What models can do for you

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Outline

- The role played by models in building understanding
- Ingredients of a model
- Models absolutely key for studies of MW
- Recent work on models of the MW's discs

What do we see?

- The more luminous stars
- We measure their
  - Positions & velocities
  - Densities in real & velocity space
- Without some hypothesis these measurements tell us nothing
  - Any distribution of stars in phase space is lawful
- Physics allows us to deduce something once we have a hypothesis
  - That what we observe is enduring/typical

The hypothesis

- It's a steady state (possibly in a rotating frame)
  - Jeans theorem
  - Jeans equations
  - Virial theorem
- The fluctuations are statistically stationary
  - Two-body scattering
  - Scattering by molecular clouds / spiral arms
- ¤CDM is correct and SPH really works
- The system is DM/baryon dominated
- Etc
- Hypotheses all start from outline concepts and proceed to numerical models

The game

- To frame a plausible hypothesis that's maximally restrictive
- To build a numerical model that makes quantitative predictions
- Typically there will be predictions for photometry / star counts / spectroscopy / kinematics so a successful model encapsulates results from many studies
- Adjust the model's parameters & discover which (if any) parameter range is compatible with the data

Open issues

- How to fit model to data?
  - How much pre-processing of data
  - Do we fit º(x) or N(J,J-K,..)?
  - Do we fit ¾, h_3, h_4 or LOSVD or spectra?
- How closely should the model fit the data?
  - Opportunity to identify noise
  - Is the model with the smallest $Â^2$ really the best (Mogorrian 2006)
  - Marginalisation over nuisance parameters, e.g., when determining BH mass
- How to learn from mistakes?
  - What do I conclude from a poor fit to the data?
  - understanding the landscape of a high-d space

Ingredients of the model

- ©(x) – on account of DM can only be constrained by models
- f(x,v) in some form
  - f(J)
  - $\{w_i\}$
  - closure hypothesis
  - N-body model
- Library of stellar atmospheres/spectra plus Hess diagram or equivalent
  - List of stellar pops with history of SFR
  - Chemical and dynamical modelling inextricably linked

Modelling the MW

- Data characterised by
- Exceptionally strong observational biases
- Large redundancy

Observational bias

- low-L stars will only be seen near the Sun
  - A major problem for N-body models (Brown Velasquez & Aguilar 2005)
  - Particles must be interpreted as groups of stars with varying m
  - Nearby groups will have many members with same (x,v)
- Most interesting regions obscured in V
  - Must model ISM/dust in parallel with stars

Redundancy

- We gather 6d data for many dynamically independent populations
- We seek
  - ©(x)
  - For each population $f(I_1, I_2, I_3)$
- If we see stars at (x_1,v_1) there must be a computable # of stars at (x_2,v_2) on same orbit
- We can guess the ages of the populations from stellar physics & all populations have evolved in the same fluctuating ©

Models with f(J) are the key

- Actions are the only integrals worth considering
- There are several ways to estimate them from (x,v)
- Analytic DFs f(J) do a remarkably good job of
  - Accounting for given data
  - Predicting subsequent data

Worked example: basic models of the discs

- © from Dehnen & Binney 1998
- Thin/thick & gas discs, bulge & dark halo
- DF a superposition of quasi-isothermal discs

$$f(J_r, J_z, L_z) = f_{\sigma_r}(J_r, L_z) f_{\sigma_z}(J_z),$$

where f_{σ_r} and f_{σ_z} are defined to be

$$f_{\sigma_r}(J_r, L_z) \equiv \left.\frac{\Omega\Sigma}{\pi\sigma_r^2\kappa}\right|_{R_c} [1 + \tanh(L_z/L_0)] \mathrm{e}^{-\kappa J_r/\sigma_r^2}$$

and

$$f_{\sigma_z}(J_z) \equiv \frac{\nu}{2\pi\sigma_z^2} \mathrm{e}^{-\nu J_z/\sigma_z^2}.$$

