The Sundiver

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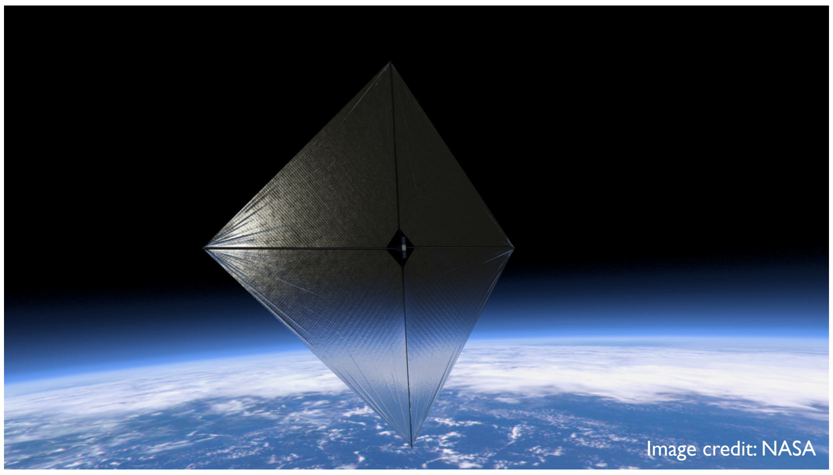


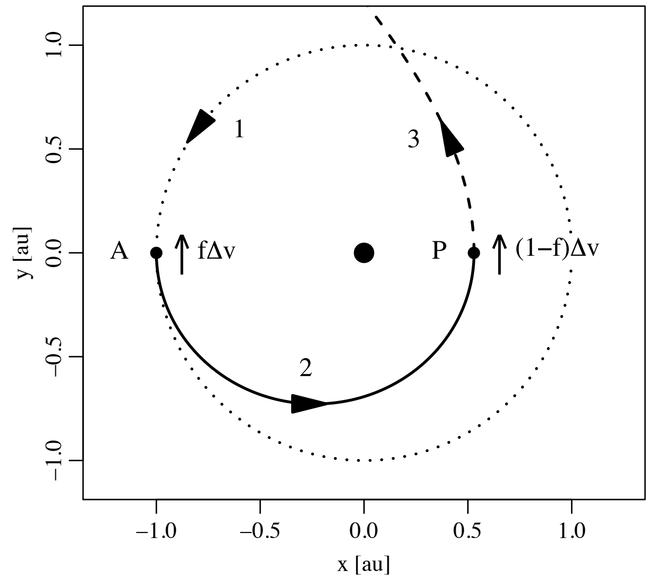
1. Goal

You are in a spacecraft in a circular orbit about the Sun and want to leave the solar system. Your spacecraft has engines that deliver a change in velocity of Δv, and a solar sail of lightness number λ. How can you use these to maximize your velocity at infinity?

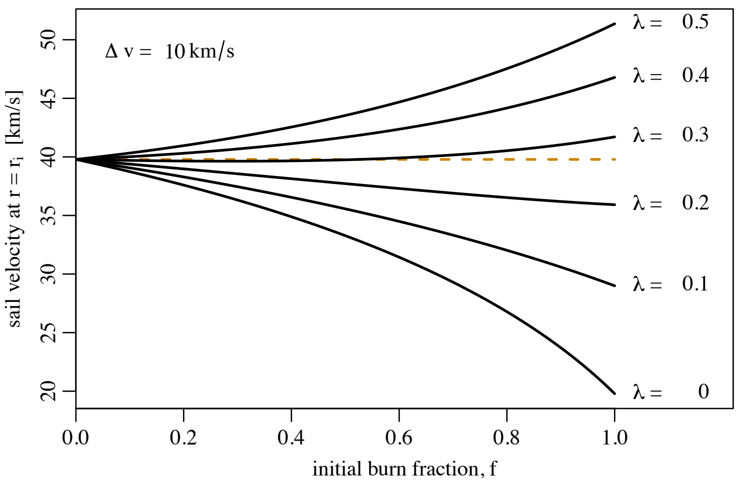
2. Solution

It seems obvious to open the sails and to use all the Δv to boost away from the Sun. But a sail is more effective the closer it is to the Sun, plus a given Δv is more efficient the faster the spacecraft (the *Oberth effect*). So it might make more sense to first use some Δv to dive closer to the Sun. But how much?

