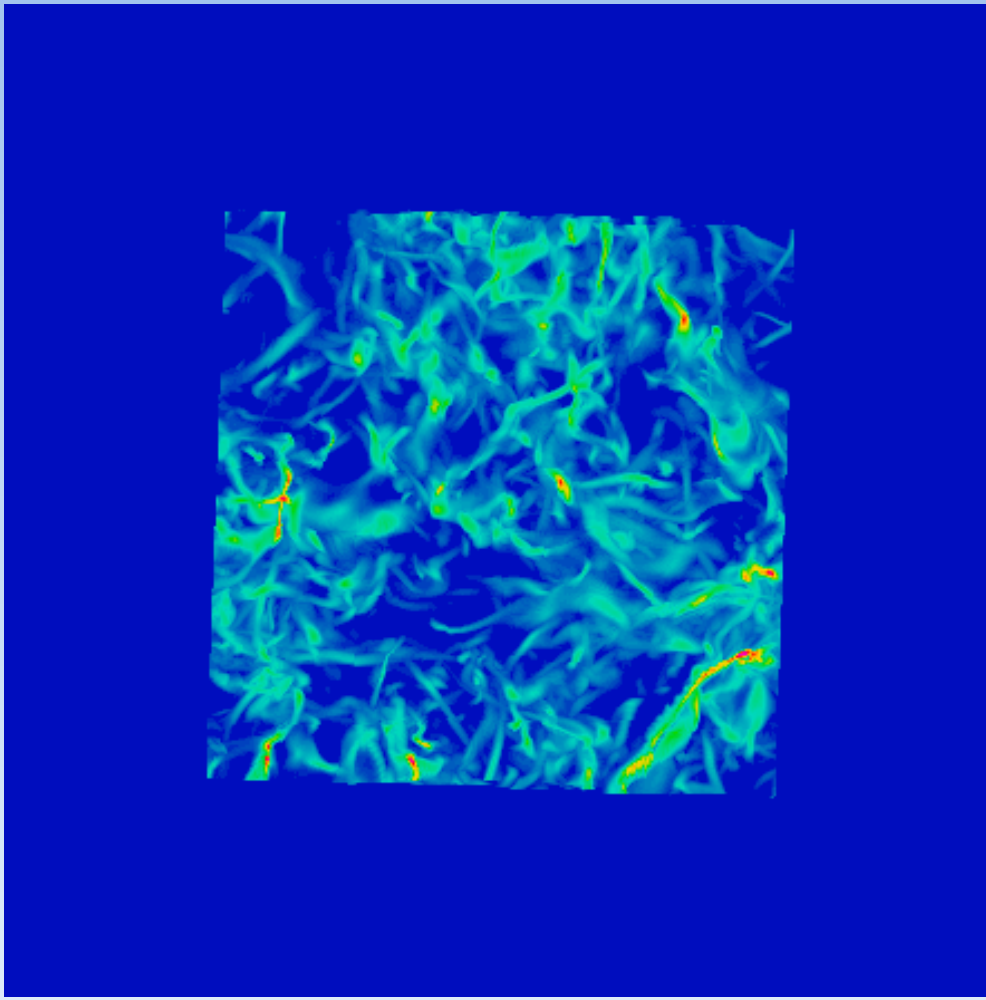
$$x \equiv \ln \frac{\rho}{\bar{\rho}} = \sum_i \ln(1 + \delta_i)$$

resulting in a log-normal volume and mass distribution:

$$f_{V,M}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(x \pm \mu)^2}{2\sigma^2}\right]$$

with $\mu = \sigma^2/2$

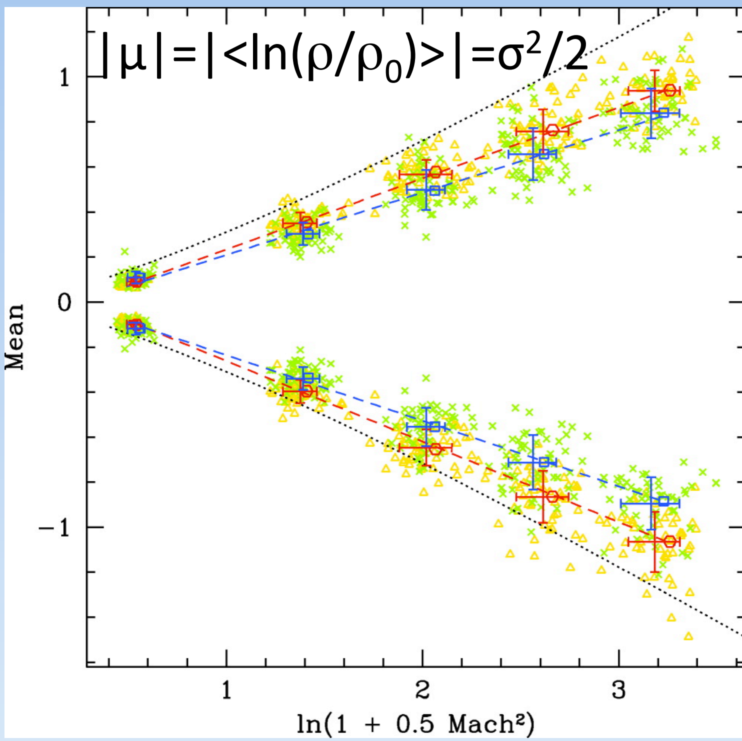
- Value of $\mu = |\langle \ln(\rho/\rho_0) \rangle|$ increases with the Mach number ⇒ more-turbulent systems have:
  - higher mass-weighted density
  - lower volume filling factor



Ostriker, Stone & Gammie (2001)

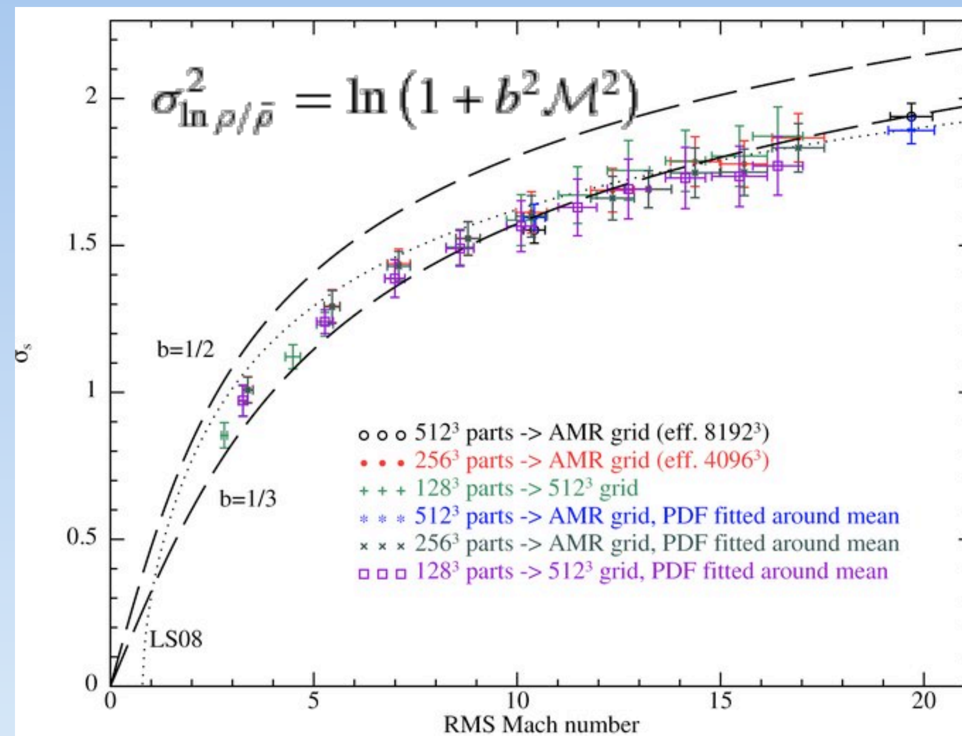
Vazquez-Semadeni 1994; Nordlund & Padoan 1999; Ostriker et al. 2001; Li et al. 2004; Kritsuk et al. 2007; Lemaster & Stone 2008; Federrath et al. 2008, Price et al. 2011

# Density variance and Mach number



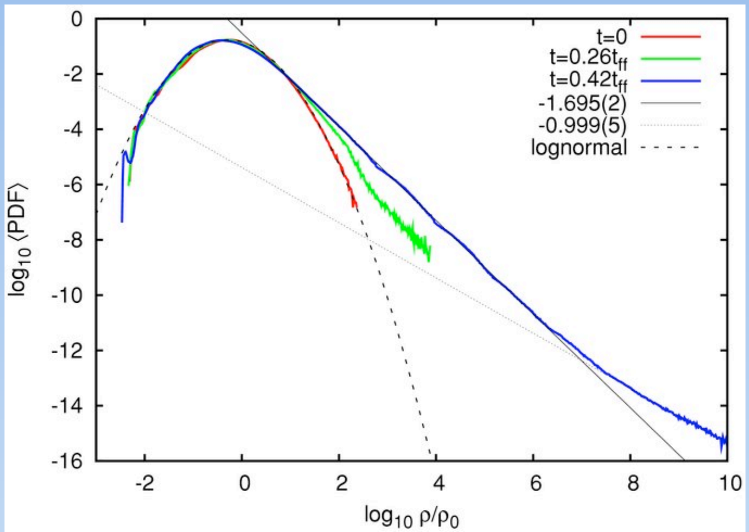

Lemaster & Stone (2008)

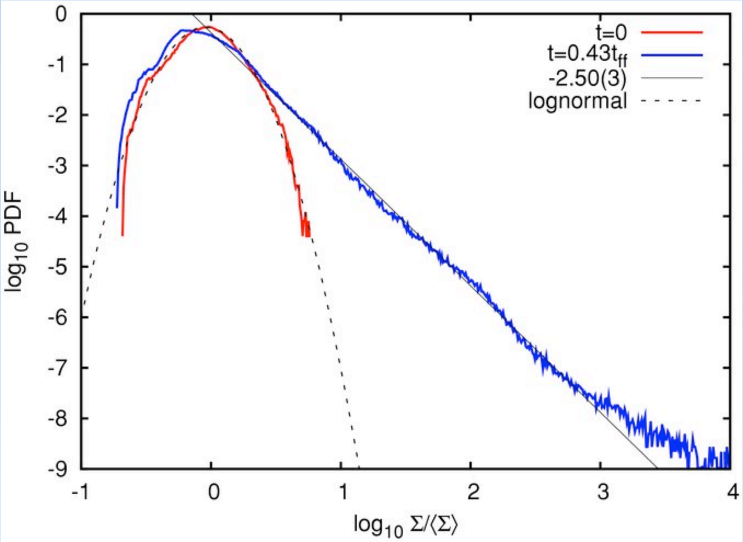
$$\mu_M = 0.32 \ln\left[1 + 0.5\mathcal{M}^2\right] - 0.10.$$



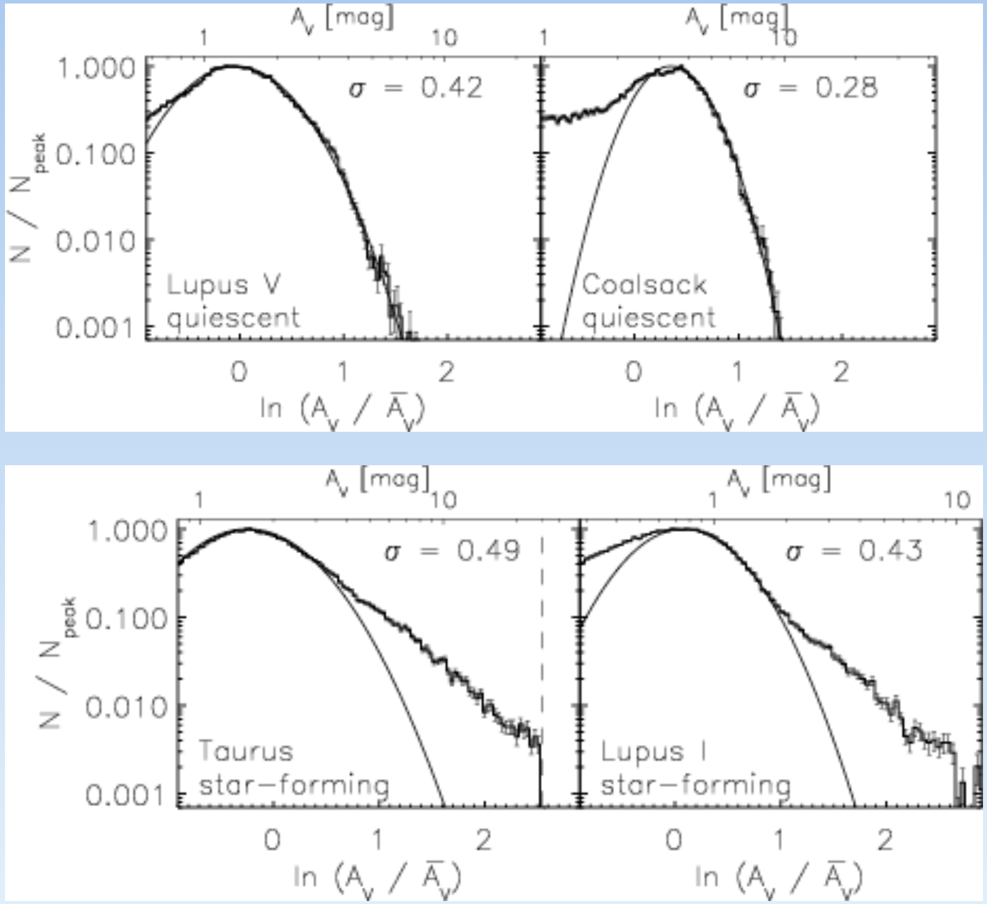

Price et al (2011)