Superpositions of quasi-isothermals

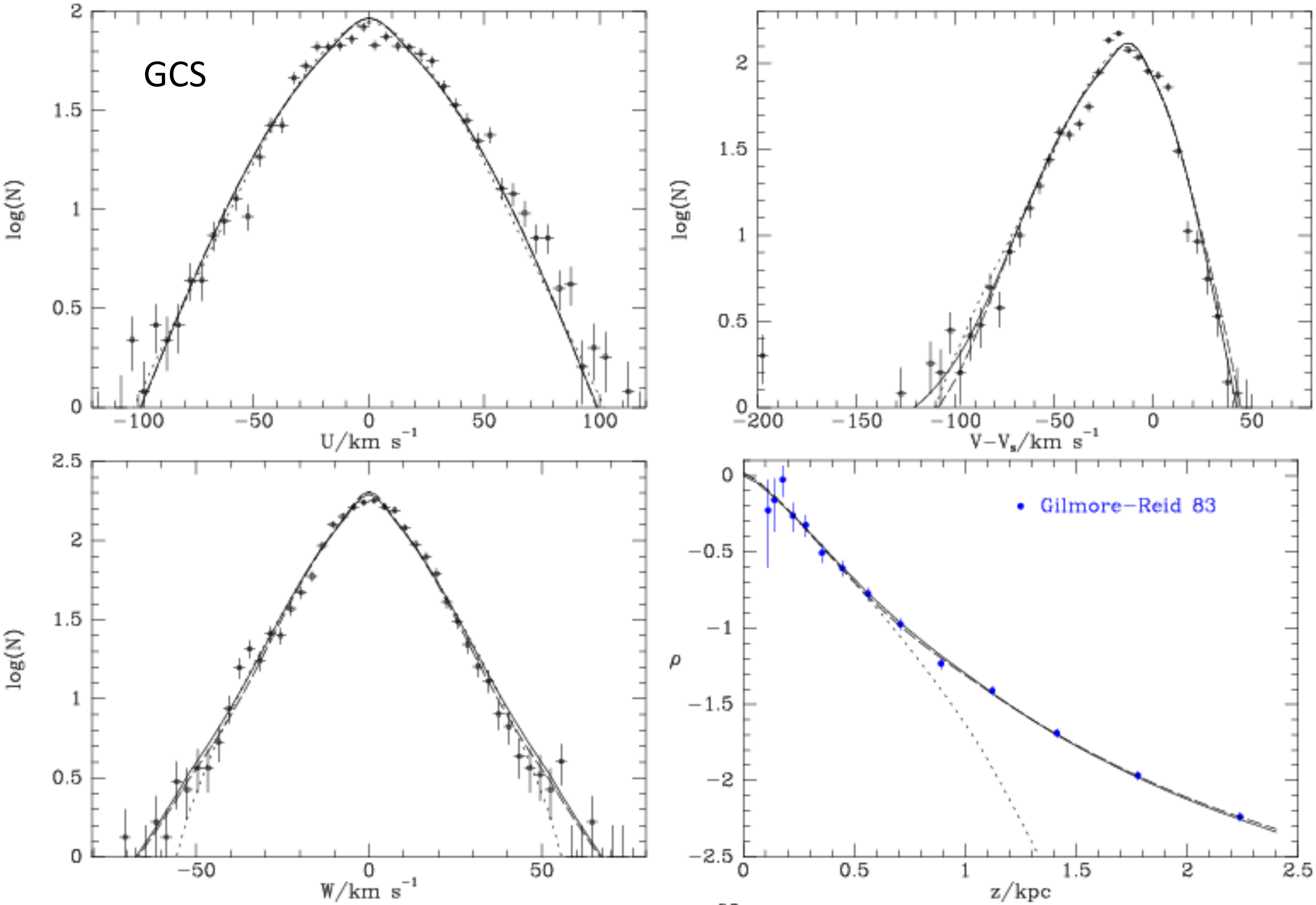
- Thick disc a single quasi-isothermal
- Thin disc one for each co-eval cohort:

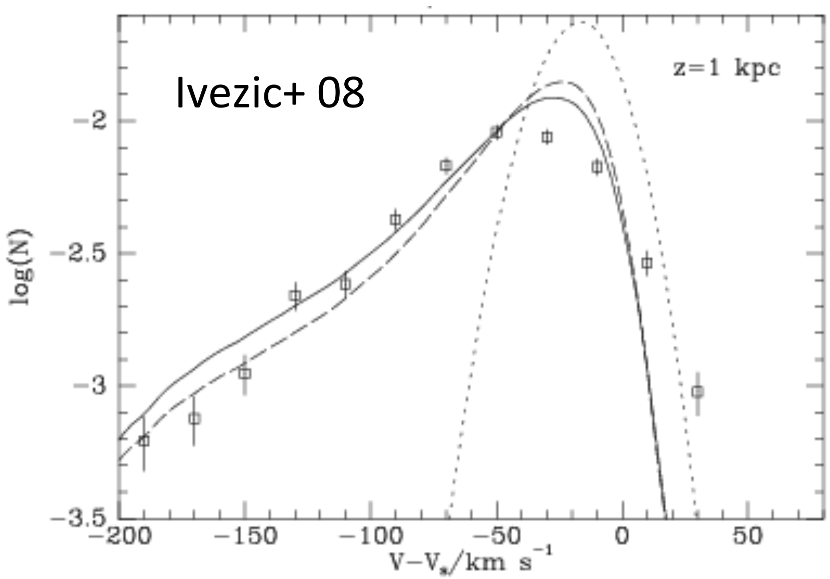
$$f_{\rm thn}(J_r, J_z, L_z) = \frac{\int_0^{\tau_{\rm m}} {\rm d}\tau\, {\rm e}^{\tau/t_0} f_{\sigma_r}(J_r, L_z) f_{\sigma_z}(J_z)}{t_0({\rm e}^{\tau_{\rm m}/t_0} - 1)}$$

