Protostellar Evolution

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Structure of protostar inside accretion shock front can be approximated by stellar structure equations with boundary conditions at accretion shock given by infalling material

Equations Spatial variable M_r (shell mass inside r)

$$M_r = \int_0^r 4\pi r^2 \rho \, dr$$

- $$\boxed{\frac{\partial r}{\partial M_r} = \frac{1}{(4\pi r^2 \rho)}} \qquad (1)$$

- Hydrostatic Equilibrium

$$-\frac{1}{\rho} \operatorname{grad} p = \operatorname{grad} \phi$$

$$\frac{\partial p}{\partial r} = -\rho \frac{G M_r}{r^2}$$

Re-write in M_r coordinate

$$\boxed{\frac{\partial p}{\partial M_r} = -\frac{G M_r}{(4\pi r^4)}} \qquad (2)$$

$$P = \frac{\rho}{\mu} R T \qquad \text{(ideal gas equation)}$$

- Radiation Transport Equation (Diffusion Equation)

$$\boxed{T^3 \frac{\partial T}{\partial r} = \frac{-3\,\varkappa\, L_{int}}{256\,\pi\,\sigma_B\, r^4}} \qquad (3)$$

- Spatial variation of L_{int} (Heat equation)

$$\boxed{\frac{\partial L_{int}}{\partial r} = \varepsilon(\rho, T) - T\, \partial s/\partial t} \qquad (4)$$

$\varepsilon(\rho, T)$ - rate of nuclear energy release per unit mass

$s(\rho, T)$ - entropy per unit mass of fluid

(For a mono-atomic gas, the entropy

$s(\rho, T) = R/\mu \ln(T^{3/2}/\rho) + s_0$

Equations for r, P, T, L_{int} +

Introduce boundary conditions

$r(0) = 0$ $\qquad$ $L_{int}(0) = 0$

$P(r_*) = P_{Ram}$ $\qquad$ (ram-ram pressure of infalling gas)

$L_* = L^t_{int} + L_{acc}$

↳ L_{int} from $r=0$ to r_*

Note: $\dot{M}$ enters the boundary conditions (should come from collapse calculations)

Often taken as "free" parameter: 10^{-6} to $10^{-5}\, M_\odot\, yr^{-1}$

Mass-Radius Relation

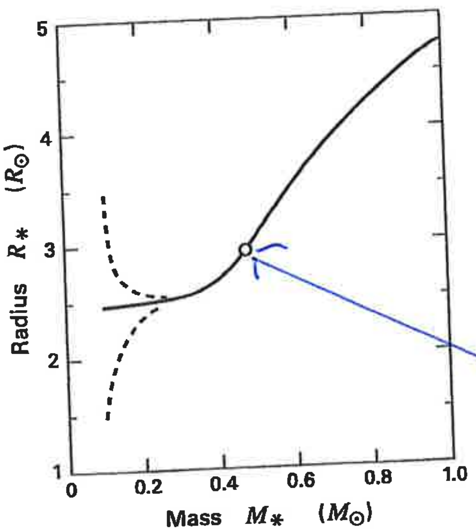
Initial size unknown but quickly converges

(a) Initially large → low infall velocity → low L_{acc} → low $s(M_r)$ → initial decrease of R_* (Opposite effect for small initial state)

(b) In later evolution mass is added; specific entropy represents the heat content of the associated mass shell; increase of s with M_r results in a protostar that swells as mass is added.
($s(M_r)$ arises naturally because with rising M_* velocity of the infalling gas and hence accretion shock gets stronger and L_{acc} increases)

$$\boxed{\text{Protostar Radius increases with time. (determined by boundary condition)}}$$

Mass-Radius Relation ($\dot{M} = 10^{-5} M_\odot \, yr^{-1}$)



fully convective interior

Palla & Stahler (2004), The Formation of Stars. p. 330

Onset of Convection

An object with $s(M_r)$ as increasing function is convectively stable.