# PDFs: power-law tail



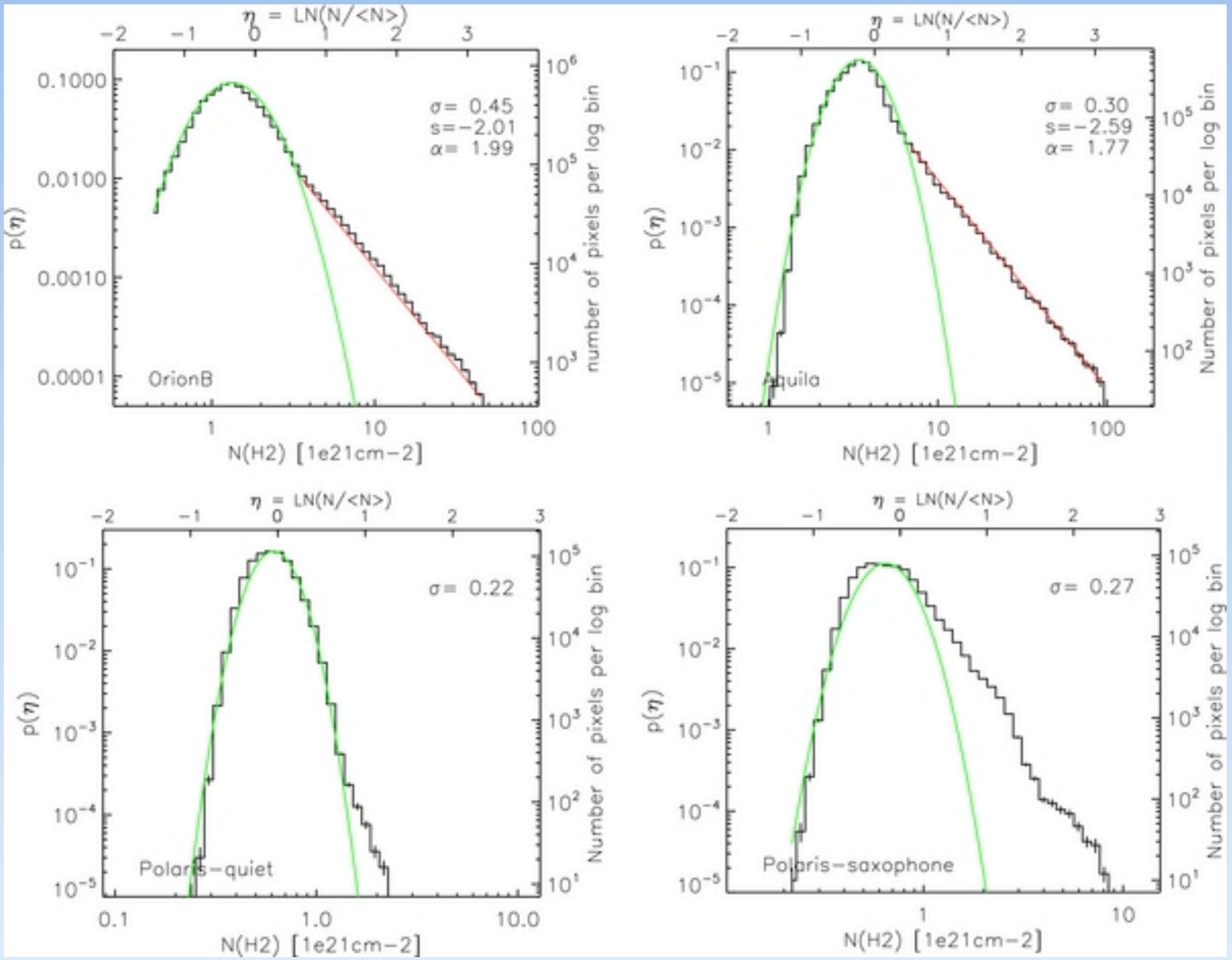

Power-law tail seen in star-forming regions




Power-law tail develops from gravitational collapse (Kritsuk et al 2011)

Kainulainen et al (2009)

# Observed column density PDFs: thermal emission




Schneider et al (2013)

# Critical density for SF

- General idea: only sufficiently dense gas, as drawn from log-normal PDF, can collapse
- Krumholz & McKee (2005): Assume turbulence is pervasive, but at scales < $L_{sonic}$ amplitude is subthermal and support is negligible; Jeans scale for given density is where thermal pressure cannot support ⇒

$\rho_{crit}$ is such that $L_{Jeans}(\rho_{crit}) = L_{sonic}$ for GMC

- Using $L_{sonic} \sim (c_s / v)\, L_{jeans}(\rho_0) / \alpha_{vir}^{1/2}$

  ➔ $\rho_{crit} / \rho_0 \sim \alpha_{vir} (v/c_s)^2$
- $SFR/M \sim \varepsilon_{core}\, t_{ff}(\rho_0)^{-1}$ (mass fraction above $\rho_{crit}$)

# Turbulent destruction

- Turbulence also destroys structure
- Gravitational collapse time for core of density $\rho$ is $t_{coll} \sim 1/(G\rho)^{1/2} \sim L_{Jeans}(\rho)/c_s$
- Core size is $s \sim c_s/(G\rho)^{1/2} \sim L_{Jeans}(\rho)$
- Turbulent "shredding" time over scale $s$ is $t_{dyn} = s/\delta v(s) = (s\, L_{sonic})^{1/2}/c_s \rightarrow [L_{Jeans}(\rho)\, L_{sonic}]^{1/2}/c_s$
- $t_{coll} < t_{dyn}$ corresponds to $L_{Jeans}(\rho) < L_{sonic}$

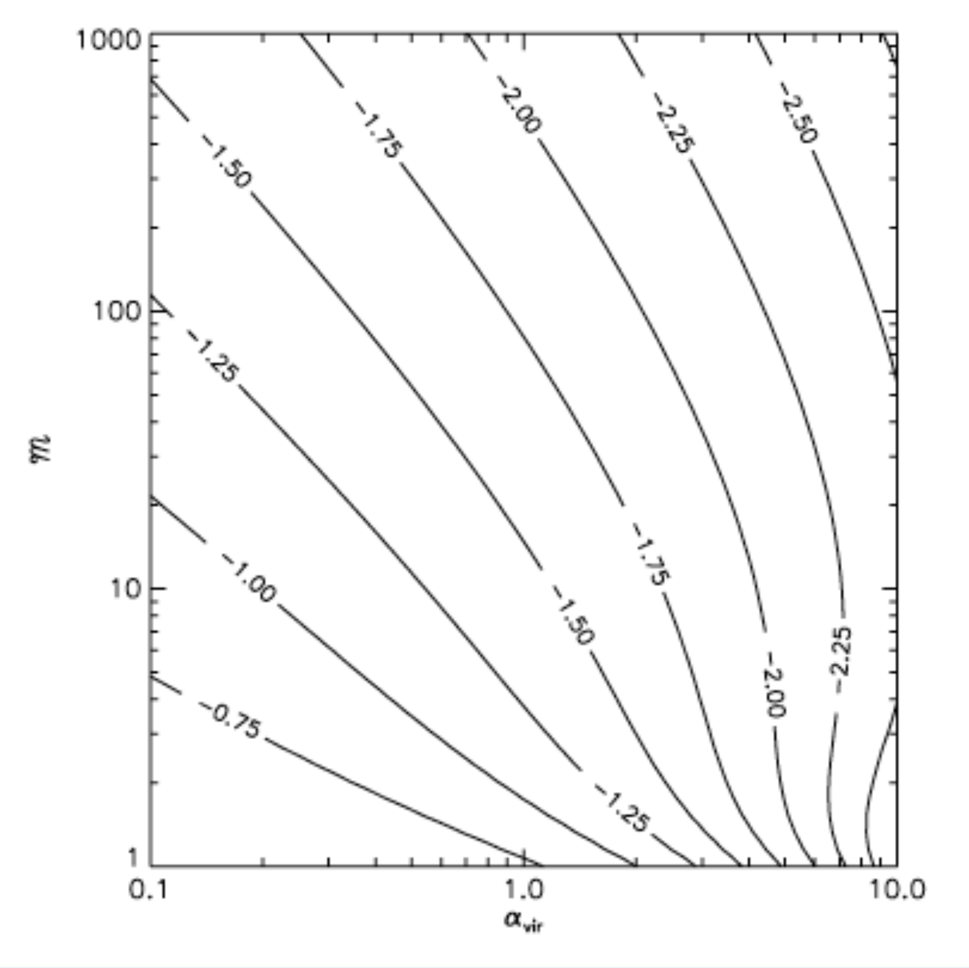
  $\Rightarrow \rho_{crit}/\rho_0 \sim \alpha_{vir}(v/c_s)^2$ as in KM05

  Note: $\rho_{crit}\, c_s^2 \sim \alpha_{vir}^2 G\Sigma^2$

# KM 2005

$$\mathrm{SFR_{ff}} = \frac{\epsilon_{\rm core}}{\phi_t}\int_{x_{\rm crit}}^{\infty} x p(x)\,dx$$
$$= \frac{\epsilon_{\rm core}}{2\phi_t}\left[1+\mathrm{erf}\left(\frac{-2\ln x_{\rm crit}+\sigma_\rho^2}{2^{3/2}\sigma_\rho}\right)\right].$$

$$\mathrm{SFR_{ff}} \approx 0.014\left(\frac{\alpha_{\rm vir}}{1.3}\right)^{-0.68}\left(\frac{\mathcal{M}}{100}\right)^{-0.32}.$$




- Weak dependence on Mach number $v/c_s$
- Low efficiency for large Mach number
- Efficiency decreases for increasing $\alpha_{vir}$
- Efficiency decreases for increasing Mach number

# Padoan & Nordlund 2011

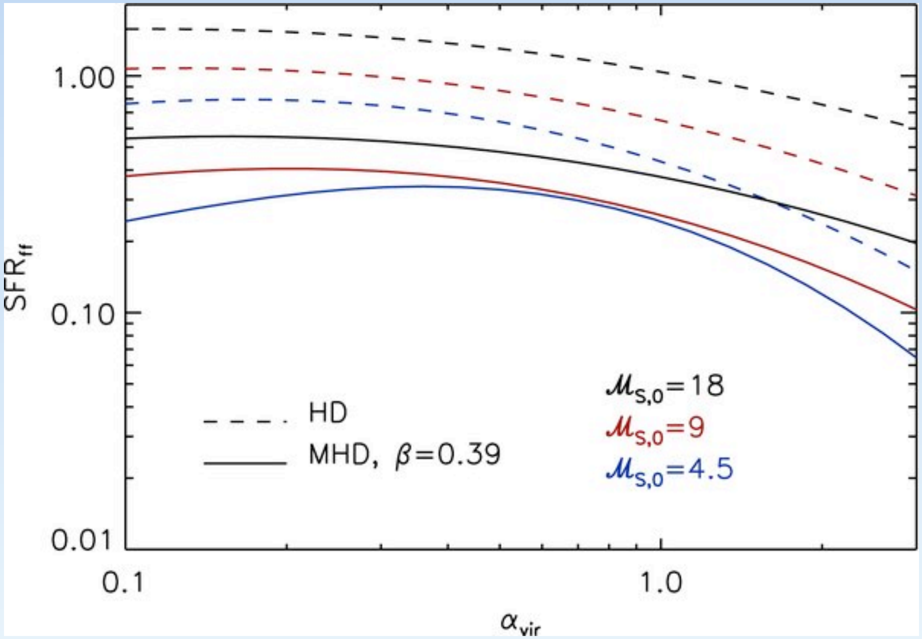
Model similar to KM05, but

$SFR \propto t_{ff}(\rho_{crit})^{-1}$ (fraction above $\rho_{crit}$) instead of

$SFR \propto t_{ff}(\rho_0)^{-1}$ (fraction above $\rho_{crit}$)