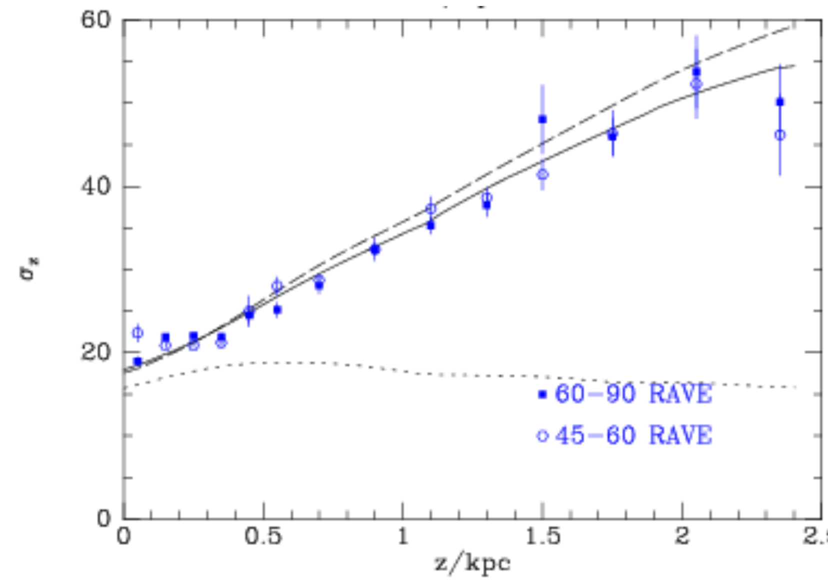
$$\sigma_r(L_z,\tau) = \sigma_{r0}\left(\frac{\tau+\tau_1}{\tau_{\rm m}+\tau_1}\right)^{\beta} {\rm e}^{q(R_0-R_{\rm c})/R_{\rm d}}$$

$$\sigma_z(L_z,\tau) = \sigma_{z0}\left(\frac{\tau+\tau_1}{\tau_{\rm m}+\tau_1}\right)^{\beta} {\rm e}^{q(R_0-R_{\rm c})/R_{\rm d}}$$

- Adopt ¿$_1$= 0.01 Gyr, ¿$_m$= 10 Gyr, ¯=0.33
- Let *amoeba* Fit ¾$_{r0}$ ¾$_{z0}$ R$_d$ q for each disc & fraction of stars in thick disc

