Lecture

Molecular cloud collapse II

Numerical Solution of Star Formation Equations

Thermal and density evolution in 1D case

(1st and 2nd hydrostatic core)

Observational signature of infall

Numerical Solution of Star Formation Equations

Protostellar Collapse

(Continuity equation, Momentum equation,
Poisson equation, Energy equation)

What are the problems: Scales, fragmentation, B loss

(1) Density needs to increase from molecular cloud density of $10^4 - 10^5\,cm^{-3}$ to $10^{24}\,cm^{-3}$ as mean solar density: 20 orders of magnitude

(2) Central temperature needs to increase from 10 K to 10^7 K in order to start fusion

(3) Specific angular momentum needs to decrease

	J/M (in $cm^2 s^{-1}$)
Molecular clump	10^{23}
Binary (P ~ 10^4 yr)	$4\times10^{20} - 10^{21}$
Binary (P ~ 10 yr)	$4\times10^{19} - 10^{20}$
Binary (P ~ 3 yr)	$4\times10^{18} - 10^{19}$
T Tauri star	10^{17}
Sun	10^{15}
Jupiter orbit	10^{20}

(4) Most of the stars are binaries or occur in higher-order hierarchical systems
→ Fragmentation

(5) Magnetic flux loss necessary
Dense core: $1\,M_{sun}$ $R_c = 0.07$ pc $B_c = 30\,\mu G$
T Tauri star: $R_1 = 5\,R_{sun}$

Flux freezing: $BR^2 = \text{const.}$

$\Rightarrow B_1 = 2 \cdot 10^7$ G

much larger than what is observed for T Tauri stars.

Next step

Formulation of all equations with the necessary material equations and boundary and initial conditions

A few remarks

- Protostellar evolution from an instable cloud core
  a) Dynamical collapse ($t \approx t_{ff}$)
  b) Accretion Phase after formation of hydrostatic core (accretion from envelope)

Pre-main sequence evolution: Energy source quasistatic contraction of core.

- Mathematical & physical problem
  a) Dynamical problem with dynamics on different scales
  b) Complex physics -> equations are non-linear
  c) Fragmentation must be treated (2D / 3D)

(a) CONTINUITY EQUATION (1)

$$\frac{d\rho}{dt} + \nabla\ (\ \rho\vec{v}) = 0$$

(b) MOMENTUM EQUATIONS (3)

$$\frac{\partial(\rho v_r)}{\partial t} + \nabla(\rho v_r \vec{v}) = -\left(\rho\frac{\partial\Phi}{\partial r} + \frac{\partial p}{\partial r}\right) + \frac{\rho}{r}(v_\Theta^2 + v_\Phi^2)$$

$$\frac{\partial(\rho v_\Theta)}{\partial t} + \nabla(\rho v_\Theta \vec{v}) = -\ \frac{1}{r}\left(\rho\frac{\partial\Phi}{\partial_\Theta} + \frac{\partial p}{\partial_\Theta}\right) - \frac{\rho}{r}\ (v_r\ v_\Theta\ -\ v_\Phi^2 \cot\Theta)$$

$$\frac{\partial(\rho A)}{\partial t} + \nabla(\rho A\vec{v}) = -\ \left(\rho\frac{\partial\Phi}{\partial_\Phi} + \frac{\partial p}{\partial_\Phi}\right)$$

$A = r \sin\Theta\ v_\Phi$ (specific angular momentum)

(c) POISSON'S EQUATION (1)

$$\nabla^2\Phi = 4\ \pi\ G\ \rho$$

(e) ENERGY EQUATION (1)

$$\frac{\partial(\rho e)}{\partial t} + \nabla(\rho e\vec{v}) + p\ \nabla\vec{v} = L$$

$L = 4\ \pi\rho\kappa\ (J - B)$: time rate of change of energy per unit volume due to radiative transfer