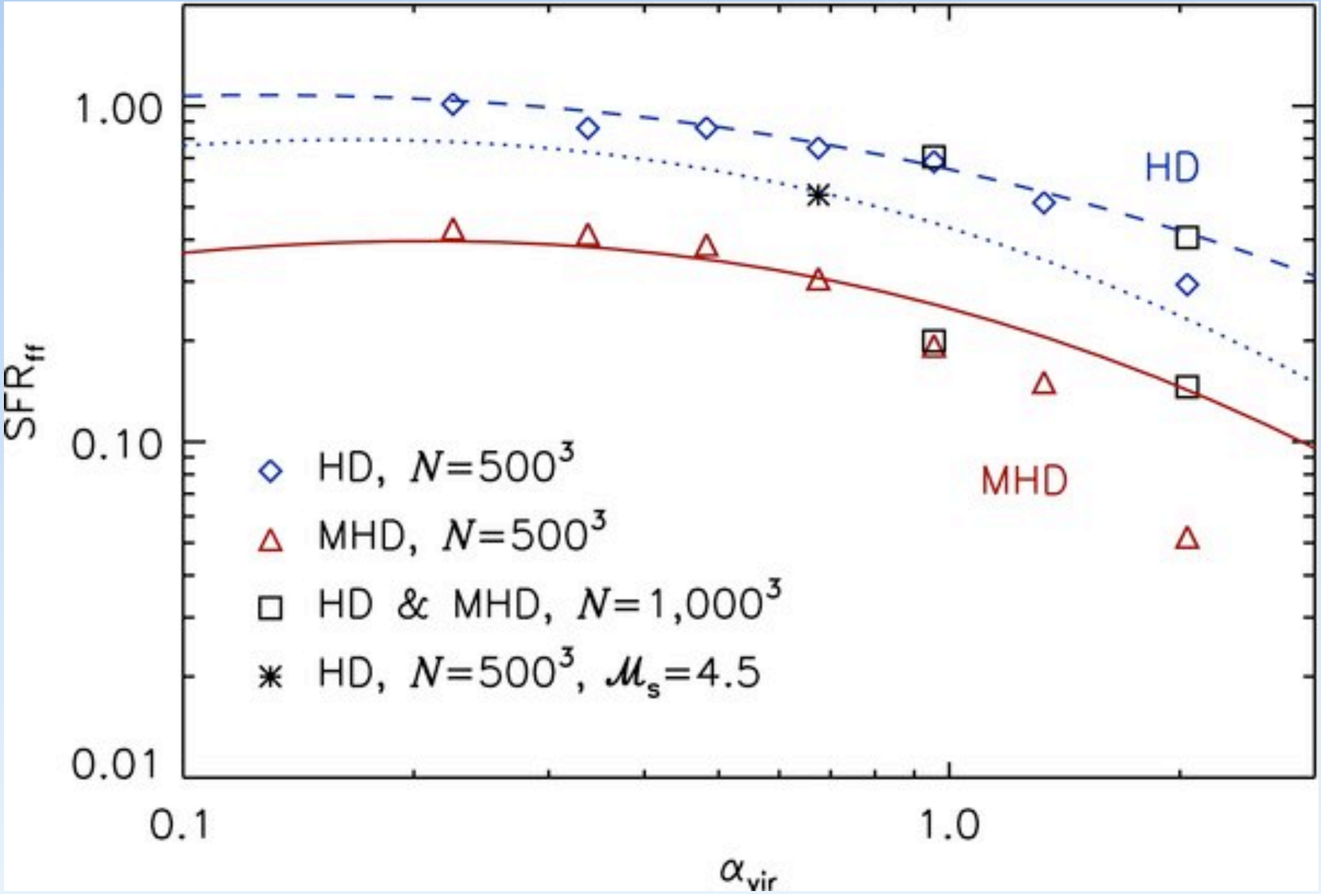
⇒ change by factor $\propto (\rho_{crit} / \rho_0)^{1/2} \sim \alpha_{vir}^{\ 1/2}(v/c_s)$

⇒ $\varepsilon_{ff}$ increases with $v/c_s$ and decreases with $\alpha_{vir}$



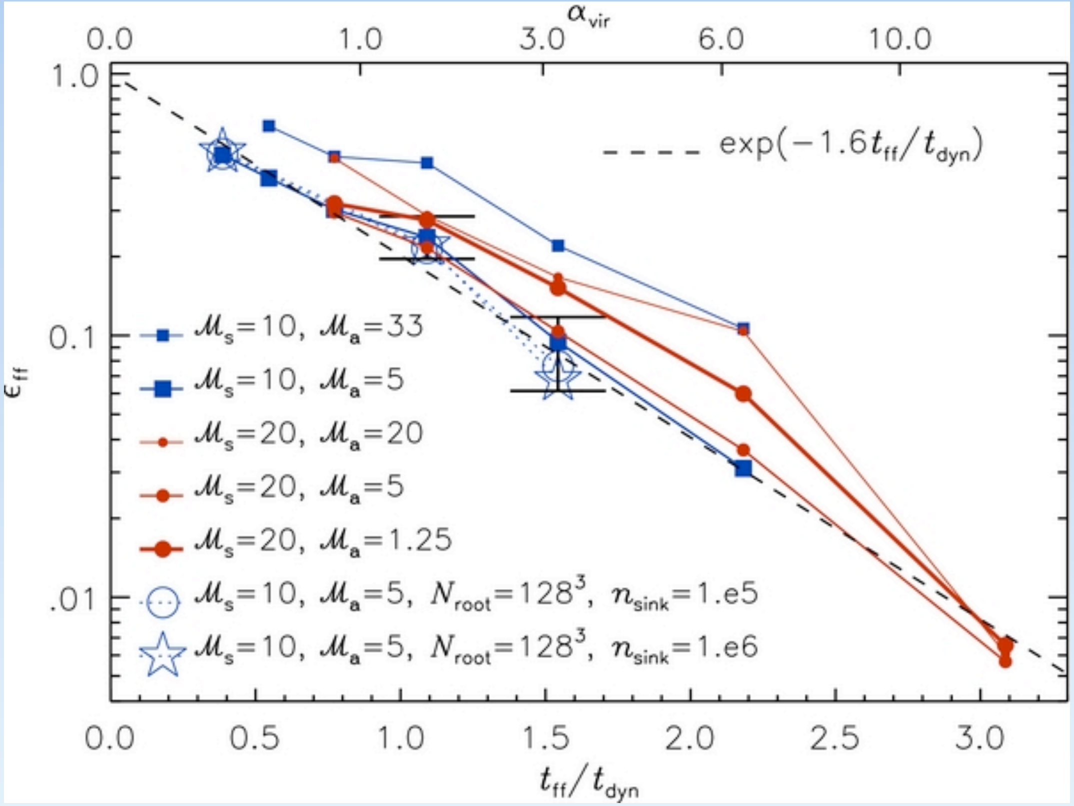
# Padoan & Nordlund 2011

- Simulations:
  - driven-turbulence HD or MHD + self-gravity
  - measure SFR per free-fall time at mean density



# Padoan et al (2012)

- Simulations extend range of $\alpha_{vir}$ , magnetic field, Mach number
- Conclude that $\varepsilon_{ff}$ depends primarily on $\alpha_{vir} \sim \left(\frac{t_{ff}}{t_{dyn}}\right)^2$




Padoan, Haugbølle,Nordlund (2012)

# Summary: $\varepsilon_{ff}$ under GMC conditions

- Simulations and models support conclusion that

  $\varepsilon_{ff}=(t_{ff}(\rho_0)/M)dM/dt$

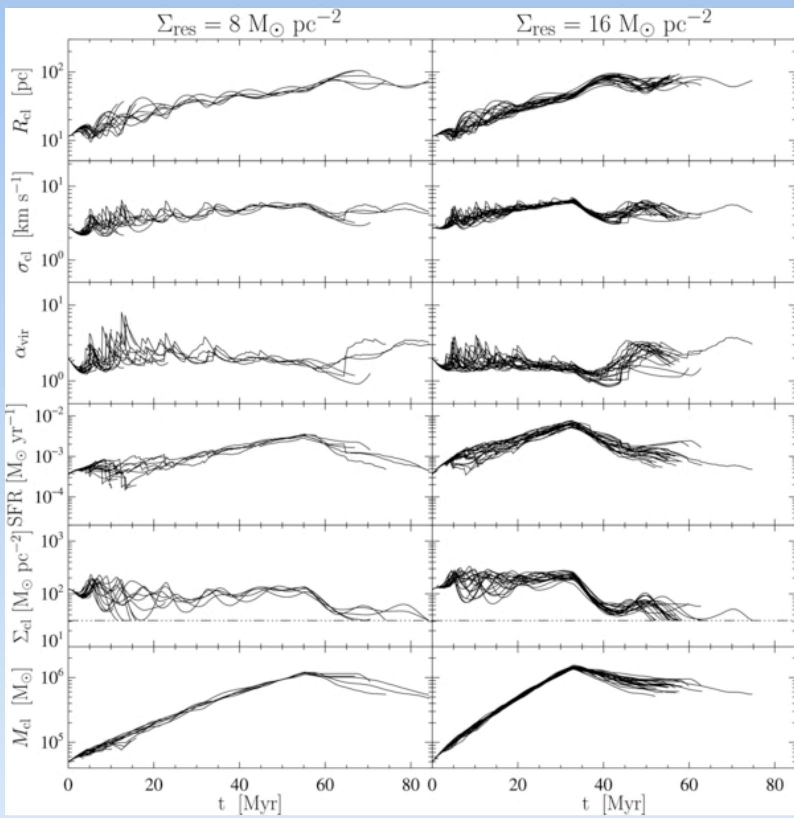
is low (~0.01 -0.1) for GMC-type conditions largely because of turbulence, secondarily from magnetic effects

Krumholz & McKee 2005; Padoan et al 2011, 2012; Hennebelle & Chabrier 2011; Federrath & Klessen 2012

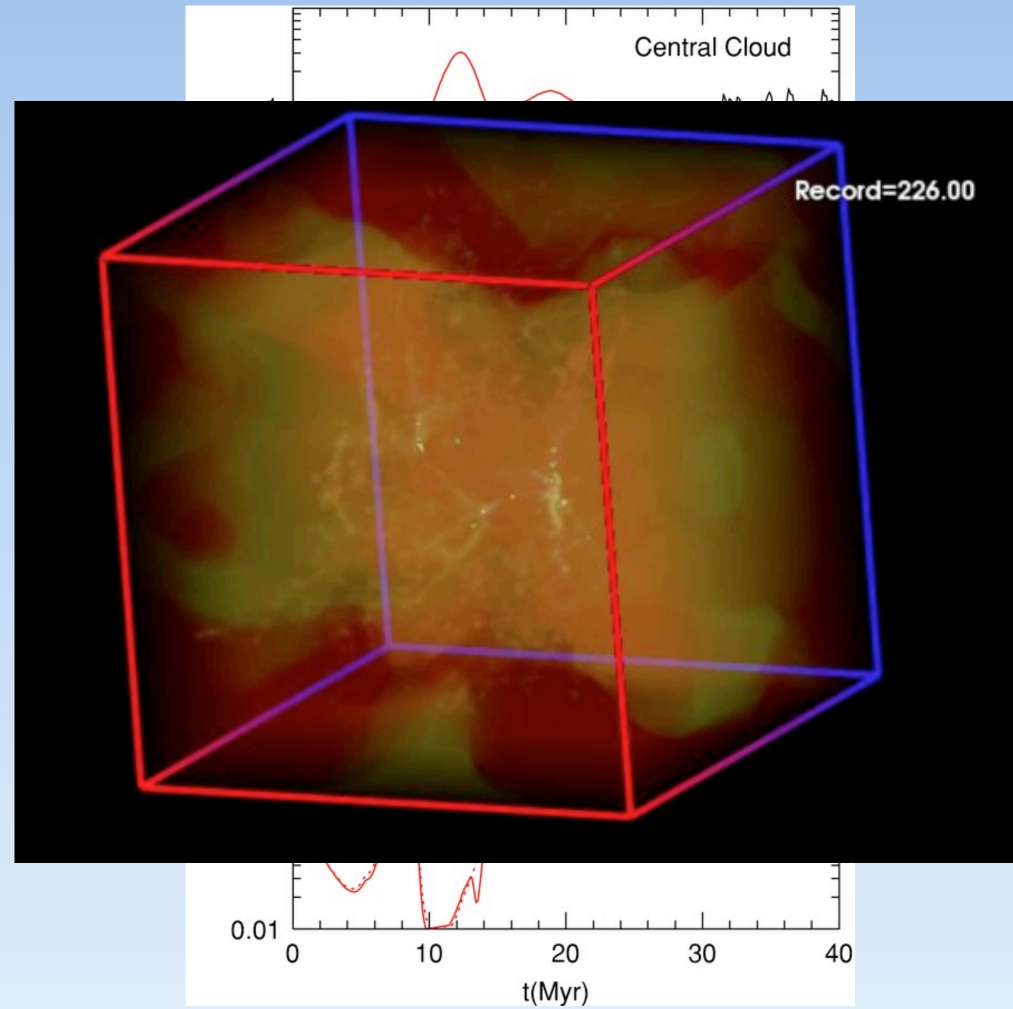
- Where turbulence is low (low $\alpha_{vir}$) , $\varepsilon_{ff}\sim1$
- Questions
  - What sets $\rho_0$, $\alpha_{vir}$, $v/c_s$ in GMCs?
  - Better: how do $\rho_0\sim M/R^3$, $\alpha_{vir}\sim v^2R/(GM)$, $v/c_s$ vary over time as GMC forms, evolves, and then disperses?

