The Star Formation Rate in Turbulent Molecular Clouds

Paolo Padoan
(ICREA - Institute of Cosmos Sciences - University of Barcelona)

Troels Haugbølle and **Åke Nordlund**
(STARPLAN and Niels Bohr Institute - University of Copenhagen)

The local SFR law - Definitions

We can express the SFR in units of the free-fall time, as the efficiency factor of a theoretical Schmidt-Kennicut law:

$$\dot{\rho}_{\rm stars} = \epsilon_{\rm ff}\, \rho_{\rm gas}/t_{\rm ff}$$

We model $\varepsilon_{\rm ff}$ **locally**, meaning in regions with size, L, smaller than the driving scale, L_0, of the ISM turbulence, $L < L_0$, so we don't worry about the large-scale driving, but only about the resulting local physical parameters ($\alpha_{\rm vir}$, $\mathscr{M}_{\rm S}$, $\mathscr{M}_{\rm A}$).

We find: $$\epsilon_{\rm ff} \sim \exp(-1.4\, \alpha_{\rm vir}^{1/2}) \sim \exp(-1.6\, t_{\rm ff}/t_{\rm dyn})$$

where $$t_{\rm ff} = (3\pi/(32 G\rho))^{1/2}, \qquad t_{\rm dyn} = R/\sigma_{\rm v,3D}$$

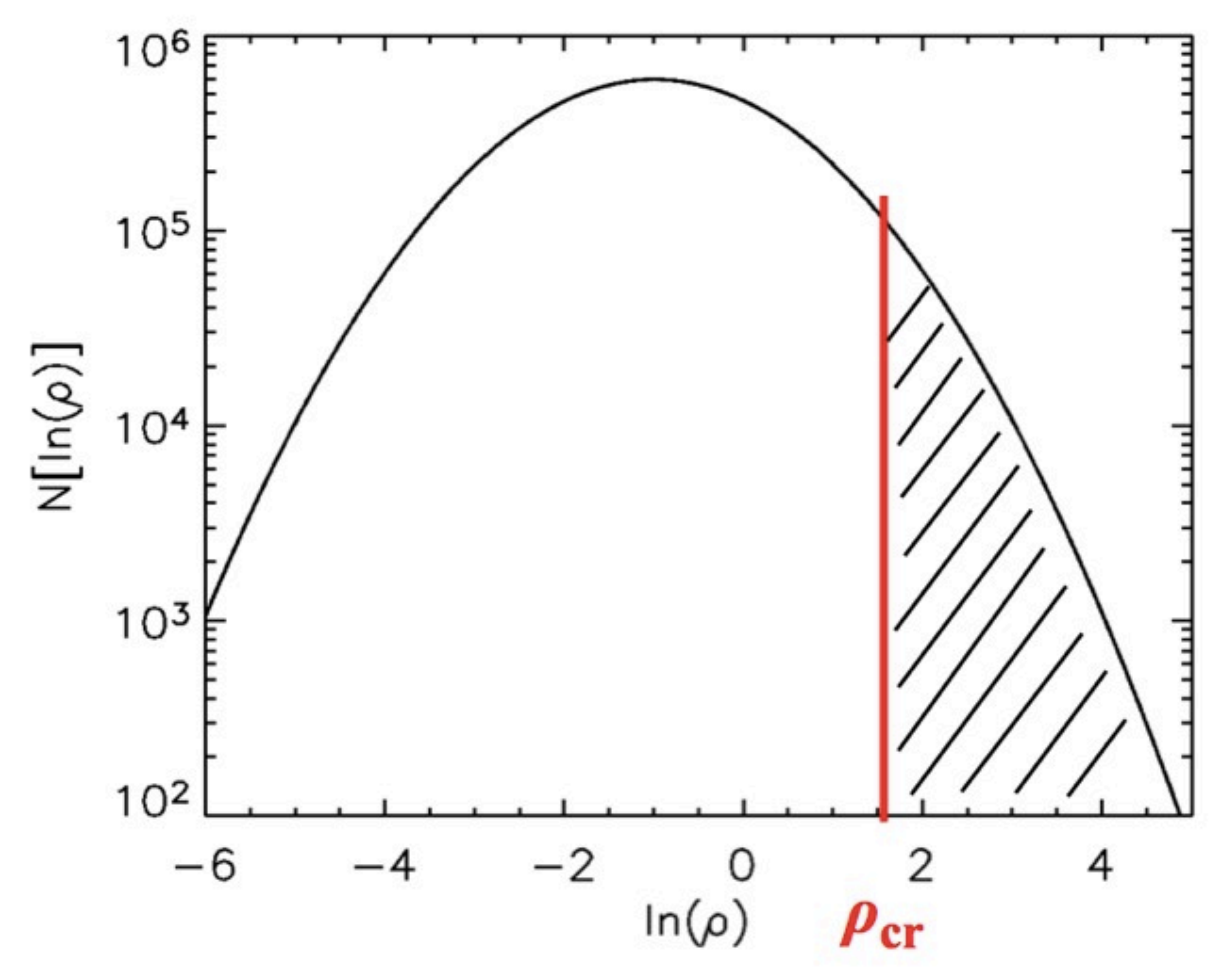
In modeling galaxy formation, how should this SFR law be applied to large scales?

The physics of the local SFR (Eve's talk, Christoph's talk)

- The main consequence of supersonic turbulence is a very intermittent PDF of gas density: a fraction of the mass ends up in very dense filaments and cores.
- The PDF is universal and depends only on the rms Mach number of the turbulence (also magnetic field, compressing driving, E.O.S., ...).
- The sonic scale and the Jeans length define a critical density for collapse.
- The SFR can be expressed as the integral of the PDF above the critical density, divided by the free-fall time of the critical density.

Krumholz and McKee (2005)
Padoan and Nordlund (2011)
Chabrier and Hennebelle (2011)
Federrath and Klessen (2012)
Hopkins (2013)

$$\frac{\rho_{\rm cr}}{\rho_0} \sim \alpha_{\rm vir}\, \mathcal{M}_{\rm S,0}^2\, \beta \qquad \mathrm{SFR}_{\rm ff} = \epsilon\, \frac{\tau_{\rm ff,0}}{\tau_{\rm ff,cr}} \int_{x_{\rm cr}}^{\infty} x\, p(x)\, dx, \qquad x = \frac{\rho}{\rho_0}$$



These analytical models give us the general picture:

1. Turbulence can slow down star formation
2. Magnetic fields reduce the SFR even further
3. The observed SFR is consistent with turbulent fragmentation
4. The fundamental parameters are those defining the turbulence with respect to gravity and thermal and magnetic pressure, $\alpha_{\rm vir}$, $\mathscr{M}_{\rm S}$, $\mathscr{M}_{\rm A}$.