(f) RADIATION EQUILIBRIUM (1)

$$\nabla\vec{H} + \rho\kappa\ (\ J - B\ (T)\ ) = 0$$

(g) FLUX (EDDINGTON APPROXIMATION) (3)

$$\vec{H} = -\ \frac{1}{3\kappa\rho}\nabla\ J$$

Initial and boundary conditions

Standard initial condition (Larson 1969)

Homogeneous sphere with const. T and density which is at rest at t = 0

Standard condition + rigid rotation ω_i:

Isothermal rotating clouds

$$\alpha_i = E_i^{th} / E_i^{grav} = \frac{5}{2} \frac{R_i \, R T_i}{G M_i \, \bar{\mu}}$$

$$\beta_i = E_i^{rot} / E_i^{grav} = \frac{\omega_i^2}{4\pi G \rho_i}$$

Adiabatic rotating cloud : $p \sim \rho^{\gamma}$; α_i, β_i

Nonisothermal clouds : $\alpha_i, \beta_i, M_i, T_i$

Boundary conditions

2 equations of second order for ϕ and J require boundary condition

- Const. J oder J from const. temperature condition
- ϕ – no mass outside protostar

Mathematical Formulation and Solution

Coordinate System

Lagrangian or Eulerian Formulation of the problem

Eulerian Formulation: Numerical Diffusion has to be minimized

Lagrangian Formulation: No numeric diffusion
(no nonlinear advection terms)

- Very complicated for multi-dimensional processes (grid can be very distorted)
- Rezoning of grids (numerical diffusion re-introduced)

Application of particle methods (instead of grid methods)

Fluid divided in cells - "particles" - which move under the action of external forces and interact

"Smoothed Particle Hydrodynamics" - SPH

(Lucy 1977, Gingold & Monaghan 1977)

Fluid divided in discrete elements. These particles have a spatial distance ("smoothing length") over which their properties are smoothed by a kernel function.

$$A(r) = \sum_j m_j \frac{A_j}{\rho_j} W(|r - r_j|, h)$$

Kernel function

Advantages:
- Large density gradients can be treated
- Boundaries can be easily included
- Grav. interaction can be easily integrated

Grid-based methods (Resolution increase)

a) Adaptive grids
Number of grid points is constant; will be increased where strong gradients occur

b) Nested grids
Individual grid points will be split; number of grid points no longer constant.

Adaptive mesh refinement (AMR)
Dynamical gridding during simulation;
Starts with coarsely resolved Cartesian grid. Then individual cells are tagged for refinement.
(e.g. mass per cell should remain constant).

GMC simulations have reached 10^{-6} effective resolution per initial radius

Solutions - Explicit and implicit methods

$$\frac{\partial \vec{u}}{\partial t} = L\vec{u} \quad ; \quad \vec{u} = \vec{u}(\vec{r}, t)$$

(L- nichtlin. Operator)

$$\vec{u}^{n+1} = \vec{u}^{n} + L\vec{u}\,(1-\varepsilon)\,\Delta t + L\vec{u}^{n+1}\,\varepsilon\,\Delta t$$

ε - Interpolation parameter
$\varepsilon = 0$ explicit solutions (Time step limitation)
$\varepsilon \neq 0$ implicit solutions (System of nonlin. algeb. equ.)

First numerical "solution" - 1D (Larson 1969)

$1 M_\odot$ $\rho(t=0) = \rho_0 = 10^{-19} g\,cm^{-3}$

$T(t=0) = T_0 = 10 K$

Result: Non-homologous evolution
(density in outer regions $\rho(r) \sim r^{-2}$)

- After t_{ff} - Formation of hydrostatic core
and free-falling envelope ($\rho(r) \sim r^{-3/2}$)