# Cloud evolution models

$M/M_{vir} = 1/\alpha_{vir} \sim GM/(R\,v^2)$



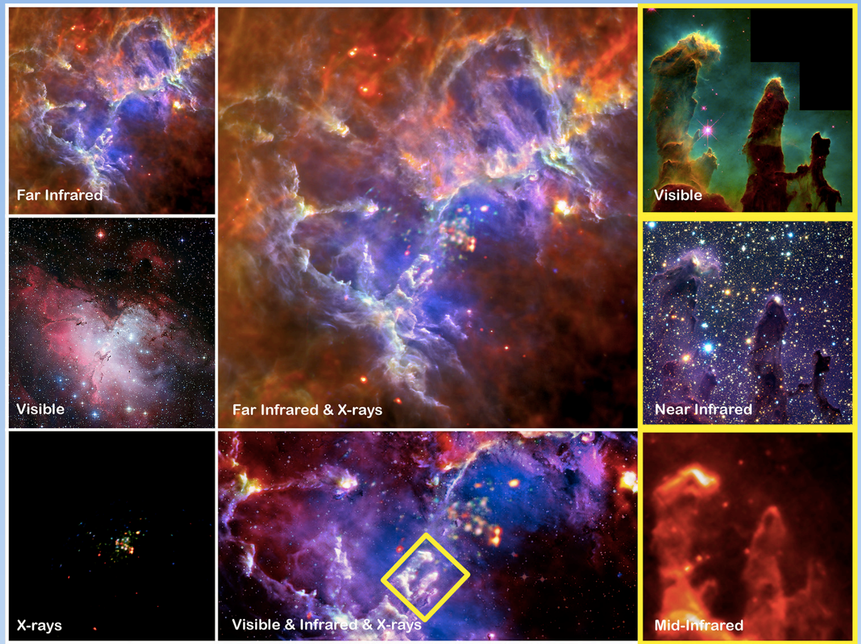

Goldbaum et al (2011): semi-analytic model with accretion & SF feedback



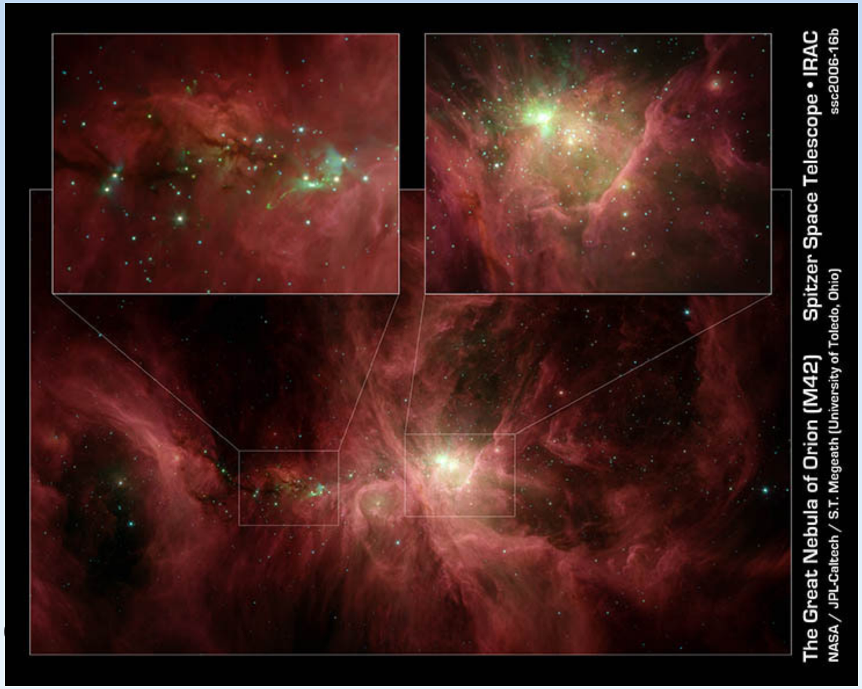

Vazquez-Semadeni et al (2010): simulations with/without feedback

# The need for feedback

- Without feedback, turbulence would dissipate, $\alpha_{vir}$ would drop, leading to $\varepsilon_{ff} \rightarrow 1$, and $\varepsilon_{GMC} \rightarrow 1$
- Collapse can be halted/turned around by:
  - Protostellar outflows (MHD)
  - HII regions (photoionization, winds)
  - Radiation pressure
  - Supernova blasts




Eagle nebula/M16




*Herschel*: Carina nebula

# Feedback mechanisms

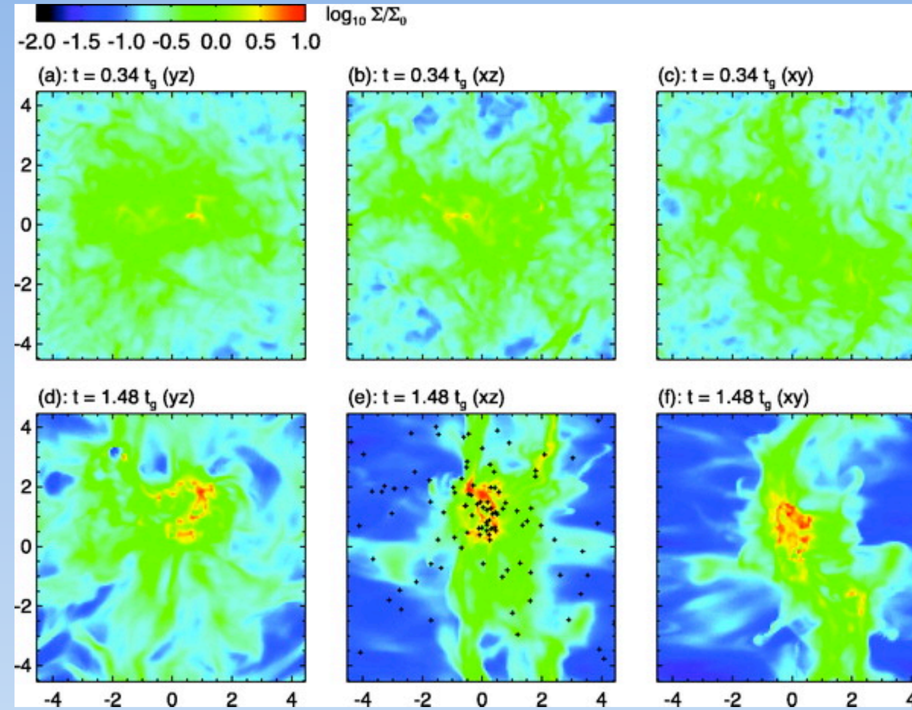
- Protostellar outflows
  - Likely important in cluster-forming clump; not enough power to support/destroy whole GMC
- HII regions
  - Likely important in moderate-mass GMCs

  → see Dale talk
- Radiation pressure
  - May be important in very massive GMCs
- Supernovae
  - May be important in GMCs (models needed!)
  - Major role in diffuse ISM ⇒ important to GMC formation rate and large-scale SFRs

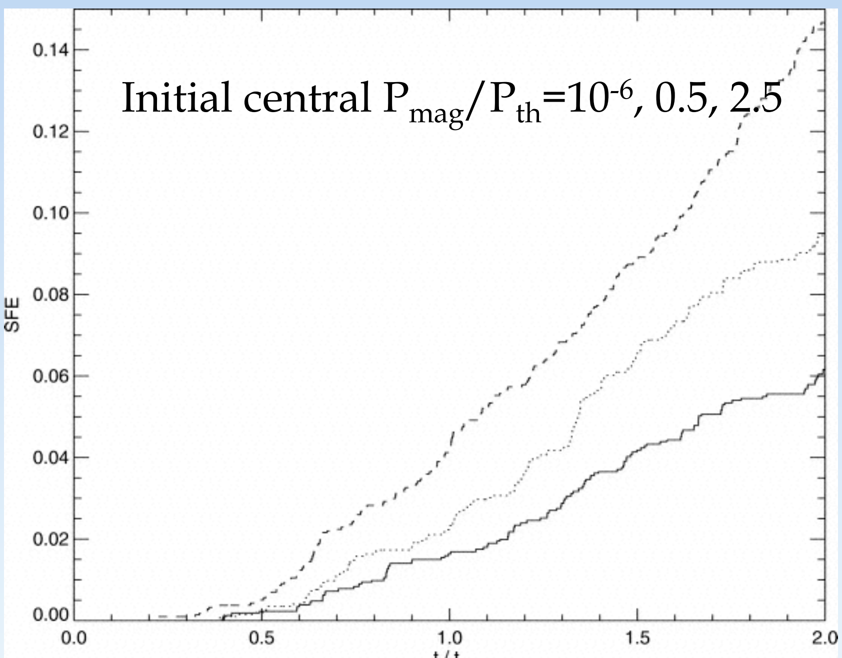
# Cluster-forming clump with outflow driving

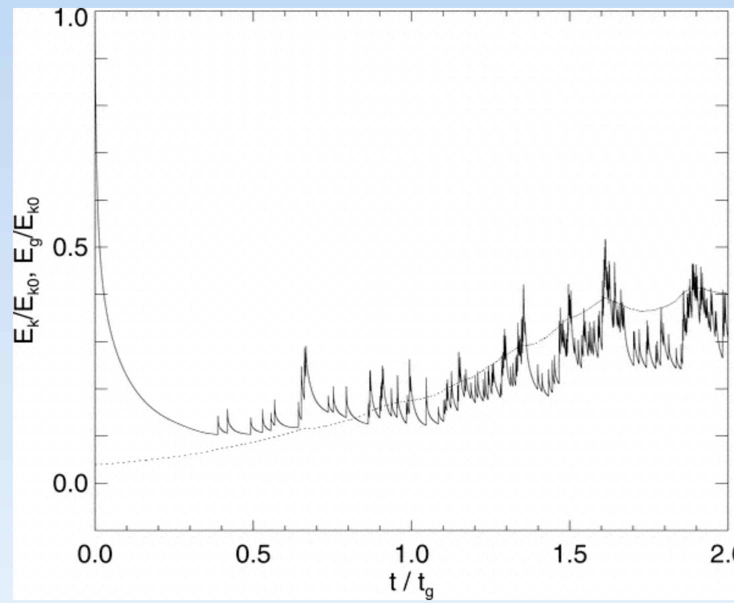
- Driven turbulence enables system to reach quasi-steady state with low $\varepsilon_{ff}$
- $\varepsilon_{ff}$ reduced to 4-8% from 20-30% over ~ 4 $t_{ff}$

(Li & Nakamura 2006, Nakamura & Li 2007, Wang et al 2010, Nakamura & Li 2011)




Nakamura & Li (2007)




Li & Nakamura (2006)

# Turbulent driving and dissipation

- Assume feedback momentum/mass is $p_*/m_*$
- Momentum input rate is

$$\dot{p}_{driv} = \frac{p_*}{m_*}\dot{M}_*$$

- Momentum dissipation rate is

$$\dot{p}_{diss} \sim \frac{vM}{t_{dyn}} \sim \frac{v^2 M}{L}$$

- Balancing, $$\dot{M}_* \sim \frac{v^2 M}{L p_*/m_*}$$

- For system in dynamical equilibrium $v^2 \sim GM_{tot}/L$

$$\Rightarrow \quad \boxed{\dot{M}_* \sim \frac{GM_{tot}M}{L^2 p_*/m_*}}$$

*Self-regulated star formation*

# Star-forming equilibrium gas cloud

- $M_{tot} \sim M \Rightarrow$ $$\dot{M}_* \sim \frac{v^2 M}{L p_*/m_*} \sim \frac{G M^2}{L^2 p_*/m_*}$$

- Combine with $$\dot{M}_* = \varepsilon_{ff} \frac{M}{t_{ff}} \sim \varepsilon_{ff} \frac{vM}{L}$$
  ($\varepsilon_{ff}$ depends on $\alpha_{vir}$, $v/c_s$ )