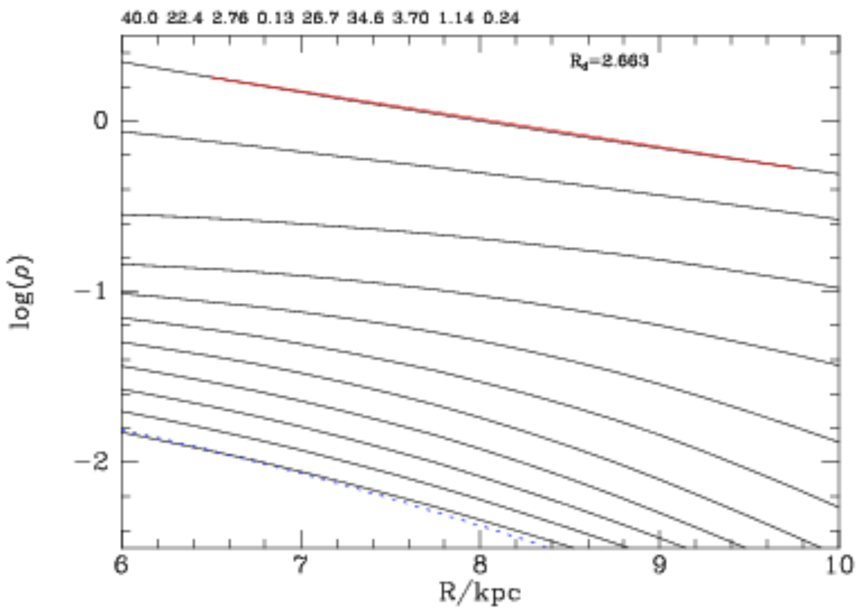

GCS
log(N)
U/km s⁻¹
V−V$_s$/km s⁻¹
W/km s⁻¹
ρ
z/kpc
Gilmore−Reid 83

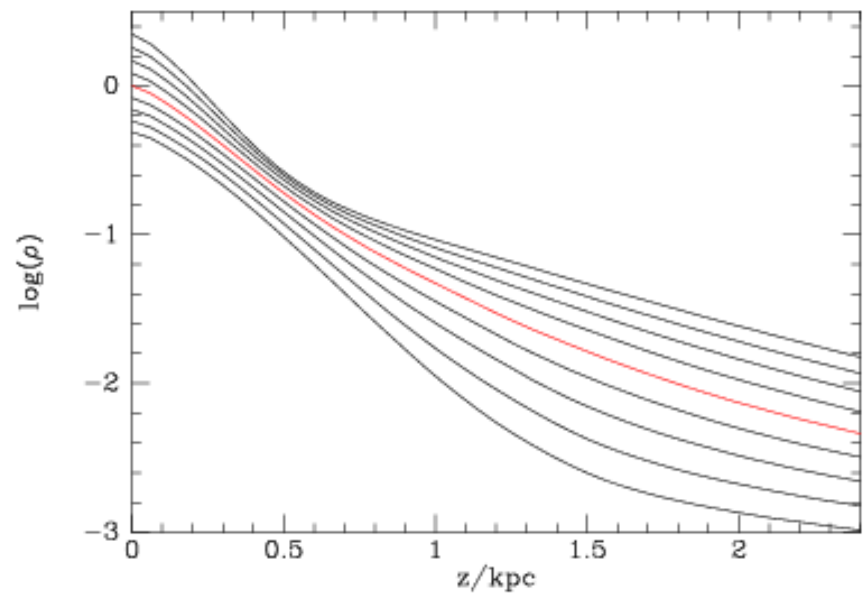




Predicted fits to SDSS and RAVE data (Ivezic 2008 & Burnett 2010)