These models are also very uncertain because the complexity of the problem forces us to make simplifying assumptions and approximations.

Assumptions, approximations and model predictions must be tested with numerical simulations, where much of the complexity can be retained.

We may even choose to adopt a local SFR law derived directly from the simulations, as long as the parameter space is well sampled.

Empirical SFR from numerical models

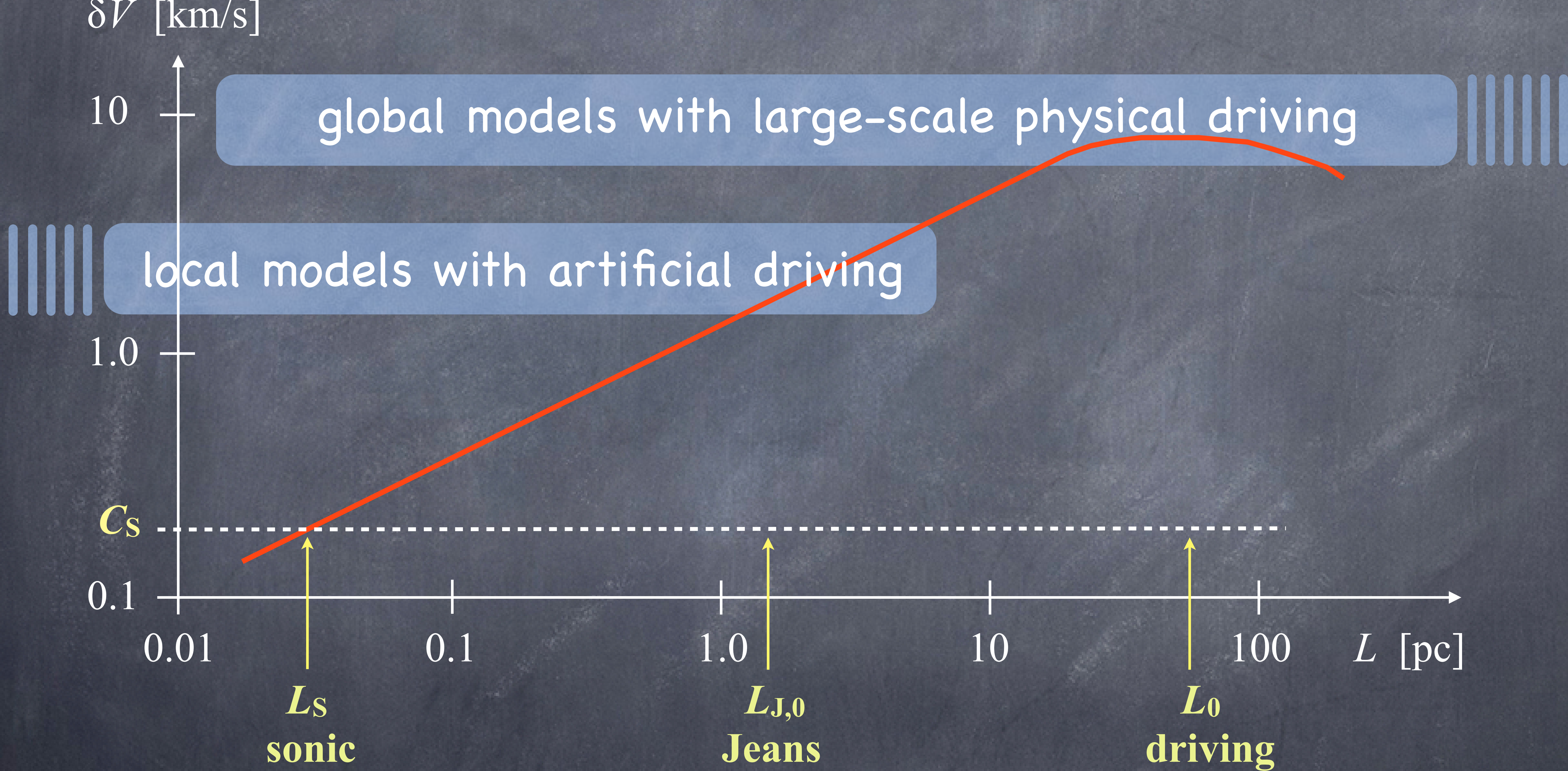
The fundamental length-scales, from large to small, are:

- driving scale, L_0 (largest turbulence turnover time)
- Jeans length, $L_{\rm J,0}$ (gravity versus thermal pressure),
- sonic scale, $L_{\rm S}$ (turbulence versus thermal pressure),
- dissipation scale (smallest turbulence turnover time)

We must include $L_{\rm J,0}$ and resolve $L_{\rm S}$, because the SFR is controlled by $L_{\rm J,0}/L_{\rm S}$ (the SFR is low because $L_{\rm S} \ll L_{\rm J,0}$, that is the turbulence fragments the gas into small pieces, so it is the local $L_{\rm J}$ in the pieces that matters, not $L_{\rm J,0}$)

Depending on the approach, we may or may not include the driving scale, L_0

The length-scales of star formation



Local models: $[L_S , L_{J,0}] \rightarrow L_{box}/dx \sim 10^3$
(can be done in unigrid, but cheaper with AMR)

Limited range of scales is good for parameter studies
$\longmapsto$ derivation of the local SFR law

Global models: $[L_S , L_0] \rightarrow L_{box}/dx > 10^5$
(require AMR)

Very large range of scales is good for statistical studies of a large sample of subregions, within a single run, with realistic boundary and initial conditions
$\longmapsto$ test for the local SFR law (intrinsic variance)
$\longmapsto$ **test the SFR law versus the size of the region**

Local models are quite idealized and must rely on random driving.

Large parameter studies with such models have been recently carried out both with uniform grid simulations and with AMR.

Unigrid: Padoan and Nordlund (2011)
AMR: Padoan, Haugbolle, Nordlund (2012),
Federrath and Klessen (2012)

Global models are a superior way to explore the parameter space, because they do not enforce artificial initial conditions, boundary conditions and driving forces on individual star-forming regions.

They are much more challenging and this is work in progress.