$\Rightarrow$

$$\varepsilon_{ff} \sim \frac{v}{p_*/m*}$$ or $$v \sim \varepsilon_{ff} \frac{p_*}{m*}$$

- Nakamura & Li (2011): cluster-forming clumps with

  $p_*/m_* \sim v_{\rm wind} \sim 100$ km/s

  $v^2 \sim GM/R$

$\Rightarrow$

$$\epsilon_{ff} \sim \frac{(GM/R)^{1/2}}{v_{\rm wind}}$$

$$\sim \frac{(G\Sigma_{clump}R)^{1/2}}{v_{\rm wind}}$$

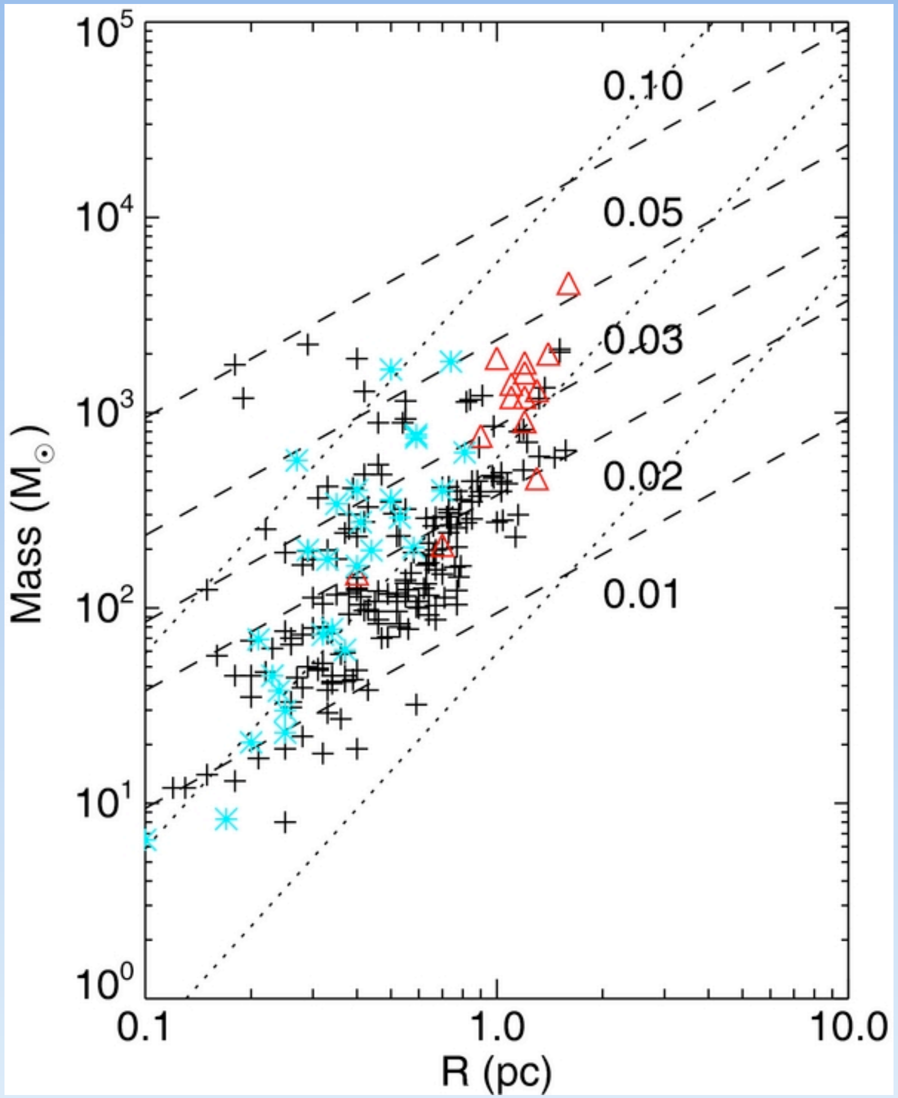
Alternatively, from

$$v \sim \varepsilon_{ff}\frac{p_*}{m_*} \rightarrow \varepsilon_{ff} v_{\rm wind}$$

using $v_{\rm wind} \sim 100$ km/s and

$\varepsilon_{ff} \sim few \times 0.01 \quad \Rightarrow$

clump evolves until $v \sim$ few km/s



# Cluster-forming cloud with radiation pressure

- Starting with $\varepsilon_{ff} \sim \frac{v}{p_*/m*}$, efficiency over cloud lifetime is

$$\varepsilon \sim \varepsilon_{ff} \frac{t_{life}}{t_{ff}} \sim \frac{v^2 t_{life}}{L p_*/m_*} \sim \frac{v^2 M_*}{L\dot{p}} \sim \frac{G\Sigma M_*}{\dot{p}}$$

- Momentum input rate from reprocessed IR is

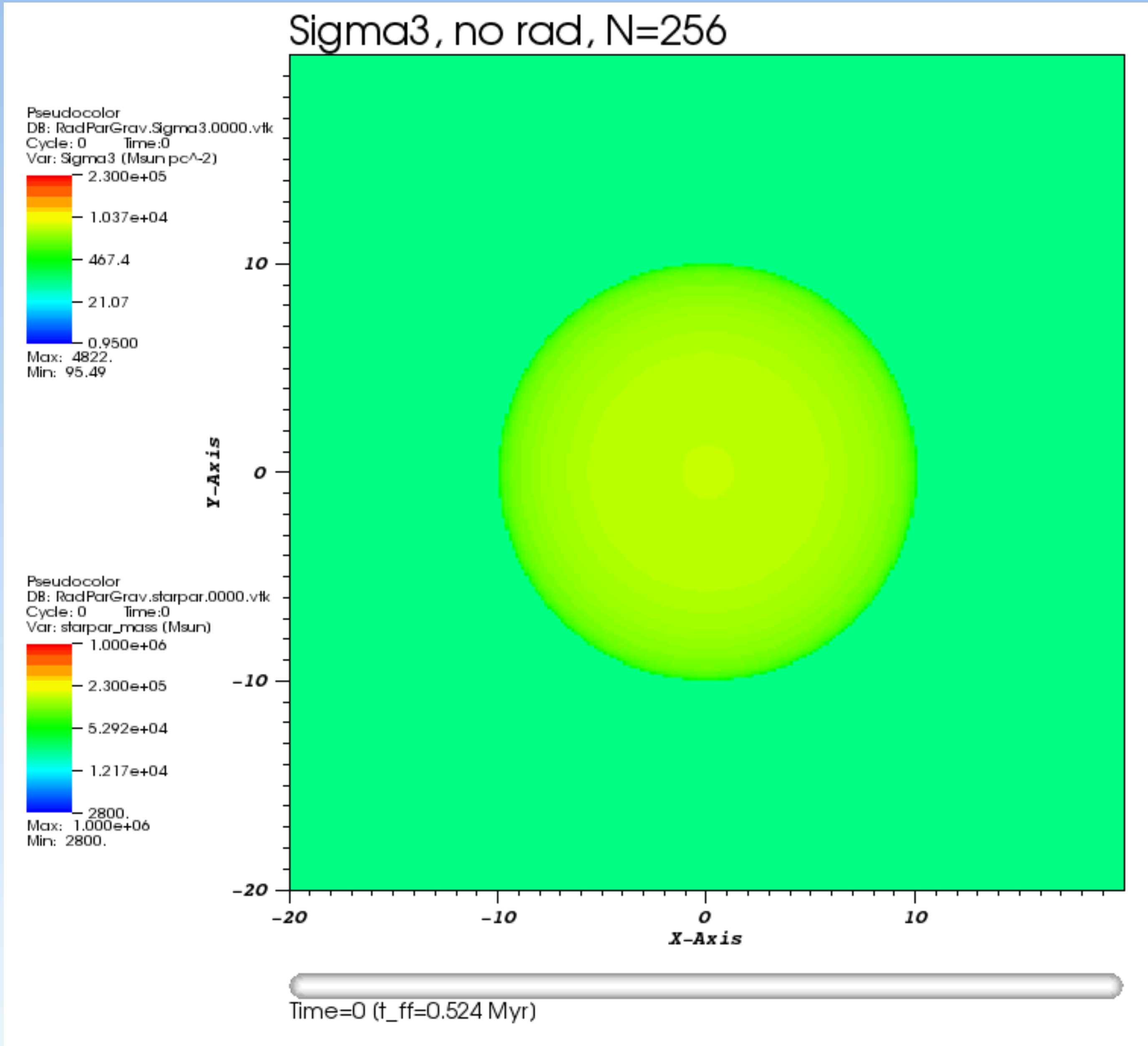
$$\dot{p} \sim \frac{\mathcal{L}_* \tau}{c} \sim \frac{M_* \Psi \kappa_{IR} \Sigma}{c}$$

Murray et al (2010), Ostriker & Shetty (2011)

$\Rightarrow$

$$\varepsilon \sim \frac{Gc}{\Psi \kappa_{IR}} \rightarrow \frac{1}{\kappa_{IR}}$$

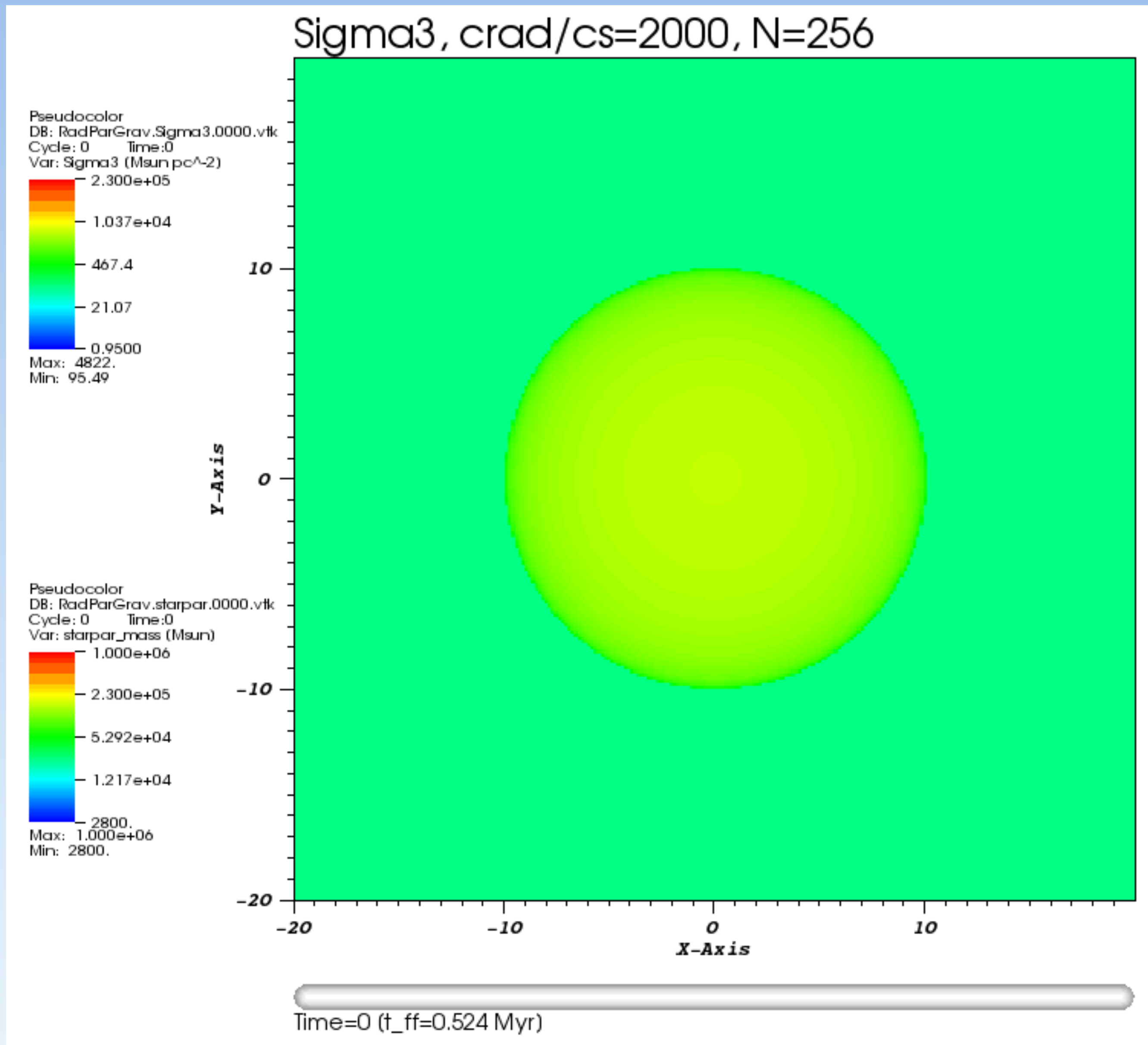
# No feedback




Skinner & Ostriker (2013)