$\rho_c = 10^{12} - 10^{13} cm^{-3}$ $T_c \approx 100 K$

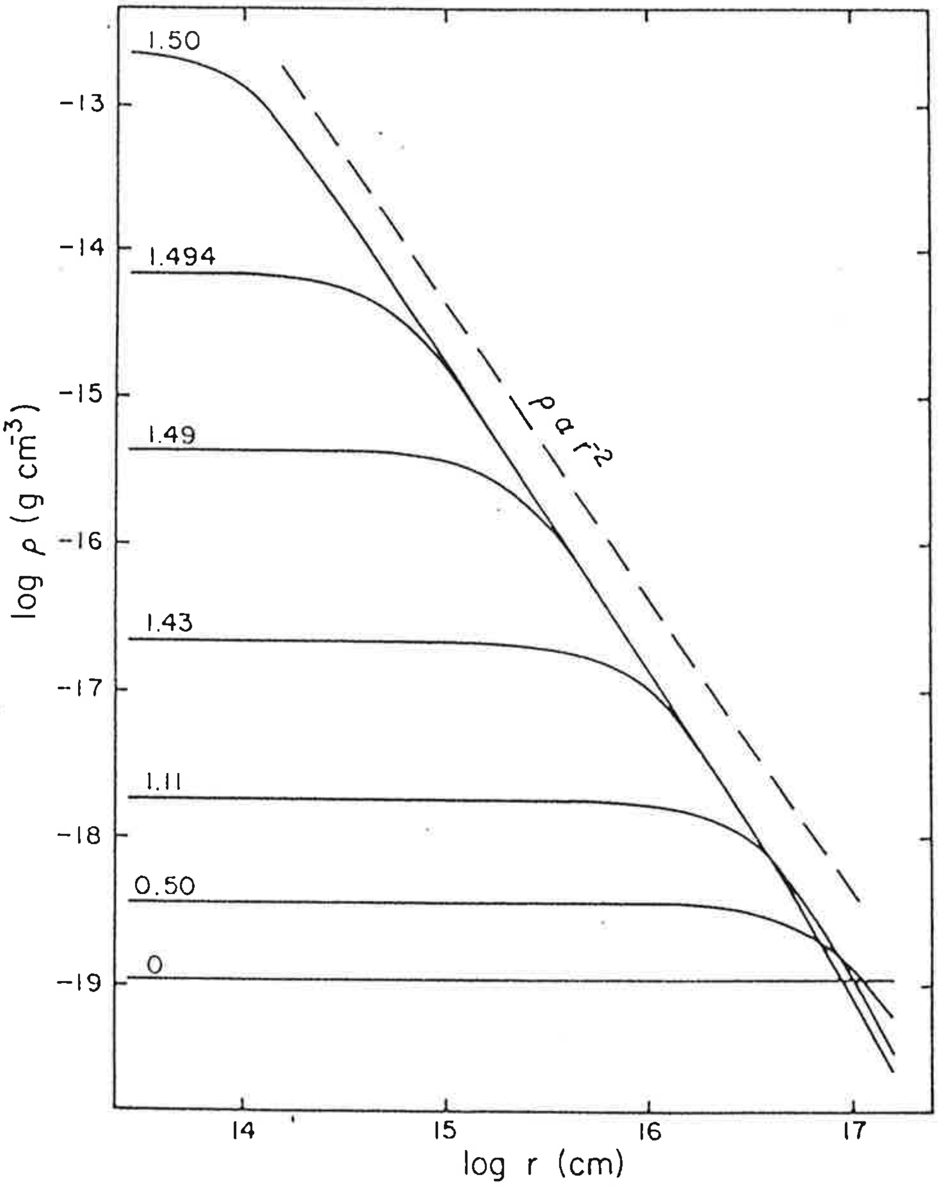
- Contraction of the core until 2000 K
is reached and $\rho_c = 10^{17} cm^{-3}$ →
Dissociation of molecular hydrogen.
(endothermic reaction; central region starts to
collapse again.

- Formation of a second hydrostatic core
at $\rho_c = 10^{23} cm^{-3}$, $T_c = 10^4 - 10^5 K$

((Core still accretes matter which goes
through shock front)

At the end: $R = 2.1 R_\odot$ $L = 1.5 L_\odot$

((Confirmed by Winkler & Newman: $2.0 R_\odot$, $1.0 L_\odot$)
(1980)



Evolution of the density distribution of a protostar of 1 solar mass, starting with $T = 10$ K and a uniform density of 1.1×10^{-19} g cm^{-3}, during isothermal collapse. The curves are labelled with the time, in units of the initial free-fall time, from the beginning of the calculation. After Larson [27].