A SIMPLE LAW OF STAR FORMATION

PAOLO PADOAN
ICREA & ICC, University of Barcelona, Marti i Franquès 1, E-08028 Barcelona, Spain; ppadoan@icc.ub.edu

TROELS HAUGBØLLE
Centre for Star and Planet Formation, University of Copenhagen, Œstervoldgade 5-7., DK-1350, Copenhagen, Denmark; haugboel@nbi.dk

ÅKE NORDLUND
Centre for Star and Planet Formation and Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, DK-2100, Copenhagen, Denmark; aake@nbi.dk

Draft version September 28, 2012

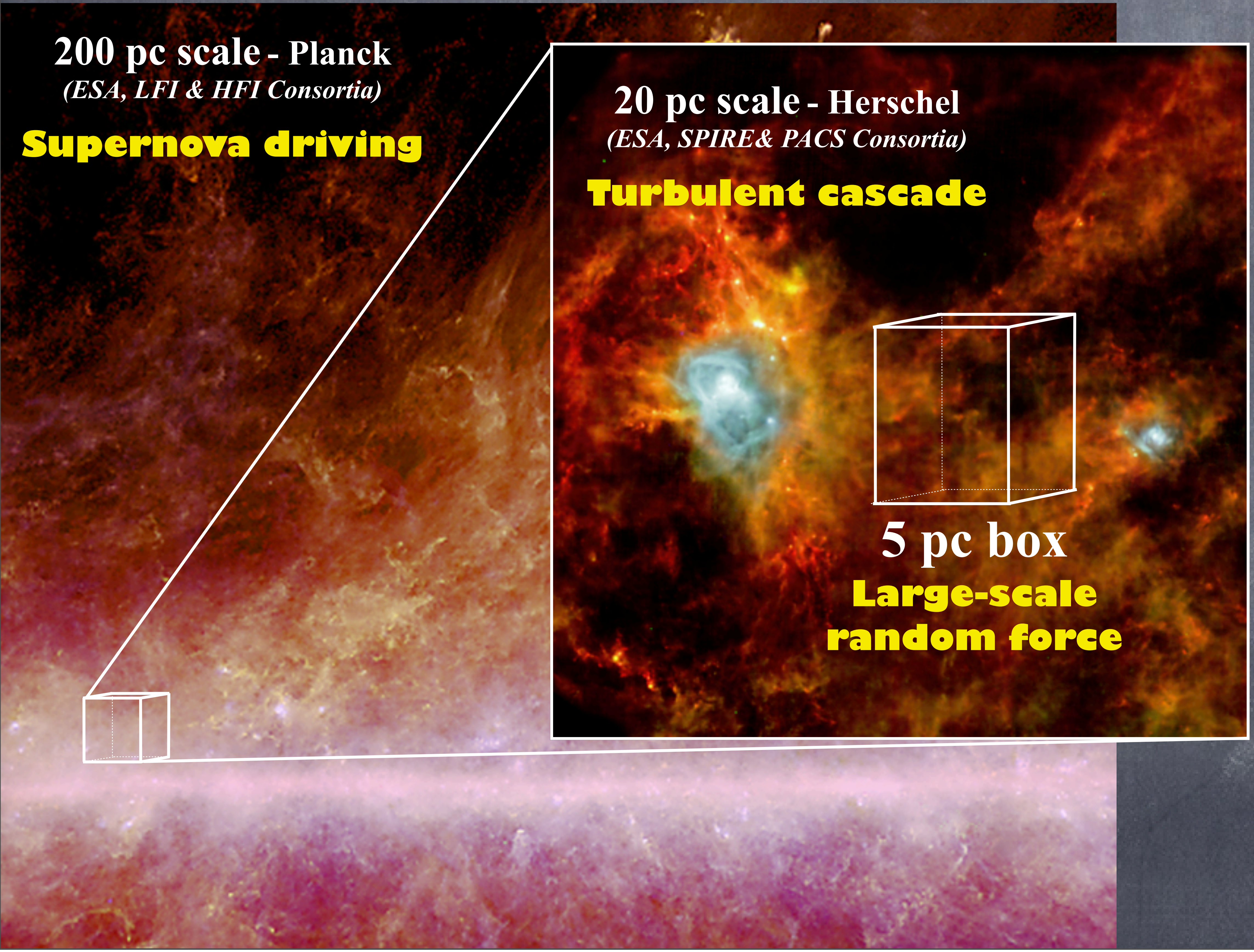
ABSTRACT

We show that supersonic MHD turbulence yields a star formation rate (SFR) as low as observed in molecular clouds (MCs), for characteristic values of the free-fall time divided by the dynamical time, $t_{\rm ff}/t_{\rm dyn}$, the alfvénic Mach number, $\mathcal{M}_{\rm a}$, and the sonic Mach number, $\mathcal{M}_{\rm s}$. Using a very large set of deep adaptive-mesh-refinement simulations, we quantify the dependence of the SFR per free-fall time, $\epsilon_{\rm ff}$, on the above parameters. Our main results are: i) $\epsilon_{\rm ff}$ decreases exponentially with increasing $t_{\rm ff}/t_{\rm dyn}$, but is insensitive to changes in $\mathcal{M}_{\rm s}$, for constant values of $t_{\rm ff}/t_{\rm dyn}$ and $\mathcal{M}_{\rm a}$. ii) Decreasing values of $\mathcal{M}_{\rm a}$ (stronger magnetic fields) reduce $\epsilon_{\rm ff}$, but only to a point, beyond which $\epsilon_{\rm ff}$ increases with a further decrease of $\mathcal{M}_{\rm a}$. iii) For values of $\mathcal{M}_{\rm a}$ characteristic of star-forming regions, $\epsilon_{\rm ff}$ varies with $\mathcal{M}_{\rm a}$ by less than a factor of two. We propose a simple star-formation law, based on the empirical fit to the minimum $\epsilon_{\rm ff}$, and depending only on $t_{\rm ff}/t_{\rm dyn}$: $\epsilon_{\rm ff} \approx \epsilon_{\rm wind} \exp(-1.6\, t_{\rm ff}/t_{\rm dyn})$. Because it only depends on the mean gas density and rms velocity, this law is straightforward to implement in simulations and analytical models of galaxy formation and evolution.