$10^6 M_\odot$ initial cloud

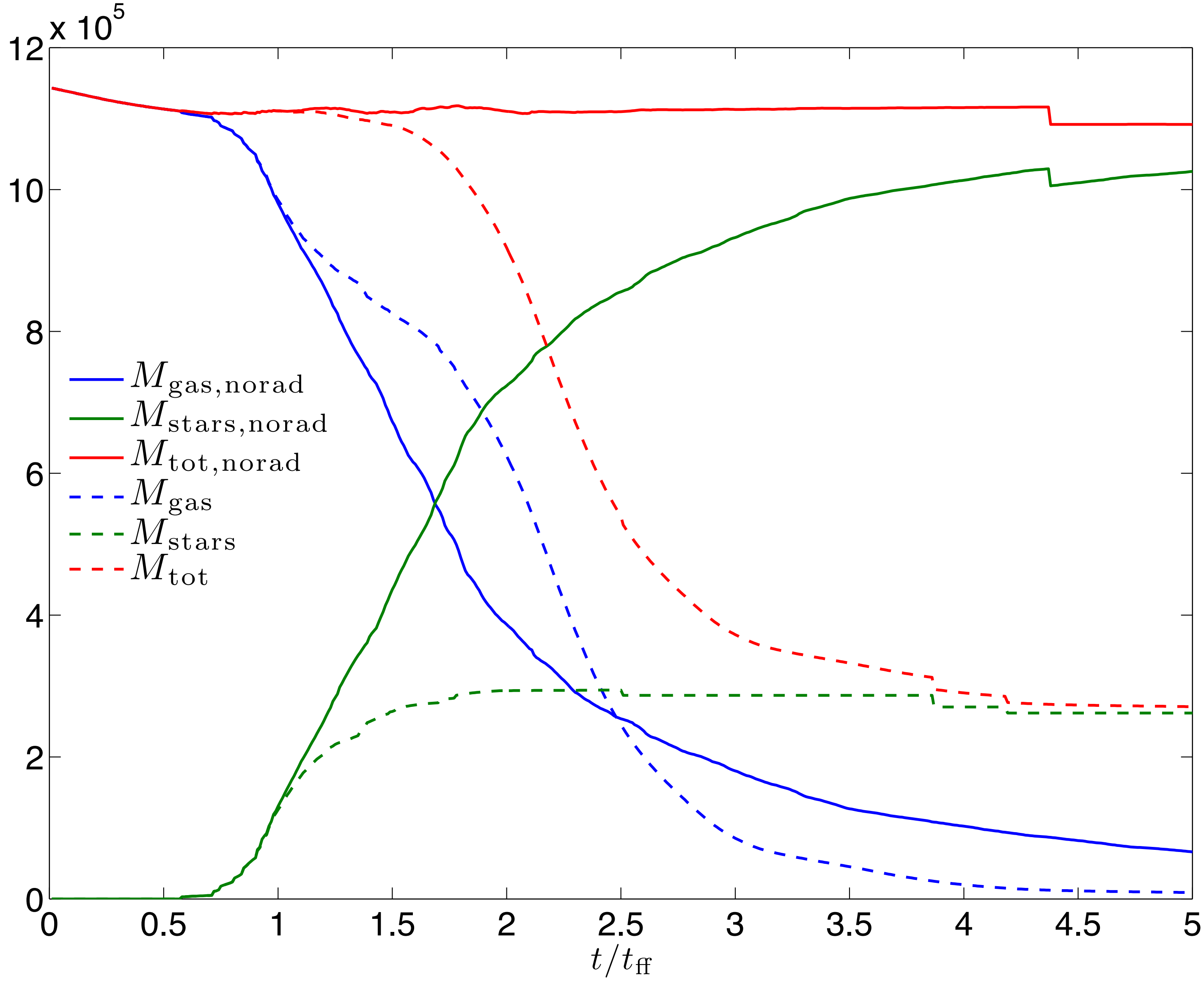
# With radiation pressure




Skinner & Ostriker (2013)

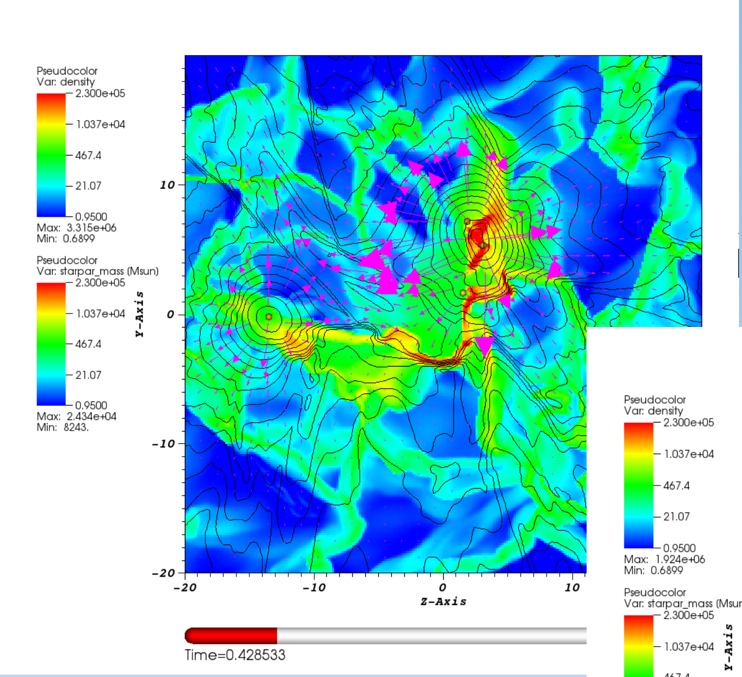
$10^6\,M_\odot$ initial cloud with sink particles and RHD

$L_* = \Psi M_*$ for subclusters




Skinner & Ostriker (2013)

$t_{ff}$=0.52 Myr


Pseudocolor
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1.037e+04
467.4
21.07
0.9500
Max: 3.315e+06
Min: 0.6899
Pseudocolor
Var: starpar_mass (Msun)
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21.07
0.9500
Max: 2.434e+04
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Z-Axis
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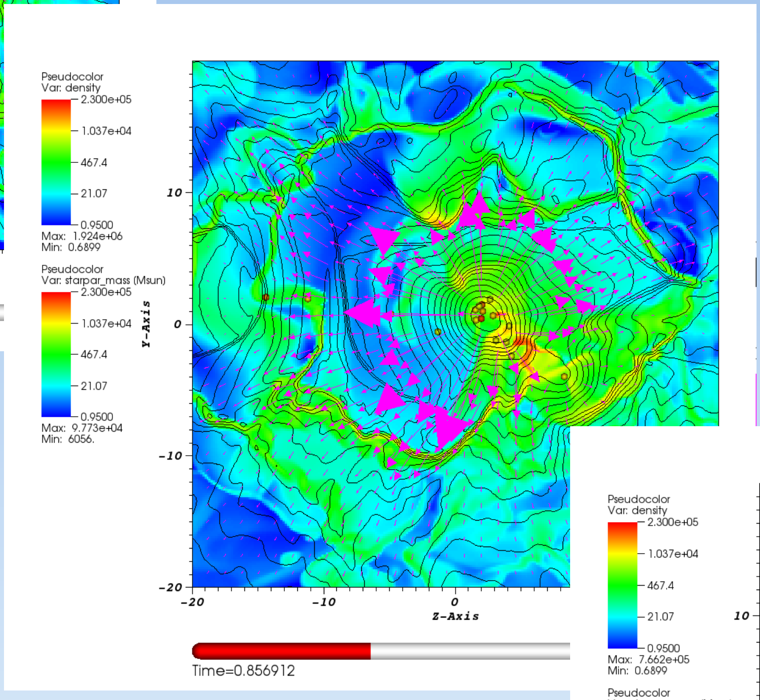
Pseudocolor
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Min: 0.6899
Pseudocolor
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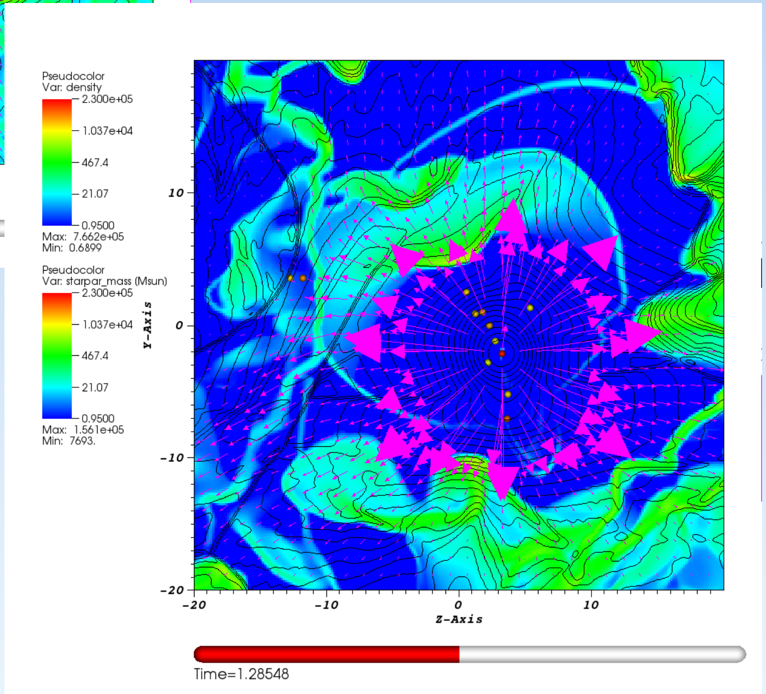
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21.07
0.9500
Max: 7.662e+05
Min: 0.6899
Pseudocolor
Var: starpar_mass (Msun)
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21.07
0.9500
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Z-Axis
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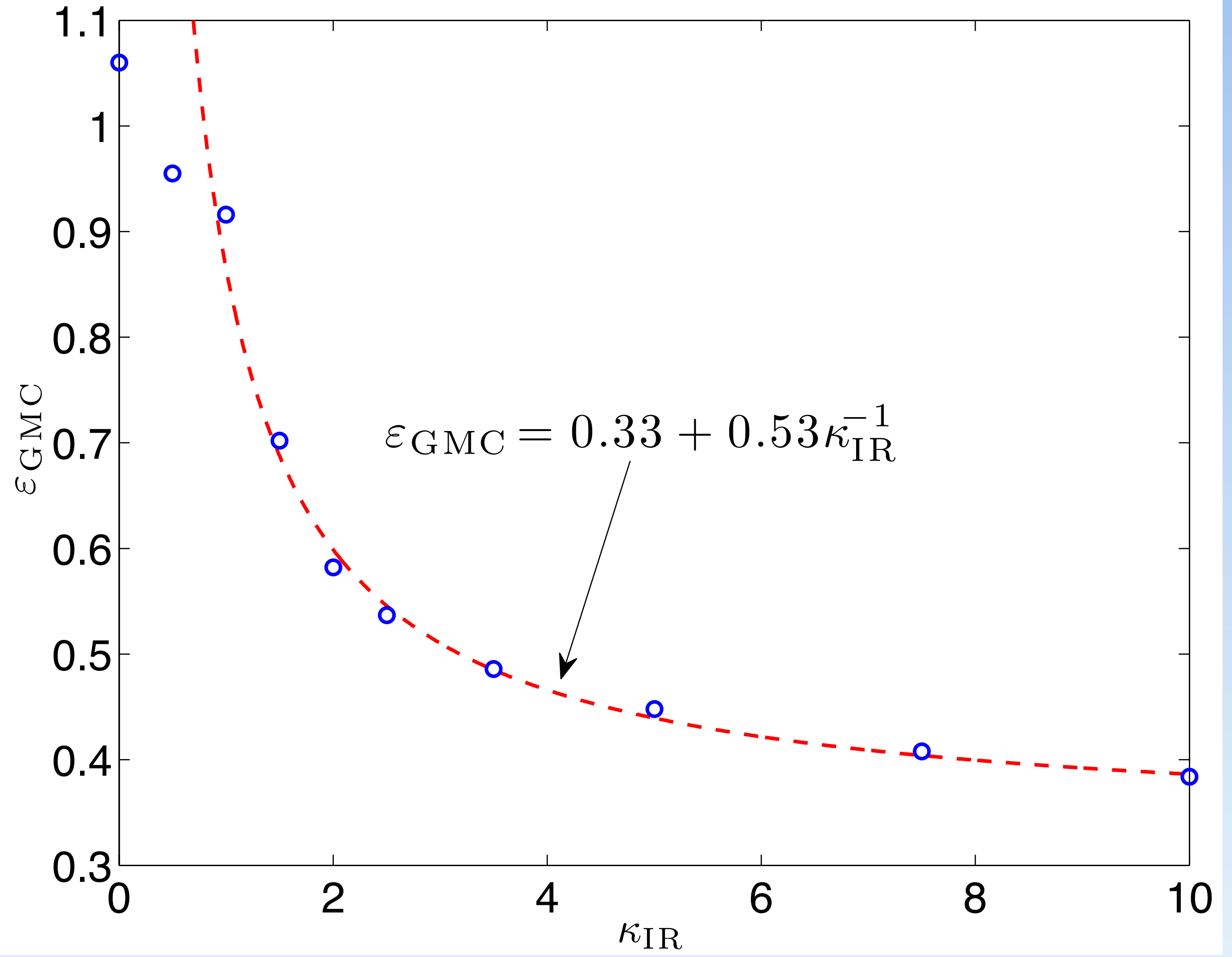