($\partial s / \partial M_r > 0$ Schwarzschild criterion)

For ordinary gases $(\partial \rho / \partial s)_P < 0$

$\Rightarrow$ Density falls with increasing entropy at constant pressure in external medium

A rising gas parcel has a higher ρ_{int} than ρ_{ext} and will fall again

$\Rightarrow$ $\boxed{\text{Object is convectively stable}}$

In course of evolution M_* / R_* increases which also leads to an increase of the temperature T (last lecture $T \approx \mu / 3R_g \; GM/R$)

Nuclear reactions (deuterium burning) begins near center. $\Rightarrow$ Convection begins because deuterium fusion produces too much L to be transported radiatively through opaque interior: $\partial s / \partial M_r < 0$

(diffusion equation no longer valid)

Deuterium burning

$^2H + {}^1H \rightarrow {}^3He + \gamma$

$\Delta E_D = 5.5$ MeV ; is highly temperature sensitive ; D fusion becomes important for 10^6 K.

Deuterium Thermostat

- Deuterium burning at 10^6 K pumps a lot of energy into the star -> leads to swelling -> Lowers T_c -> lowers deuterium burning
- Steady supply of new D from infalling gas via convection necessary to maintain thermostat
  For sufficiently high $\dot{M}$ (10^{-5} $M_\odot$/yr) R_* proportional to M_* and set by "protostellar" physics

Short consideration

Fully convective star -> For R_*, M_* obtain T_c:

$T_c = 0.54\ G M_* \mu / k R_*$

(Chandrasekhar 1939; Introd. to Stellar Structure)

- Completely ionised gas $\mu = 0.62\, m_H$
- $T_c = 1 \times 10^6$ K

Relation for $0.01\, M_\odot \leq M_* \leq 2\, M_\odot$

$R_* / R_\odot \approx 0.15 + 7.6\ (M_* / M_\odot)$

(partly degeneracy considered)

↳ Theoretical birthline

- If accretion stops object of a certain mass has a certain radius, relatively independent of details of star formation
- $\dot{M}$ determines start of D burning, but is not responsible for relation

Evolution for objects with $M_* > 2M_\odot$

a) Formation of a radiative barrier and end of central hydrogen burning

- Matter added to core and inner T increases; Opacity mostly from ff emission (Kramers opacity) scales with $\varkappa_{ff} \propto \rho\, T^{-7/2}$ → strong decrease with T; radiation can transport energy again
- Critical L_{crit} (maximum value to be carried by diffusion) is

$$L_{crit} \sim M_*^{11/2}\, R_*^{-1/2} \qquad \text{(Palla p. 348)}$$

For growing protostar L_{crit} sharply rises and gets larger than L_{int}

⇒ $\boxed{\text{Convection disappears and protostar gets radiative barrier}}$

Deuterium Shell Burning

- As long as accretion continues → D accumulates in a shell ; M_*/R_* continues to rise → T increases → Base of D mantle reaches 10^6 K → D burning starts

- D burning in shell injects heat and rises the entropy S of the outer layers → further swelling of the protostellar radius

If we continue to add mass → Convection & swelling gradually disappears → Star becomes nearly fully radiatively stable

$$\boxed{L_{crit} = L_{int} \approx 1\, L_\odot \left(\frac{M_*}{1 M_\odot}\right)^{11/2} \left(\frac{R_*}{1 R_\odot}\right)^{-1/2}}$$

(radiative protostar)

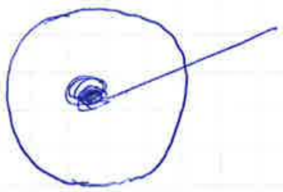
- For $M \gtrsim 3 M_\odot$ interior $L \sim 100 L_\odot$ (higher than shell burning or L_{acc})

- Between 5 and 6 $M_\odot$ ⇒ $L \sim 10^3 L_\odot$

Luminosity comes from gravitational contraction!

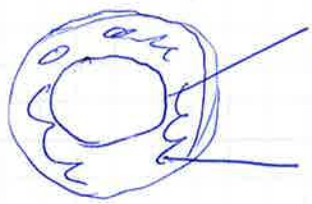
Summary of stages

a)



Central burning; fully convective

b)