Subject headings: ISM: kinematics and dynamics — stars: formation — magnetohydrodynamics — turbulence

Wednesday, June 26, 13

Local models below the physical driving scale



A chunk of a MC:

Periodic Box
Random forcing
Isothermal E.O.S.
Self-gravity
Sink particles

Adaptive Mesh Refinement Simulations

(Pleiades at NASA/Ames and Supermuc at Leibnitz Research Center)

AMR code: Our own extra scalable version of Ramses, with a hybrid layout, using OpenMP within individual nodes, and MPI between nodes.

Numerical setup:

- Periodic boundary conditions
- Random, large-scale force
- Uniform initial magnetic and density fields
- Random initial velocity field
- Isothermal equation of state
- Root grid: up to 32^3 - 128^3
- AMR levels above root grid: 8 ($32{,}768^3$)
- Sink particles, $\rho_{\rm sink} = 10^5 \langle \rho \rangle$

Parameter space probed with 45 simulations:

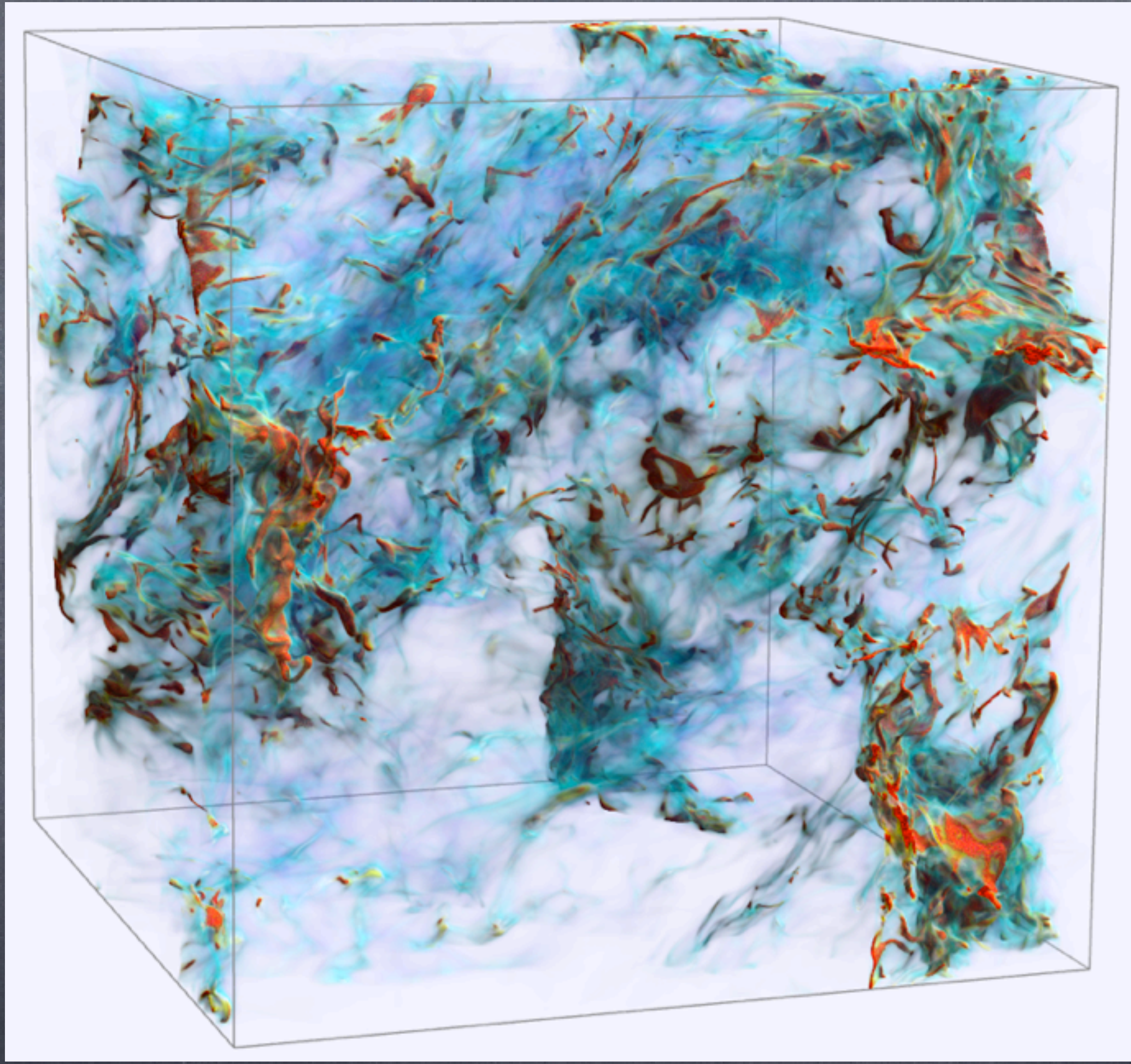
$\mathcal{M}_{\rm S}$ =10, 20

$\mathcal{M}_{\rm A}$ =33, 20, 5, 1.25