1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
2
4
6
8
10
$\varepsilon_{\rm GMC}$
$\kappa_{\rm IR}$
$\varepsilon_{\rm GMC} = 0.33 + 0.53\kappa_{\rm IR}^{-1}$

# Gas-dominated disk system

Ostriker & Shetty (2011)

$$\dot{M}_* \sim \frac{G M_{tot} M}{L^2 (p_*/m_*)} \rightarrow \qquad \frac{\dot{M}_*}{L^2} \sim \frac{G M^2}{L^4 (p_*/m_*)}$$

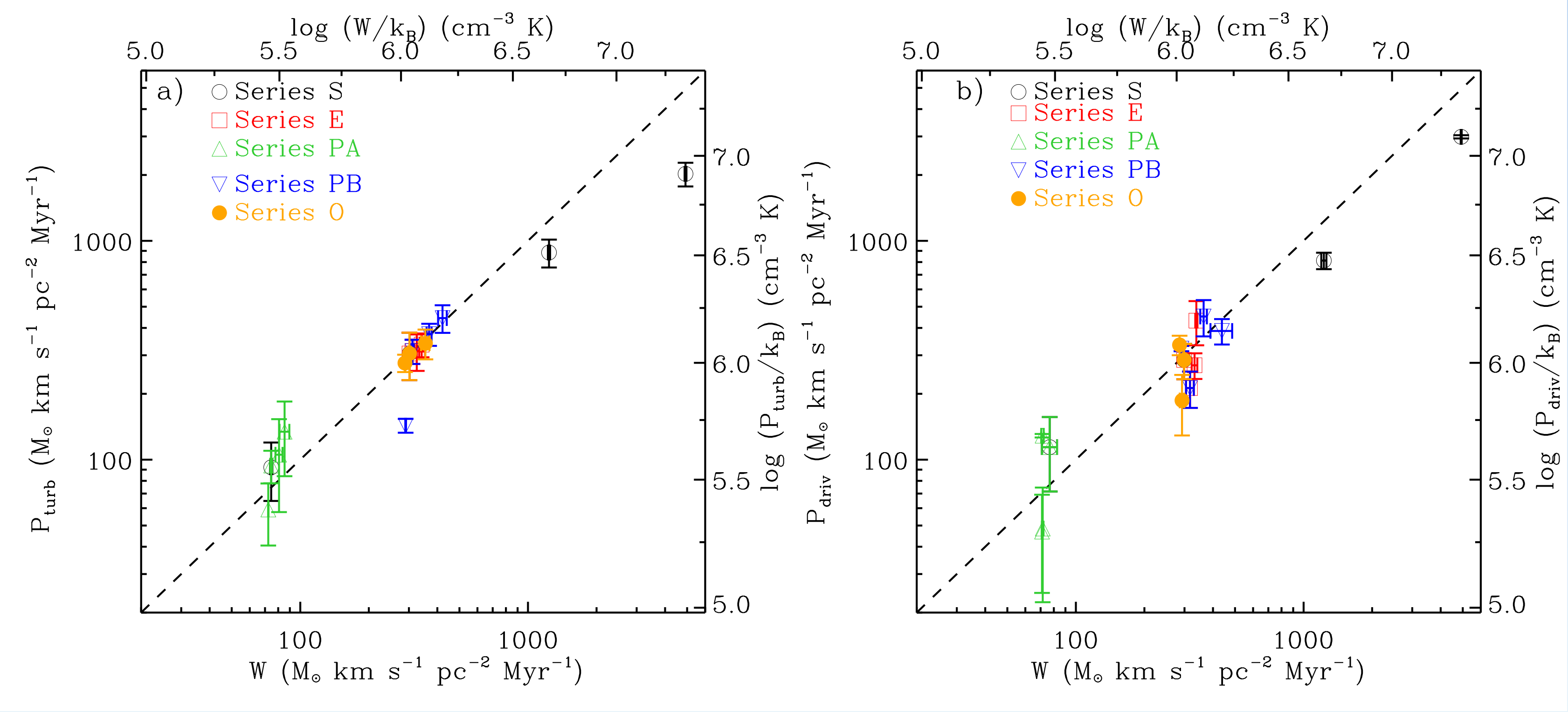
$$\Rightarrow \qquad \Sigma_{\rm SFR} \sim \frac{G\Sigma^2}{p_*/m_*}$$

- Star formation rate per unit area is *independent* of details of turbulence
- Disk thickness and internal dynamical time must evolve until momentum feedback rate matches vertical weight
- As for other systems with $\alpha_{\rm vir} \gtrsim 2$, expect low $\varepsilon_{ff}$ , and $v$ related to feedback: $v \sim \varepsilon_{ff} \frac{p_*}{m*}$

# Starburst regime simulations

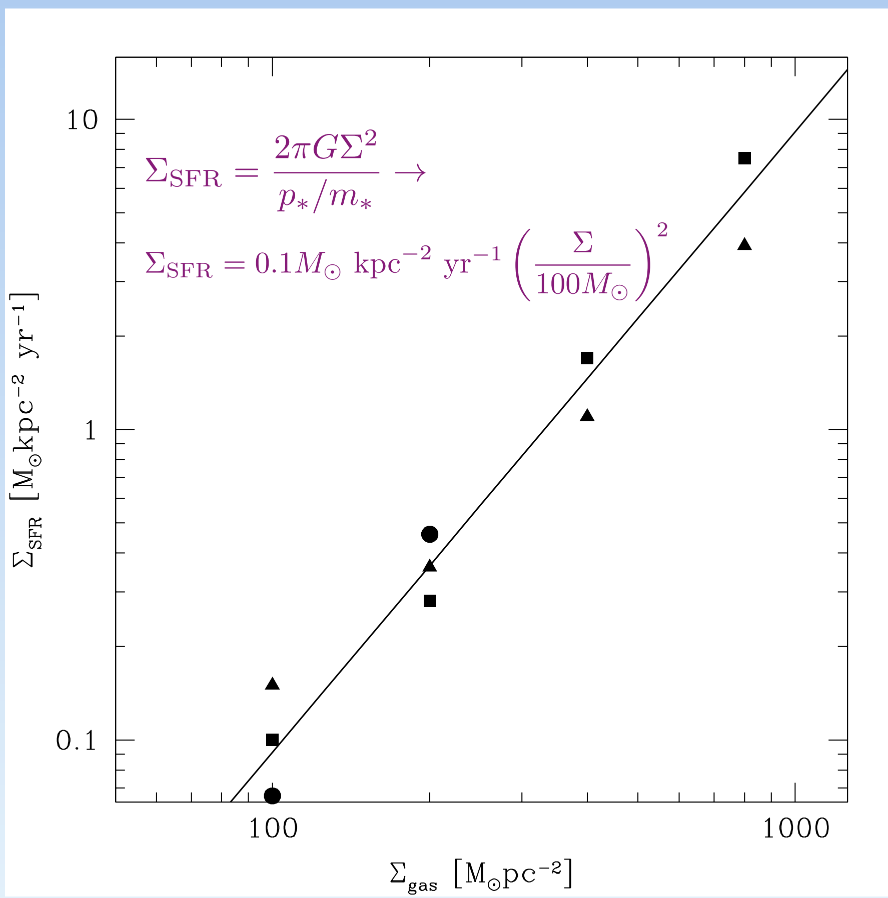
Shetty & Ostriker (2012)

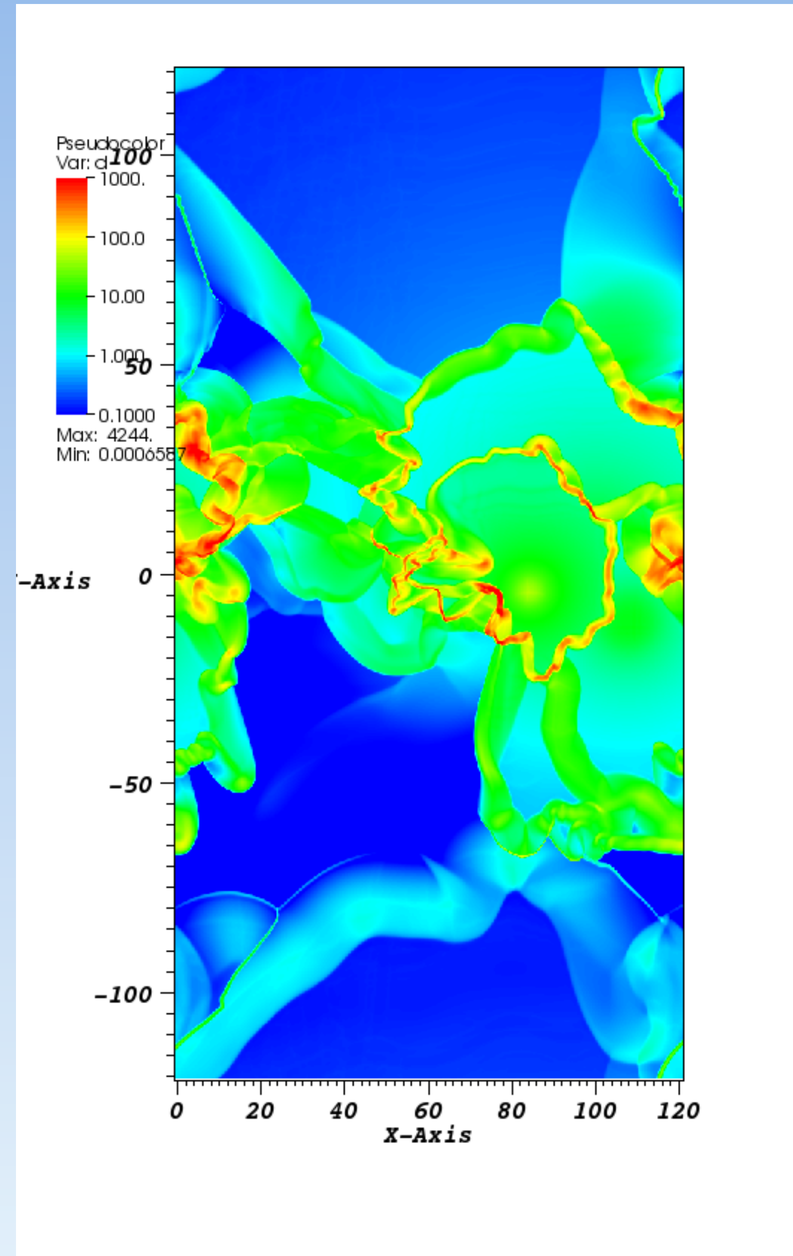
- Feedback-driven, turbulence-dominated equilibrium:
  - $P_{turb} \approx W \approx \pi G \Sigma^2/2 \approx (1/4)(p_*/m_*)\Sigma_{SFR}$
  - $\varepsilon_{ff} \sim 0.005\text{-}0.01$ insensitive to other conditions
  - $v_z \sim$ 5-10 km/s $\propto p_*/m_*$



Radiative SN remnants (Cioffi et al 1988; Blondin et al 1998, Thornton et al 1998):