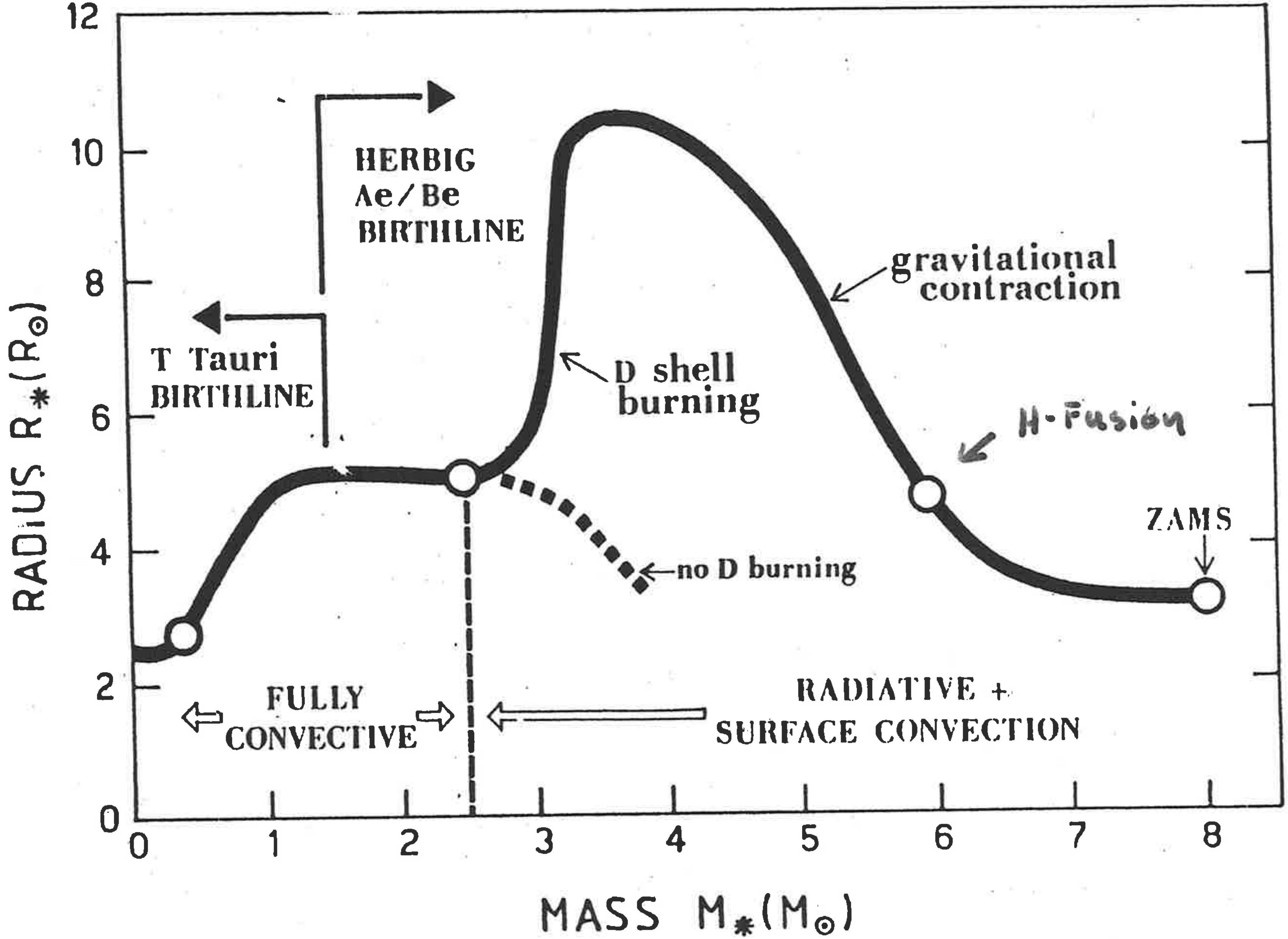


Radiative barrier

Shell burning / convection

Gravitational contraction phase

- when more mass added to star $L_{rad} \sim M_*^{11/2} R_*^{-1/2}$ increases rapidly through contraction; Protostar reaches relaxed state and homologous contraction.
- T increases up to 10^7 K → hydrogen burning starts; ZAMS reached at about $8\,M_\odot$
- Stars with $L_{rad} > L_{acc}$ ($M_* \approx 4 M_\odot$) occur directly on radiative tracks (relaxation in protostar phase)


HERBIG
Ae/Be
BIRTHLINE
T Tauri
BIRTHLINE
D shell
burning
gravitational
contraction
H-Fusion
ZAMS
no D burning
FULLY
CONVECTIVE
RADIATIVE +
SURFACE CONVECTION
RADIUS R$_*$ (R$_\odot$)
MASS M$_*$ (M$_\odot$)
0
2
4
6
8
10
12
0
1
2
3
4
5
6
7
8