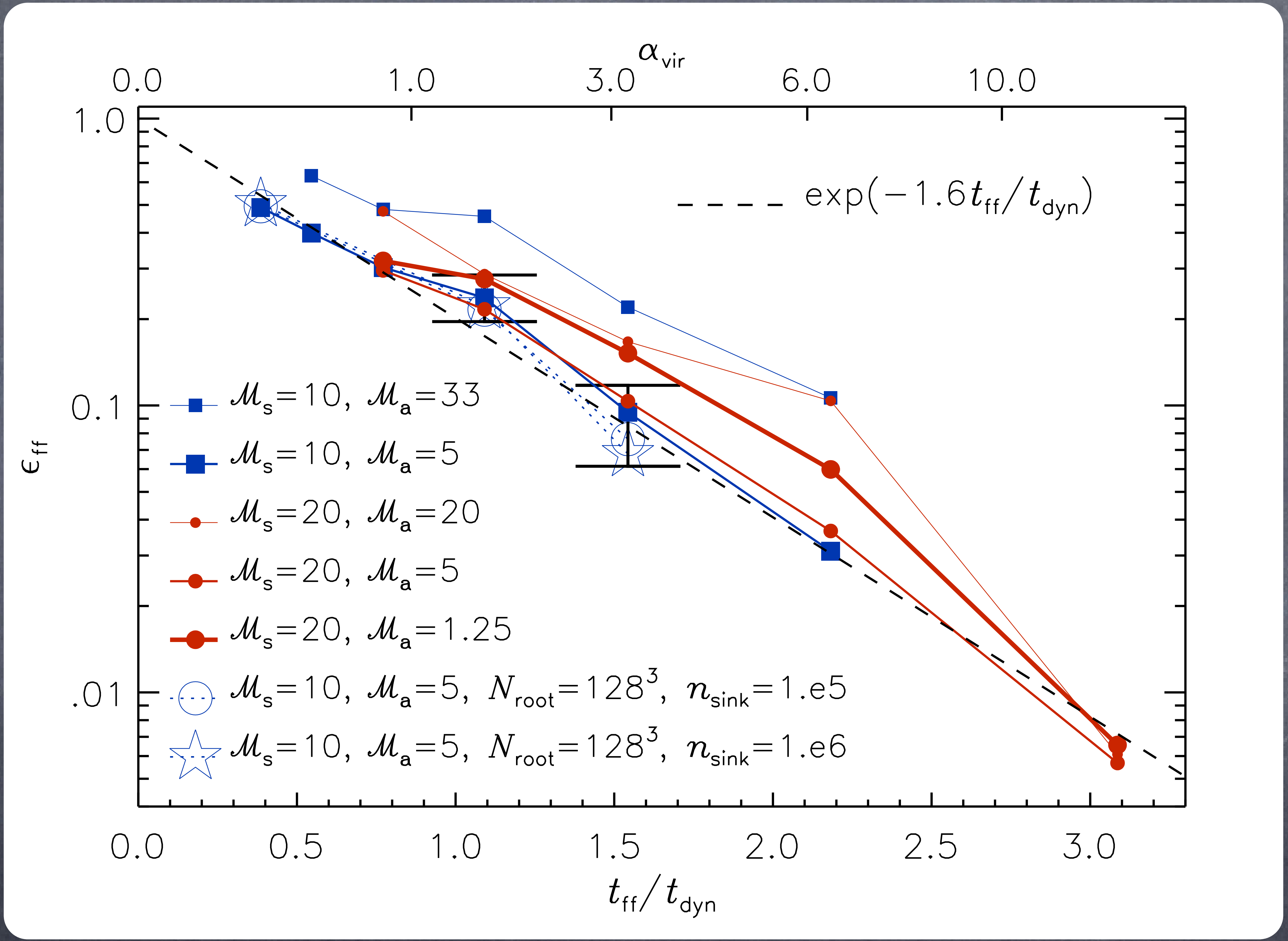
$t_{\rm ff}/t_{dyn}$ =0.4 - 3.1

($\alpha_{\rm vir}$ = 0.22 - 13.5)

1000 $M_\odot$ in a 5 pc box - 60 AU resolution



The minimum SFR



Results

1. $\varepsilon_{\rm ff}$ decreases exponentially with increasing values of $t_{\rm ff}/t_{dyn}$
2. $\varepsilon_{\rm ff}$ has almost no dependence on $\mathcal{M}_{\rm S}$
3. $\varepsilon_{\rm ff}$ has a weak dependence on $\mathcal{M}_{\rm A}$

For characteristic values of $\mathcal{M}_{\rm S}$ and $\mathcal{M}_{\rm A}$ in molecular clouds, we can therefore express the local SFR law as:

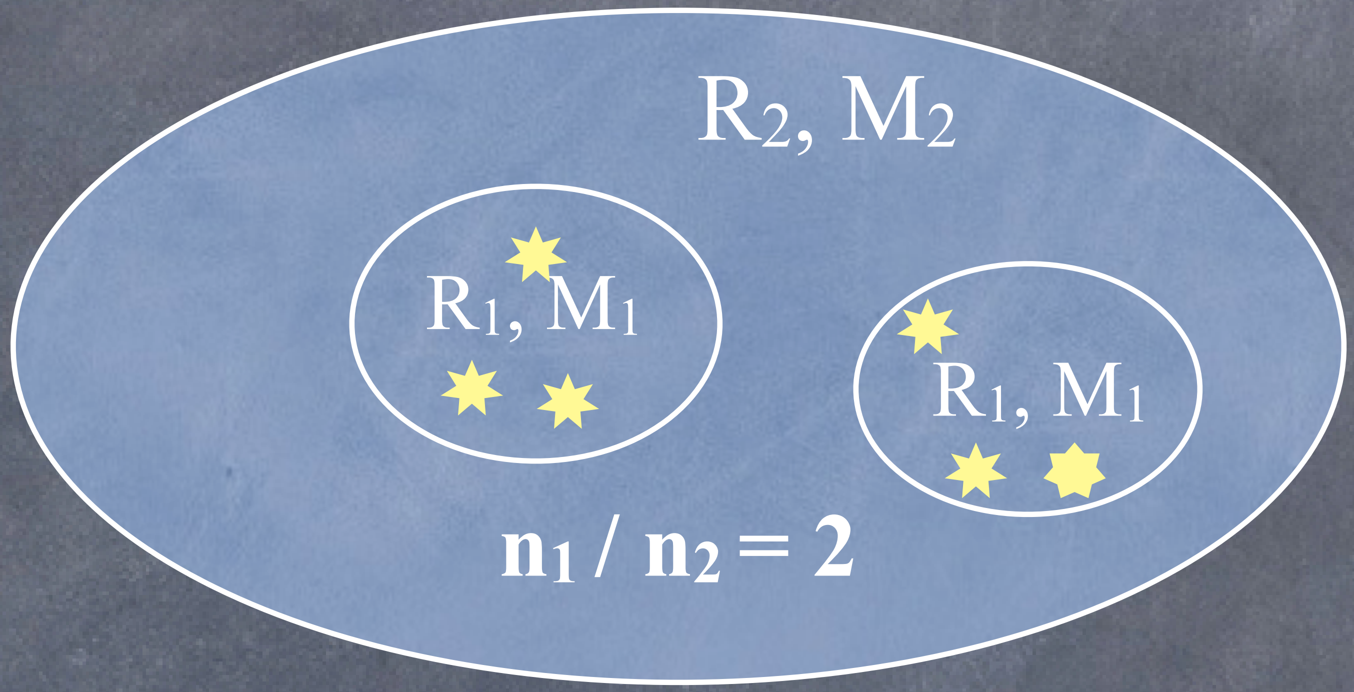
$$\epsilon_{\rm ff} \approx \epsilon_{\rm wind} \exp(-1.6\, t_{\rm ff}/t_{\rm dyn})$$

This law only depends on the density and rms velocity of a star forming region, so it is easy to implement. But we need to test if/how it applies to different scales, including nested hierarchical structure, in order to use it as a sub-grid model for galaxy formation simulations.