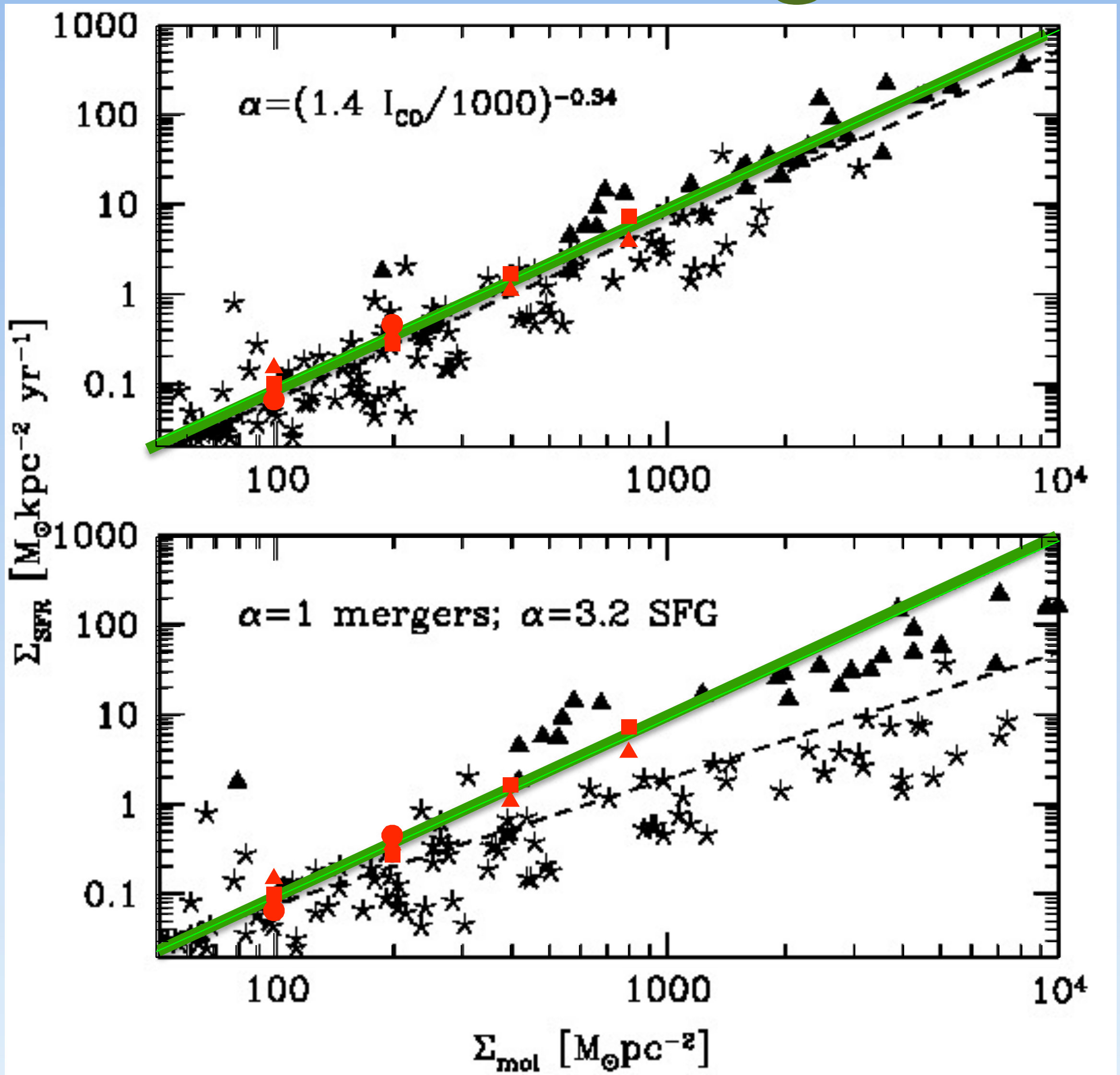
$$\frac{p_*}{m_*} \approx 3000 \mathrm{km\,s^{-1}} \left(\frac{E_{\rm SN}}{10^{51}\mathrm{erg}}\right)^{0.94} \left(\frac{n_0}{1\mathrm{cm}^{-3}}\right)^{-0.12} \left(\frac{m_*}{100 M_\odot}\right)^{-1}$$




Ostriker & Shetty (2011)

# Starburst regions




Ostriker & Shetty (2011)

Data from Genzel et al (2010) sample; two different CO →$H_2$ conversion factors α

→ See Shetty et al 2011a,b and Narayanan et al (2011,2012) for $X_{CO}$ dependencies

$$\Sigma_{SFR} = 0.1 M_\odot \text{ kpc}^{-2}\text{yr}^{-1}\left(\frac{\Sigma_{gas}}{100 M_\odot \text{ pc}^{-2}}\right)^2$$

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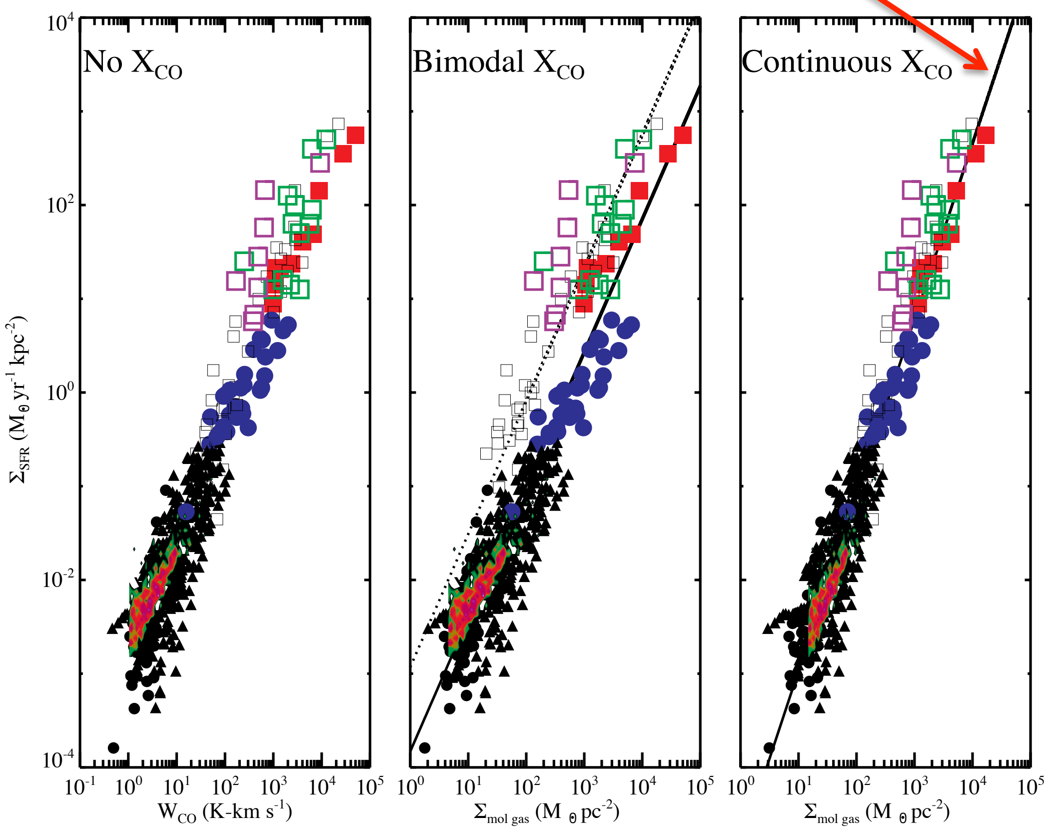
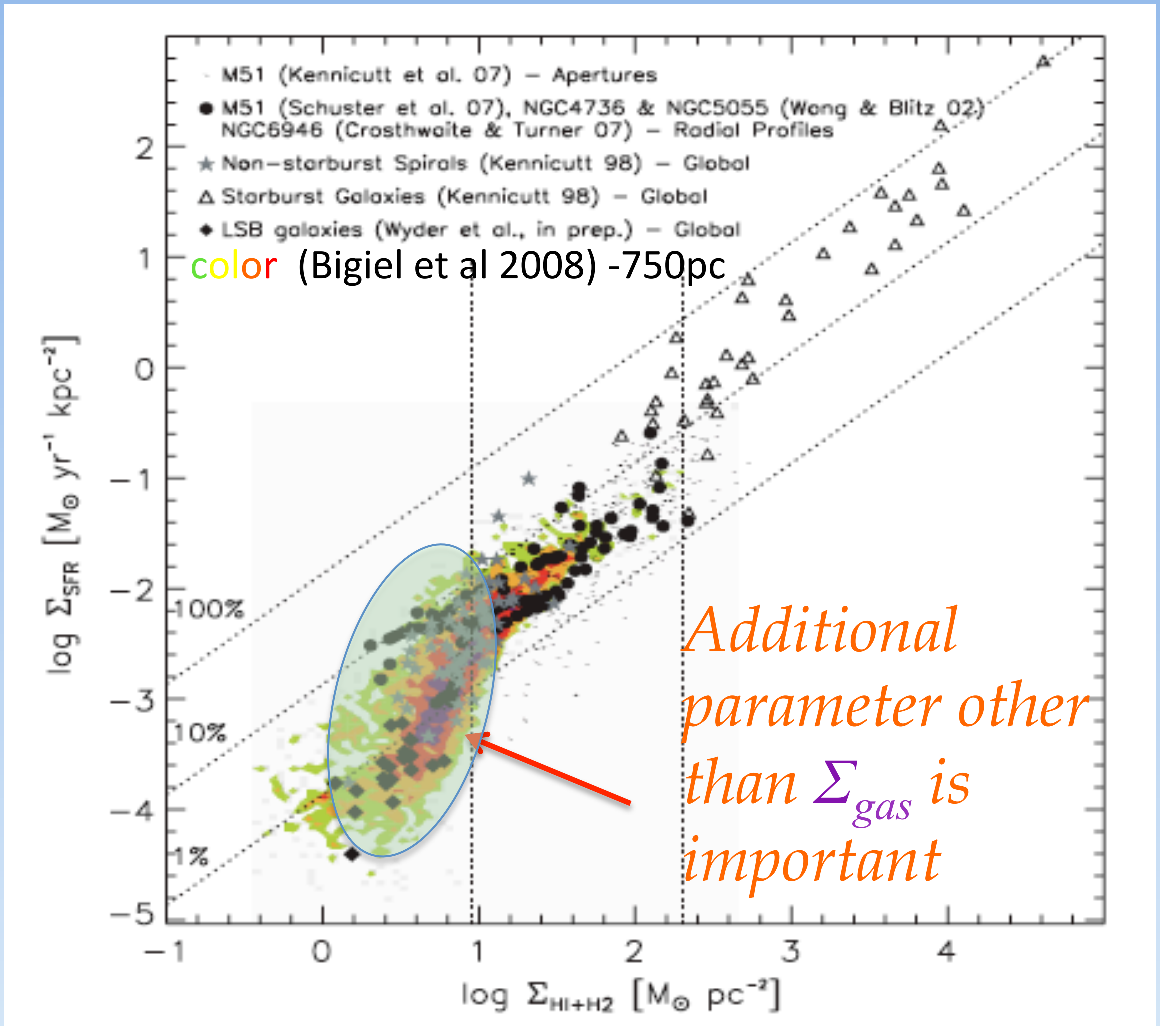



Narayanan, Krumholz, Ostriker, & Hernquist (2012)




$$\Sigma_{SFR} = 2 \times 10^{-3} M_\odot \mathrm{kpc}^{-2} \mathrm{yr}^{-1} \left( \frac{\Sigma}{10 M_\odot \mathrm{pc}^{-2}} \right) \left( \frac{\rho_*}{0.1 M_\odot \mathrm{pc}^{-3}} \right)^{1/2}$$

Ostriker et al (2010); Kim, Kim, & Ostriker (2011,2013)

cf: Wong & Blitz 2002; Blitz & Rosolowsky 2004, 2006; Leroy et al (2008)

# Gravity and turbulence for externally-confined gas

- Gravitational free-fall time (gas):

$$t_{\rm ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 43{\rm Myr}\left(\frac{n_H}{1\ {\rm cm}^{-3}}\right)^{-1/2}$$

- Dynamical crossing time:

$$t_{\rm dyn} = \frac{1}{(4\pi G\rho_*)^{1/2}} = 13{\rm Myr}\left(\rho_*/0.1M_\odot\ {\rm pc}^{-3}\right)^{-1/2}$$

- Ratio:

$$\frac{t_{\rm ff}}{t_{\rm dyn}} = \left(\frac{3\pi^2\rho_*}{8\rho}\right)^{1/2} = 3.3\left(\frac{\rho_*/0.1M_\odot\ {\rm pc}^{-3}}{n_H/1\ {\rm cm}^{-3}}\right)^{1/2}$$

# Summary

- Gas consumption rate per free-fall time $\varepsilon_{ff}$ is expected to be low in turbulence-dominated systems with $\alpha_{vir} \sim 1\text{-}2$
- Keeping net efficiency low over lifetime requires preventing continuing contraction and increase of $t_{ff}$
- Secular contraction is limited by various SF feedback mechanisms at a range of scales that offset dissipation
- System with both driving/dissipation balance and force balance has

$$v \sim \varepsilon_{ff} \frac{p_*}{m*} \qquad L \sim \frac{GM_{tot}}{v^2} \qquad \dot{M}_* \sim \frac{GM_{tot}M}{L^2 p_*/m_*}$$

- Details of feedback processes and evolution determine $p_*/m_*$ and resulting SFR, size, and velocity dispersion
- Starburst disks are particularly simple, with $\Sigma_{\rm SFR} \sim \frac{G\Sigma^2}{p_*/m_*}$