Thermal Evolution of Cloud (1D)

A&A proofs: manuscript no. Paper

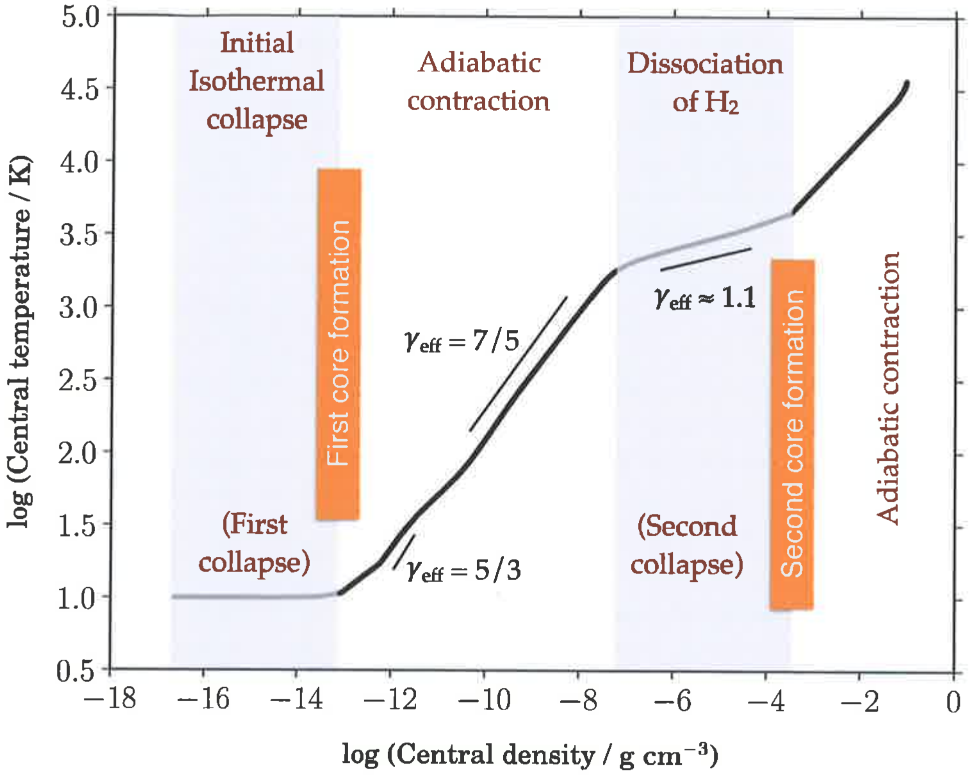


Fig. 3: Thermal evolution showing the first and second collapse phase for a 1 $M_\odot$ cloud at an initial temperature of 10 K. The change in effective gamma $\gamma_{\rm eff}$ indicates the importance of using a more complex gas equation of state.

a) Cloud collapses under own gravity
(optically thin, isothermal)

b) Cloud becomes optically thick / heats up
$(10^{-10}\ g/cm^3)$
takes $\approx 10^4$ yrs ; $R_c \approx 3$ AU , $M_c \approx 3\times10^{-2} M_\odot$

Temperature Estimate $(W = -2U)$

$$-GM^2/R = -3\, M R_g T/\mu$$

$$T = \frac{\mu}{3R_g}\,\frac{GM}{R}$$

$$= 850\,K \left(\frac{M}{5\times10^{-2}M_\odot}\right)\left(\frac{R}{5\,AU}\right)^{-1}$$

c) Once the temperature reaches 2000 K
H_2 molecule begins to dissociate

($\gamma_{eff} = 1.1 < \gamma_{eff} = 4/3$ for stability)