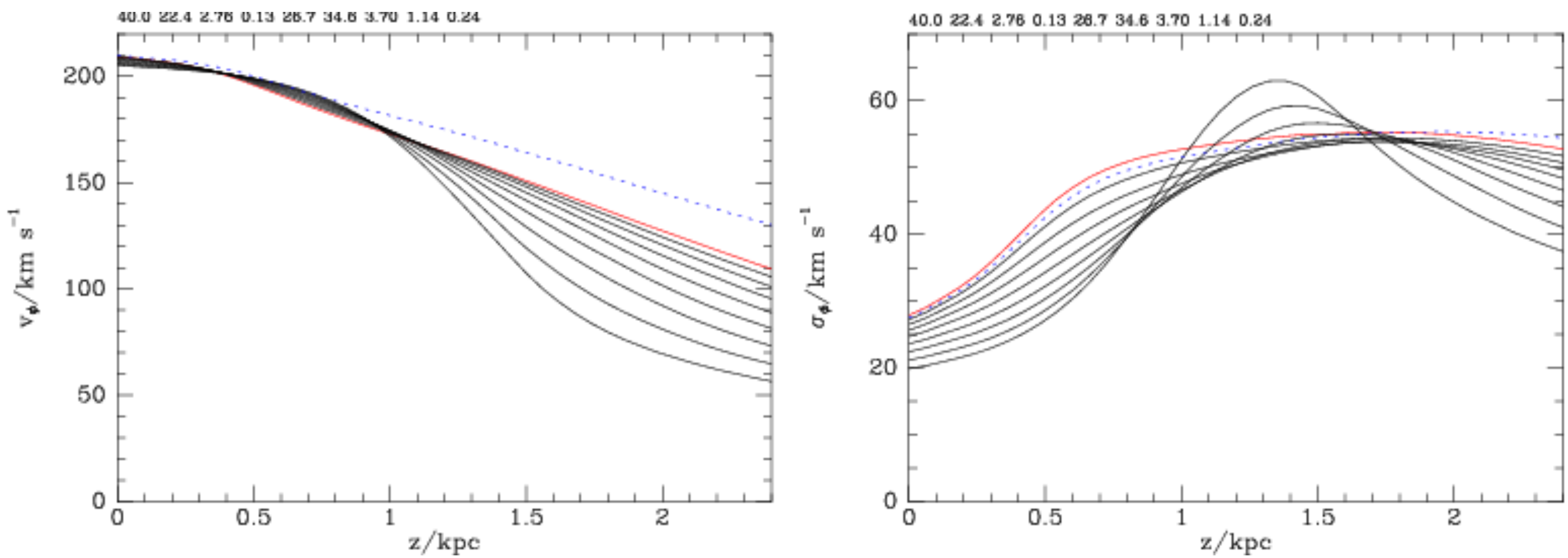
Global structure





Thin & thick discs individually exponential
But vertical scaleheight of thin disc $z_0(R)$
Results in rapid variations in $R_d(z)$

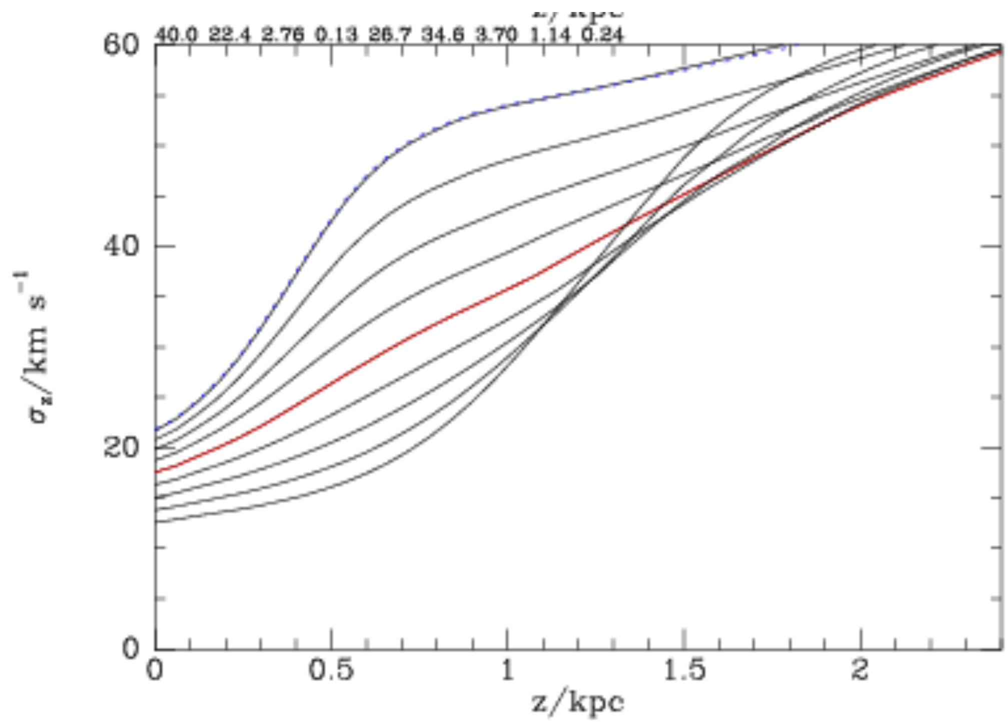
VÁ(R,z) & ¾Á(R,z)



Curves for R=6 kpc in red

Sharp transition at z=1kpc t slowly rotating thick disc

¾$_Z$(R,z)



Curve for R=8kpc in red

Transition between thin & thick discs again at z=1kpc

R=6kpc a very steep gradient in thin disc & shallow gradient in thick

The parameters

		(a)	(b)	(c)
Thin	σ_{r0}	47.3	40.0	38.6
	σ_{z0}	17.2	22.4	22.7
	R_{d}	1.91	2.76	2.66
	q	0.13	0.13	0.138
Thick	σ_{r0}	24.7	26.7	29.2
	σ_{z0}	26.9	34.6	32.6
	R_{d}	5.45	3.70	3.68
	q	0.26	1.14	1.08
	f_{thk}	0.18	0.24	0.265
χ^2		5.85	7.25	10.1

Conclusions

- Measurements are too fragmentary to be useful without a model
- A model is based on a hypothesis and is the means by which we test the hypothesis
- Models
  - pull together disparate data
  - exploit dynamics to predict data not yet taken
- Several important issues are still open regarding how we fit models to data & draw inferences from the fit
- Models key for studies of MW
- Models with f(J) are best
- Some pleasing/intriguing aspects of models with analytic DFs of the Galactic discs
- Next step is to add chemistry
- The technology should be applied to external galaxies