Hierarchical structure and the local SFR law

Is our result self-consistent in the hierarchy of ISM structures?

n_i is the number of sub-regions of size R_i within a larger region of size R_{i+1}



The SFR at two different scales in a hierarchical structure must satisfy:

$$SFR_2 = n_1/n_2\, SFR_1 \;\;\Rightarrow\;\; \epsilon_{\rm ff,2}\frac{M_2}{t_{\rm ff,2}} = \epsilon_{\rm ff,1}\frac{M_1}{t_{\rm ff,1}}\frac{n_1}{n_2}$$

There are indications that $\varepsilon_{\rm ff}$ does not depend on scale, because t_{ff}/t_{dyn} (or α_{vir}) is constant if both Larson relations are satisfied. Then the condition is satisfied.

From the definition of the free-fall time, assuming fractal structure with dimension D, and a power law mass-size relation,

$$n \sim R^{-D}, \quad M \sim R^{\beta}$$

we get

$$\epsilon_{\mathrm{ff},2} = \epsilon_{\mathrm{ff},1} \left(R_2/R_1\right)^{D-(\beta-1)3/2}$$

Given that we found that $\varepsilon_{\mathrm{ff}}$ has an exponential dependence on parameters, this condition can be satisfied for all R only if $\varepsilon_{\mathrm{ff}}$ does not depend on scale. This implies

$$D = (\beta - 1)3/2$$

Notice that in the simple case of mass conservation,

$$M_2 = M_1 \frac{n_1}{n_2}$$

we would get the simple result

$$D = \beta$$

Evidently, when selecting star-forming regions we are not conserving mass, in the sense that as we go to smaller regions we leave out an increasing amount of gas that is not involved in star formation.

From the scale independence of $\varepsilon_{\rm ff}$, and using v ~ R $^{\alpha}$, we also get

$$\beta = 2\alpha + 1 \approx 2$$

and therefore

$$D = 3\alpha \approx 1.5$$

In summary:

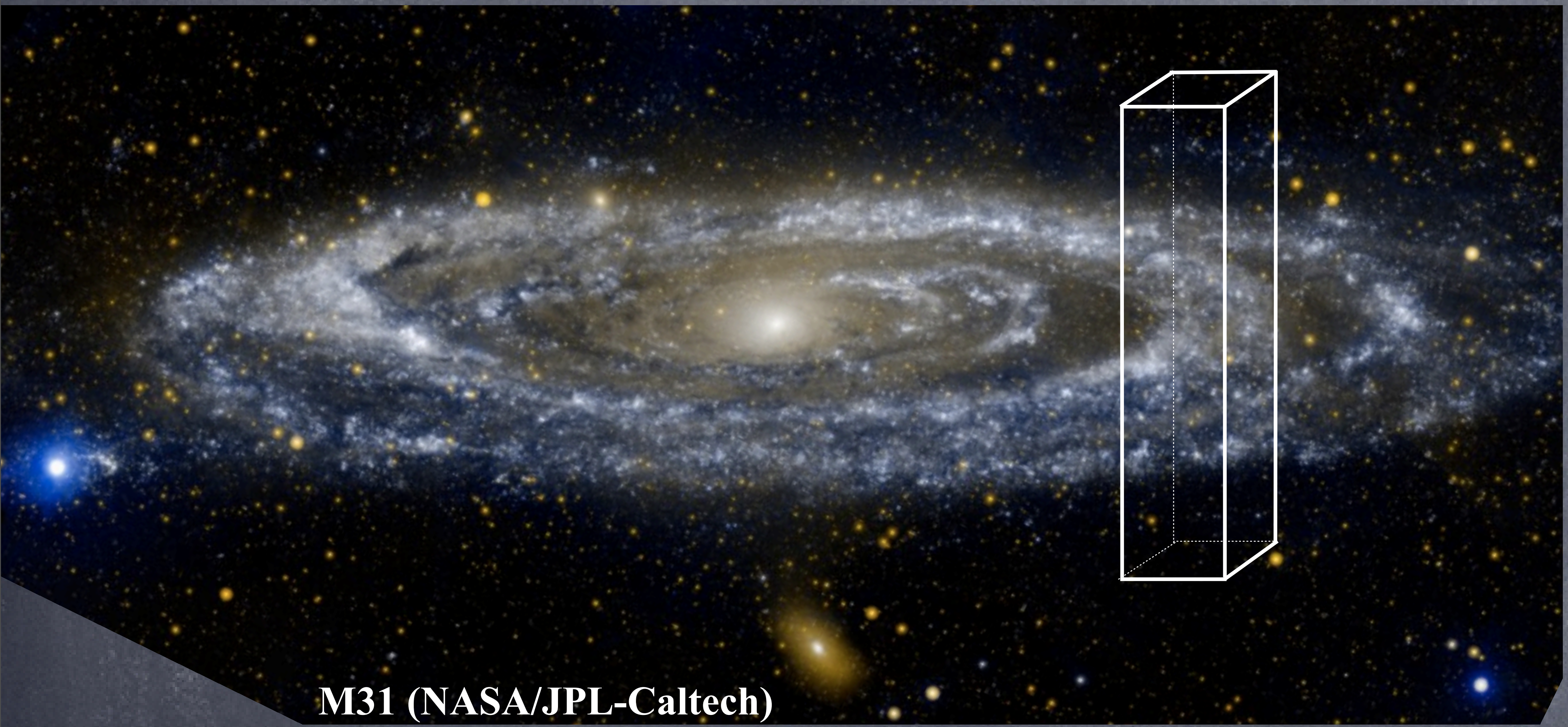
When we select star-forming regions on different scales in the hierarchy, $\varepsilon_{\rm ff}$ must be scale independent, the mass fraction of gas involved in star formation must decrease with scale (SFE increases towards smaller scales), and such regions must be organized in a very filamentary structure (D=1.5), and their column density should be roughly independent of scale.

So maybe our local SFR law can be applied to any scale below the driving scale, but we really need to check that.

This requires "global" numerical models including the SN driving scale.

IMPORTANT NOTE (why millions of cpu hrs on supercomputers?): The numerical models should include a level of complexity and realism much higher than the analytical models we want to test. Over-simplified numerical models (2D, no magnetic fields, insufficient resolution) do not really test the assumptions of equally over-simplified analytical models, and they too easily match the analytical predictions......