Note thermal energy per H_2 molecule is small compared to dissoc. energy

$$U = \frac{3}{2} \frac{R_g \cdot T}{\mu} M \quad ; \quad N = \frac{X \cdot M}{2 m_H}$$

$\Rightarrow$ Energy / Molecule $= 3 k_B T / X =$
0,74 eV

Dissociation energy is 4,48 eV

(Compression energy goes in dissociation
$\rightarrow$ no longer increase of T)

d) After all hydrogen is dissociated
$\rightarrow$ Second hydrostatic core forms

$R \approx 1.8 \times 10^{-2}$ AU $\quad M \approx 4.6 \times 10^{-3} M_\odot$

Accretion of gas onto protostar

Gas reaches star with free-fall speed which causes an accretion shock front ($T > 10^6$ K, UV + X-rays)

$$L_{acc} = G M_* / R_* \, (dM/dt)$$

$$= 61 L_\odot \, (dM/dt \, / \, 10^{-5} M_\odot/yr) \cdot (M_* / 1 M_\odot) \, (R_* / 5 R_\odot)^{-1}$$

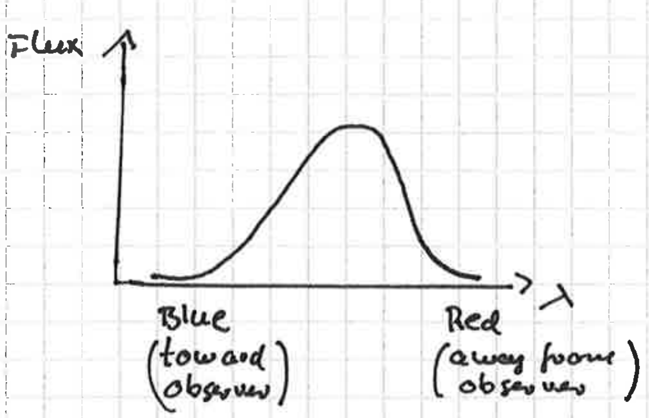
Additional energy from contraction and early nuclear fusion are negligible compared to L_{acc} for low- to intermediate-mass stars

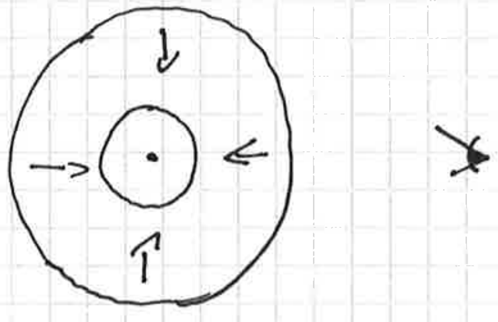
Def.: Low-mass protostar - Mass gaining star with L from accretion shock screened by an envelope

- Opacity gap (no dust)
- Inner dust sublimation radius (~1 AU)
- Effective warm radiating surface observable at MIR wavelength ("dust photosphere") (few AU)
- Outer optically thin envelope (100-1000 AU)

Line profile of collapsing cloud

(1)

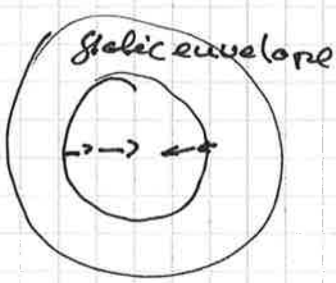




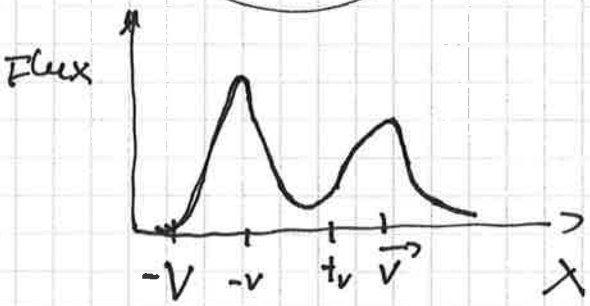
Temperature & Intensity in center is maximum

(optically thin emission)