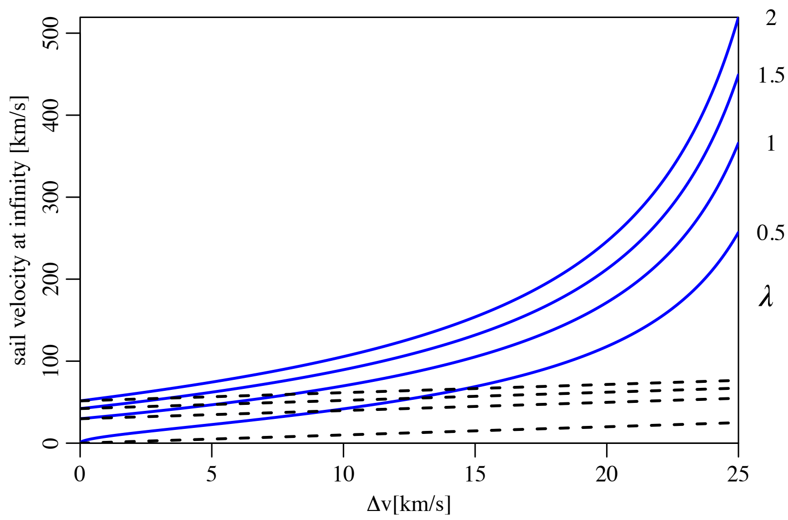
To solve this analytically we keep the sail pointed at the Sun, which ensures that orbits are Keplerian. As shown on the right, the spacecraft starts off (1) with r_i = 1 au (and so velocity v_i = 29.8 km/s). A fraction f of the fuel is then used in a retrograde burn at A to dive (2) towards the Sun (*Hohmann transfer orbit*). At perihelion (P) the sail is deployed and the rest of the fuel is used in a prograde burn to escape (3). The objective is to find the value of f that maximizes the velocity at infinity (equivalently at $r = r_i$) for given λ and Δv.



3. Results



The figure above shows how the velocity of the spacecraft back at r = 1 au varies with f for various λ for Δv = 10 km/s. If λ is small we should use f=0, i.e. no dive. But for larger λ we should use all the propellant to do the deepest dive possible. The figure below shows the velocity at infinity as a function of Δv for various λ. The solid blue lines show the full dive (f=1); the dashed lines the corresponding results for no dive (f=0).



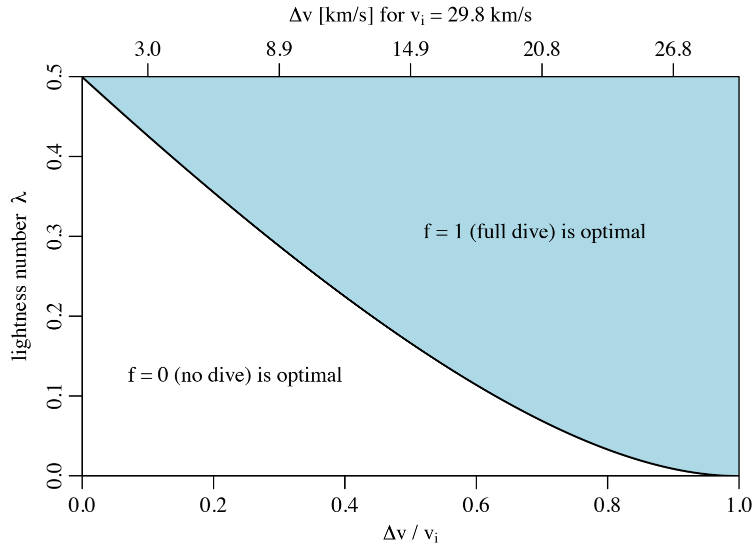
Solar sails

A solar sail is propelled by the pressure of photons from the Sun. It is characterized by its *lightness number*, the ratio of the radiation acceleration to the gravitational acceleration

$$\lambda = \frac{L_\odot}{2\pi G c M_\odot \sigma} = \frac{1.5\times10^{-3}}{\sigma\,[kg\,m^{-2}]}$$

where σ is the mass per unit surface area of the sail spacecraft. If the sail is pointed at the Sun and λ=1 (in which case σ is a tenth the value of plastic food wrap), then the forces cancel.

4. Conclusion



We should only ever use f=0 (no dive) or f=1 (deepest dive). Which to do depends on what λ and Δv we have available, as shown above. This applies for any size initial orbit. In practice the depth of the dive is limited by not hitting the Sun. Furthermore, using more burns, or pointing the sail away from the Sun (to produce non-Keplerian orbits) can achieve larger final velocities (but take longer to achieve).

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