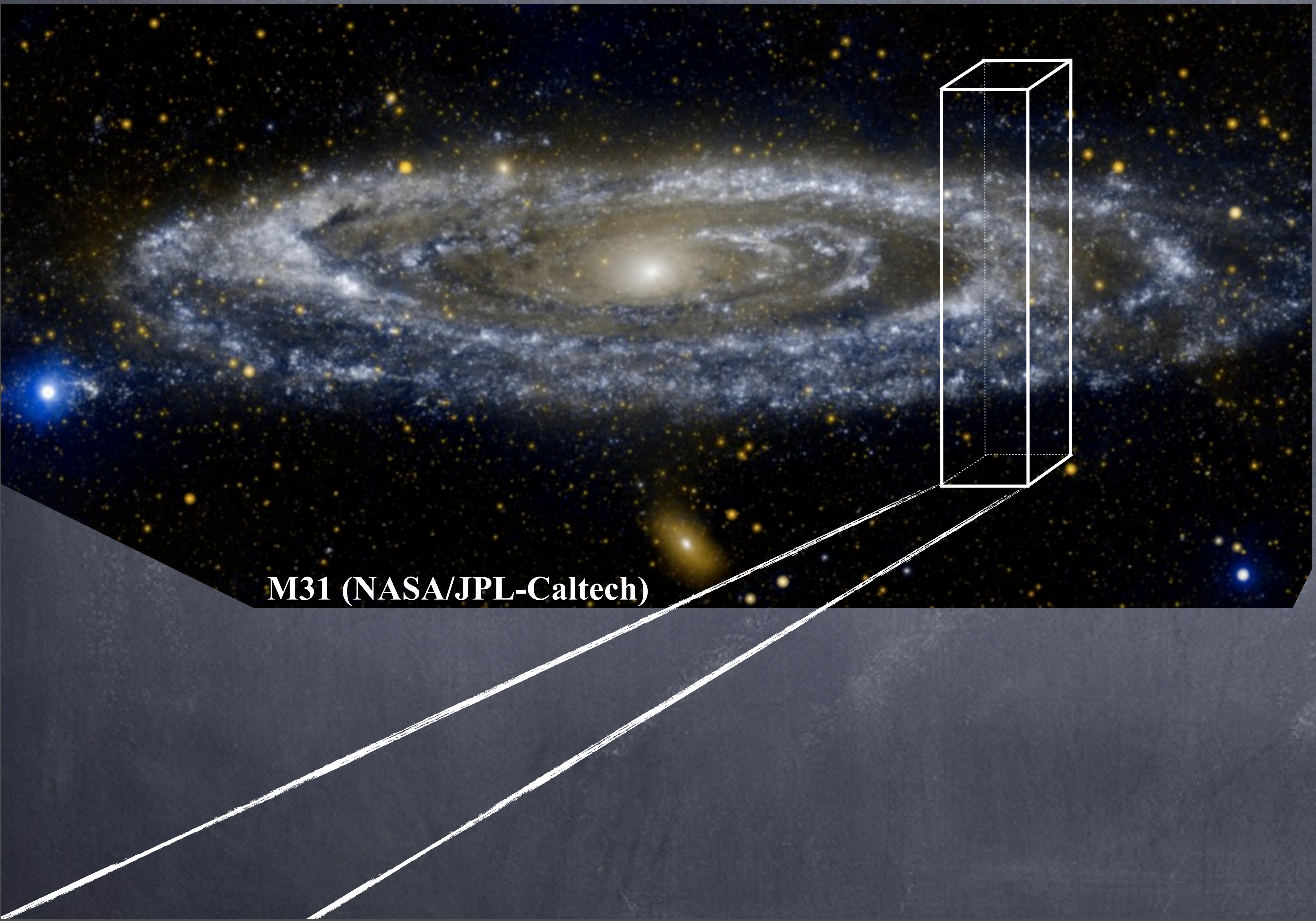
Global models including the driving scale



A chunk of a galaxy:

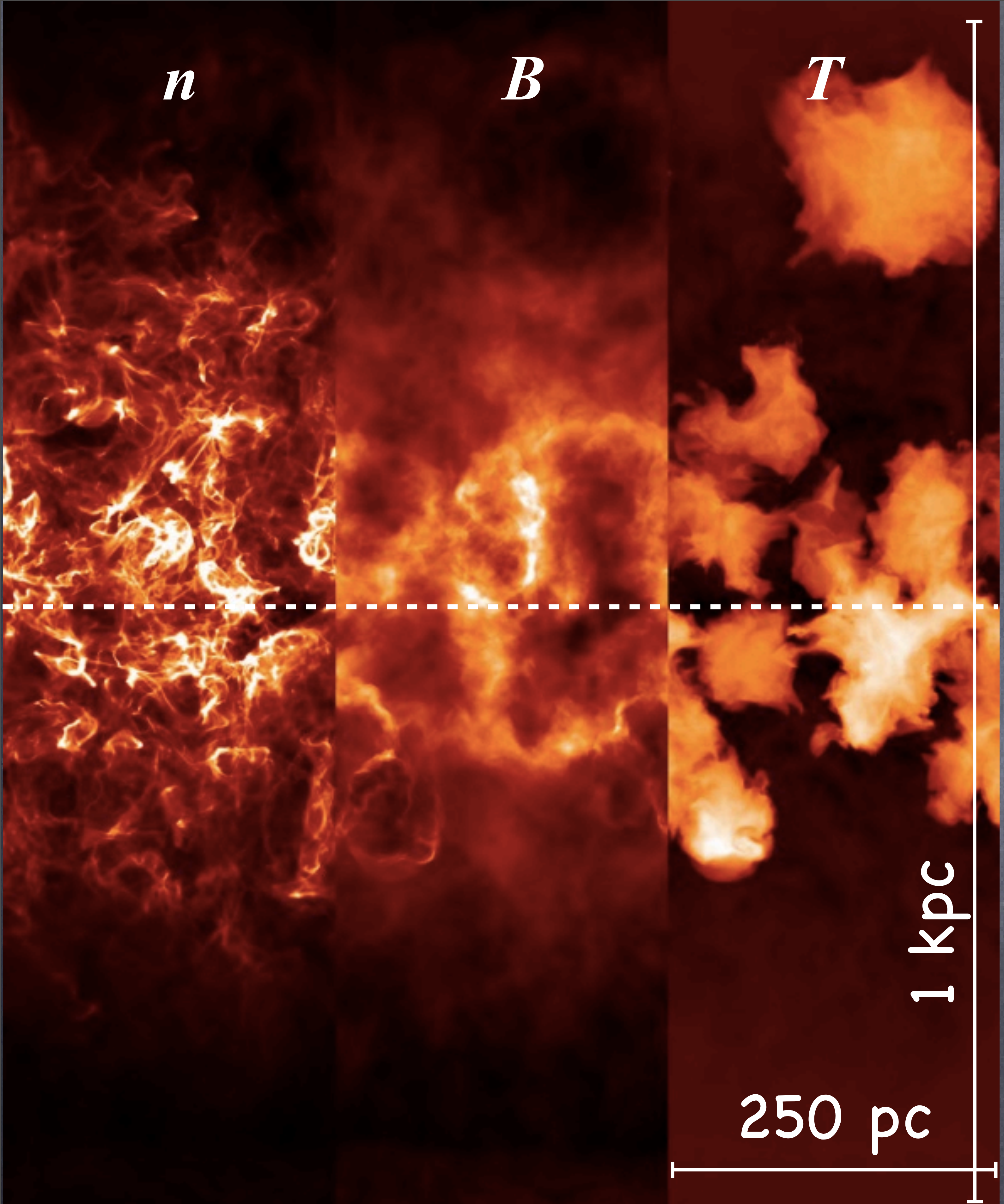
Supernova driving
Heating and cooling
Galactic potential
Self-gravity
Sink particles