(2) Profile from core (collapsing) with static envelope



Inside-out-collapse: $v(r) \sim r^{-0.5}$ (free-falling) inside a static envelope

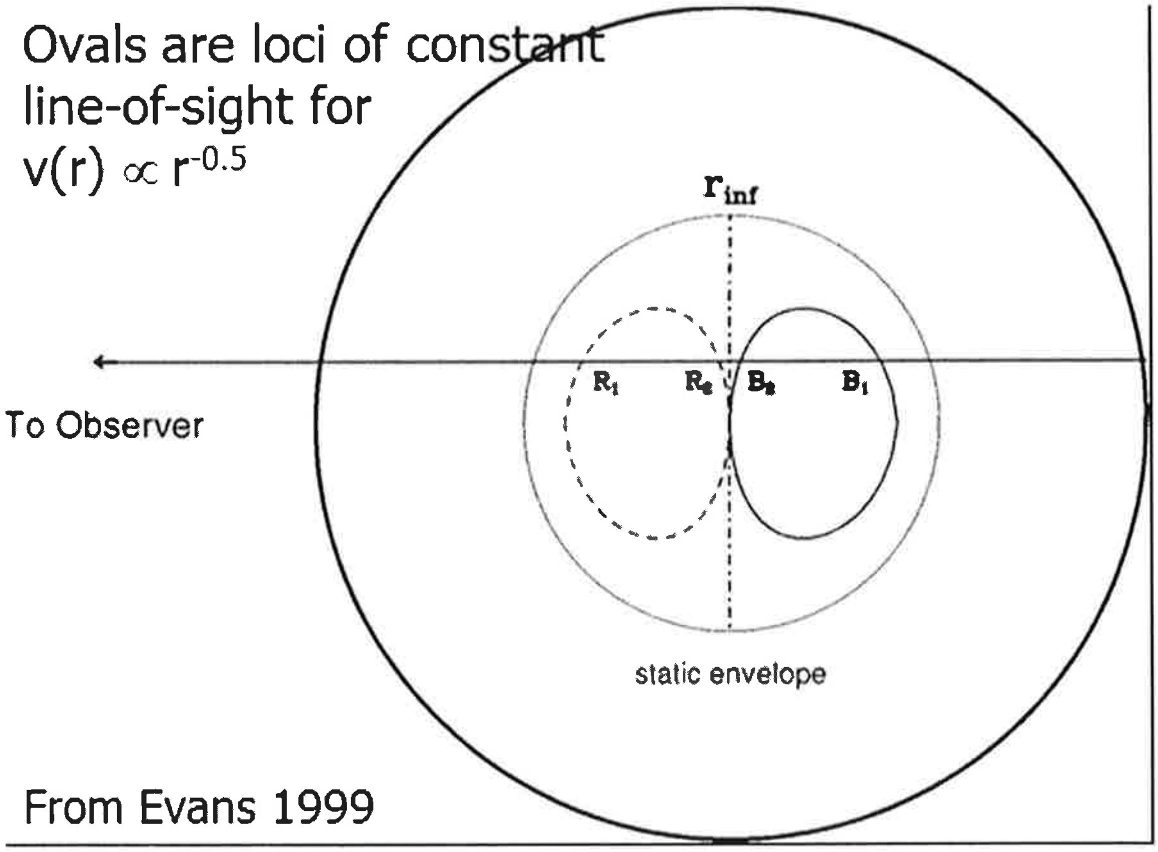


Redshifted absorption (absorption only on observer's side)

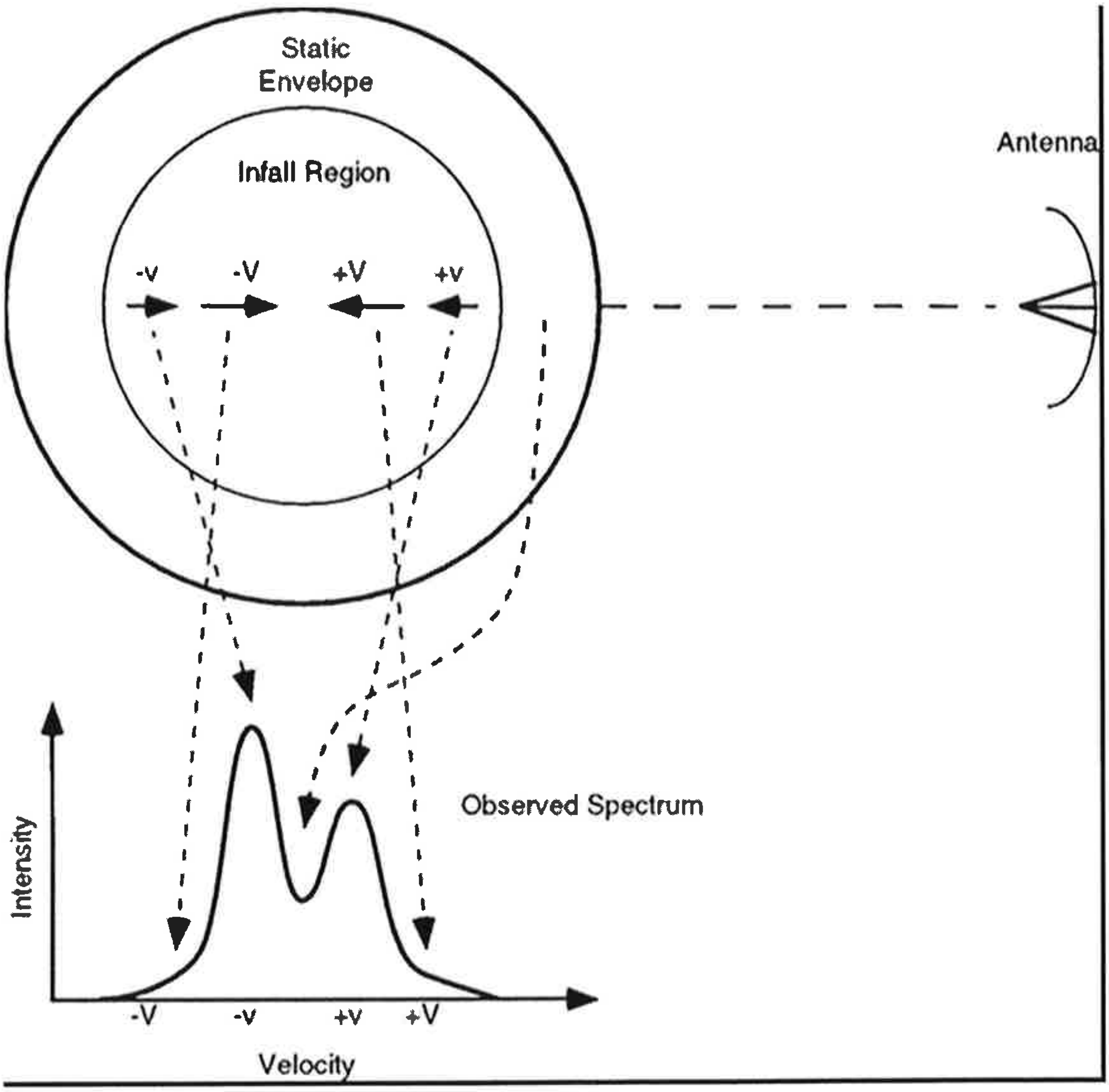
good example B 335.

(always opt. thin + opt. thick line to be observed)

Skewed to the blue. Each line of sight intersects the loci of constant line-of-sight velocity at two points. Points closer to center have higher Tex. Point R1 will obscure R2, but B2 lies in front of B1.



Ovals are loci of constant line-of-sight for $v(r) \propto r^{-0.5}$
r_{inf}
To Observer
R_1
R_2
B_2
B_1
static envelope
From Evans 1999



Static Envelope
Infall Region
Antenna
-v
-V
+V
+v
Observed Spectrum
Intensity
-V
-v
+v
+V
Velocity