Global models including the driving scale



A chunk of a galaxy:

Supernova driving
Heating and cooling
Galactic potential
Self-gravity
Sink particles

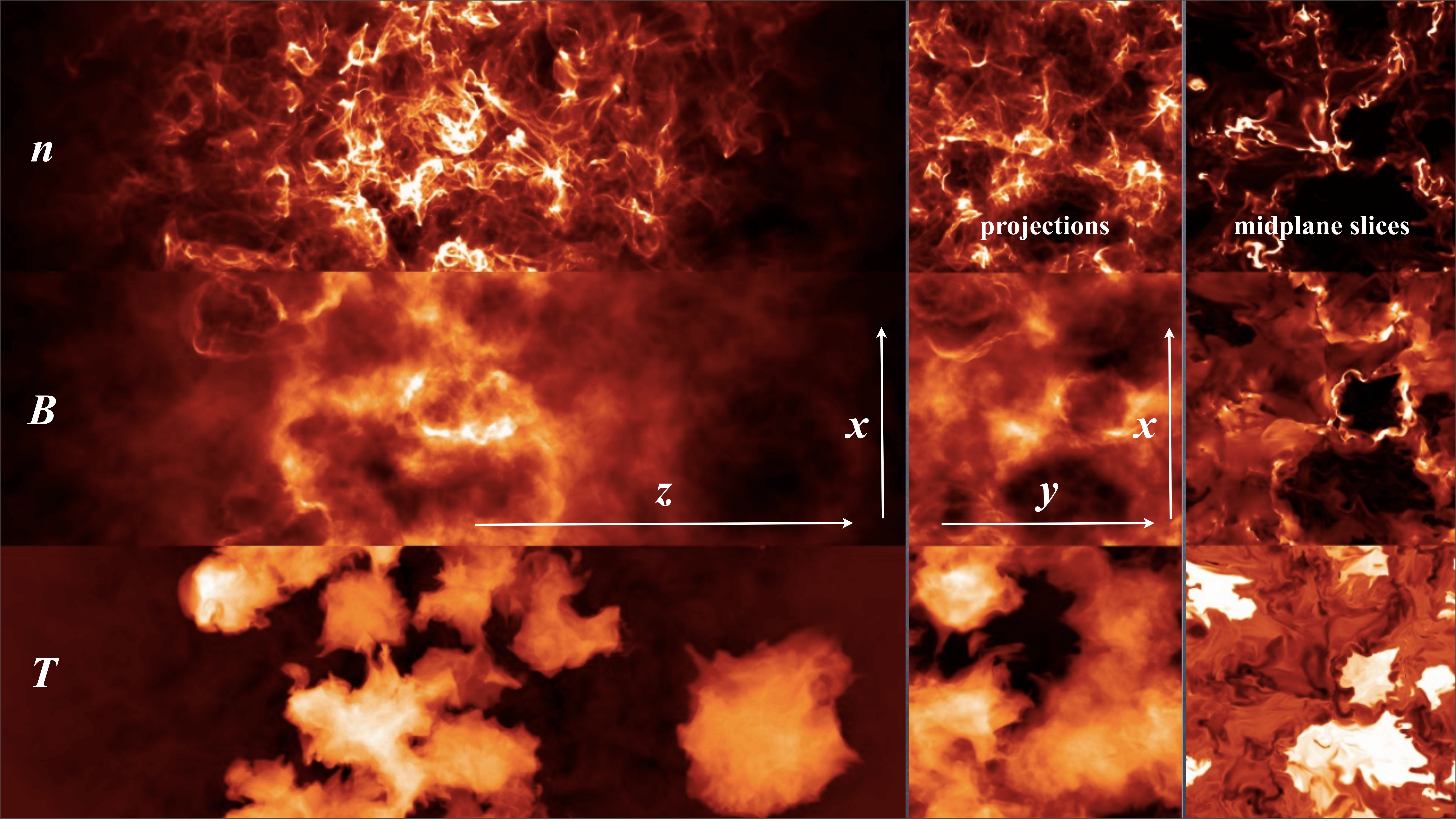


Range of scales: $1\text{-}32\times10^3$ pc – 10^{-2} pc

(17M cpu hr - PRACE grant on Supermuc)

Best so far: dx=2 pc (Hill et al. 2012)

- We can measure the SFR in many different clouds, with different $\alpha_{\rm vir}$, $\mathcal{M}_{\rm S}$, $\mathcal{M}_{\rm A}$
- We can derive the SFR law, SFR = f($\alpha_{\rm vir}$, $\mathcal{M}_{\rm S}$, $\mathcal{M}_{\rm A}$).
- We can test the scale invariance


n
B
T
projections
midplane slices
x
z
x
y

Conclusions

1. We understand how turbulent fragmentation leads to the observed low star formation rate, and we can model that
2. We have derived a local SFR law directly from local numerical models, with a parameter study based on many AMR simulations
3. The SFR per free-fall time has a simple exponential dependence on the virial parameter
4. This local law only depends on density and rms velocity of a star-forming region, and should be applicable to any scale below the driving scale, thus easy to use in modeling galaxy formation
5. Global numerical models including the SN driving scale will provide a necessary test for this law and its dependence on scale.