Dust Dynamics in the Turbulent ISM

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Basic assumption in Astrophysics: **Dust/Gas~0.01 (Z/Zsun)**

However, ***the inertia of dust grains cannot be neglected***, and it is well known that inertial particles in turbulent flows are strongly clustered.

Theoretical studies of clustering:
Elperin et al. (1998), Balkovsky et al. (2001), Zaichik et al. (2003)

Simulations: Sundaram and Collins (1997), Hogan and Cuzzi (1999,2001,2003), Reade and Collins (2000), Wang et al. (2000), Zhou et al. (2001), Collins and Keswani (2004), Falkovich et al. (2004), Bec et al. (2006), Cencini et al. (2006)

Experiments: ***Channel flows:*** Fessler et al. (1994), Eaton and Fessler (1994); ***Wind tunnel turbulence:*** Aliseda et al. (2002); ***In situ droplet sampling of clouds:*** Kostinski and Shaw (2001), Pinsky and Khain (2003)

Turbulence clustering may explain rain droplet formation:
Jameson & Kostinski (2000), Falkovich et al. (2002), Vallancourt et al (2002), Celani et al (2005)

The only astrophysical application: Hogan and Cuzzi (1999, 2001, 2003):

Turbulent clustering in protoplanetary disks --> chondrule formation.

But they don't study the clustering below the Kolmogorov dissipation scale (they instead assume the clustering grows with Reynolds number, based on an extrapolation in disagreement with recent simulations by Collins and Keswani 2004).

Summary

- Numerical simulations of inertial particles in compressible isothermal turbulence (assuming the sub-Kolmogorov velocity field is locally linear), **512^3 computational zones and 512^3 particles**

- Clustering measured directly from particle distribution and pair correlation function (+ extrapolation)

- Assumption that clustering is suppressed only on the scale of Brownian diffusion --> ***Huge dust density fluctuations***

Caveats

- No magnetic field forces on gas or grains
- No back reaction on the fluid, no mass-loading effects
- Ghost particles, no collisions

Timescales

Friction time:

Small particles --> Epstein regime:

$$\frac{\lambda}{a} = \mathrm{Kn} > \frac{4}{9} \quad \Rightarrow \quad \tau_{\mathrm{fr}} = \frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{g}}} \frac{a}{C_{\mathrm{S}}}$$

Large Particles --> Stokes regime:

$$\frac{\lambda}{a} < \frac{4}{9} \quad \Rightarrow \quad \tau_{\mathrm{fr}} = \frac{4}{9} \frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{g}}} \frac{a^2}{C_{\mathrm{S}} \lambda}$$

Epstein: ISM $(\mathrm{Kn} \sim 10^{14} - 10^{20})$ and disks (up to cm size particles)
Stokes: BDs and planetary atmospheres, large particles in disks

Kolmogorov (viscous dissipation) time
(smallest timescale in the turbulent flow)

$$\tau_{\mathrm{K}} = \left(\frac{\nu}{\epsilon}\right)^{1/2} = \left(\frac{C_{\mathrm{S}} / \sigma n}{u_0^3 / L_0}\right)^{1/2}$$

Stokes number:

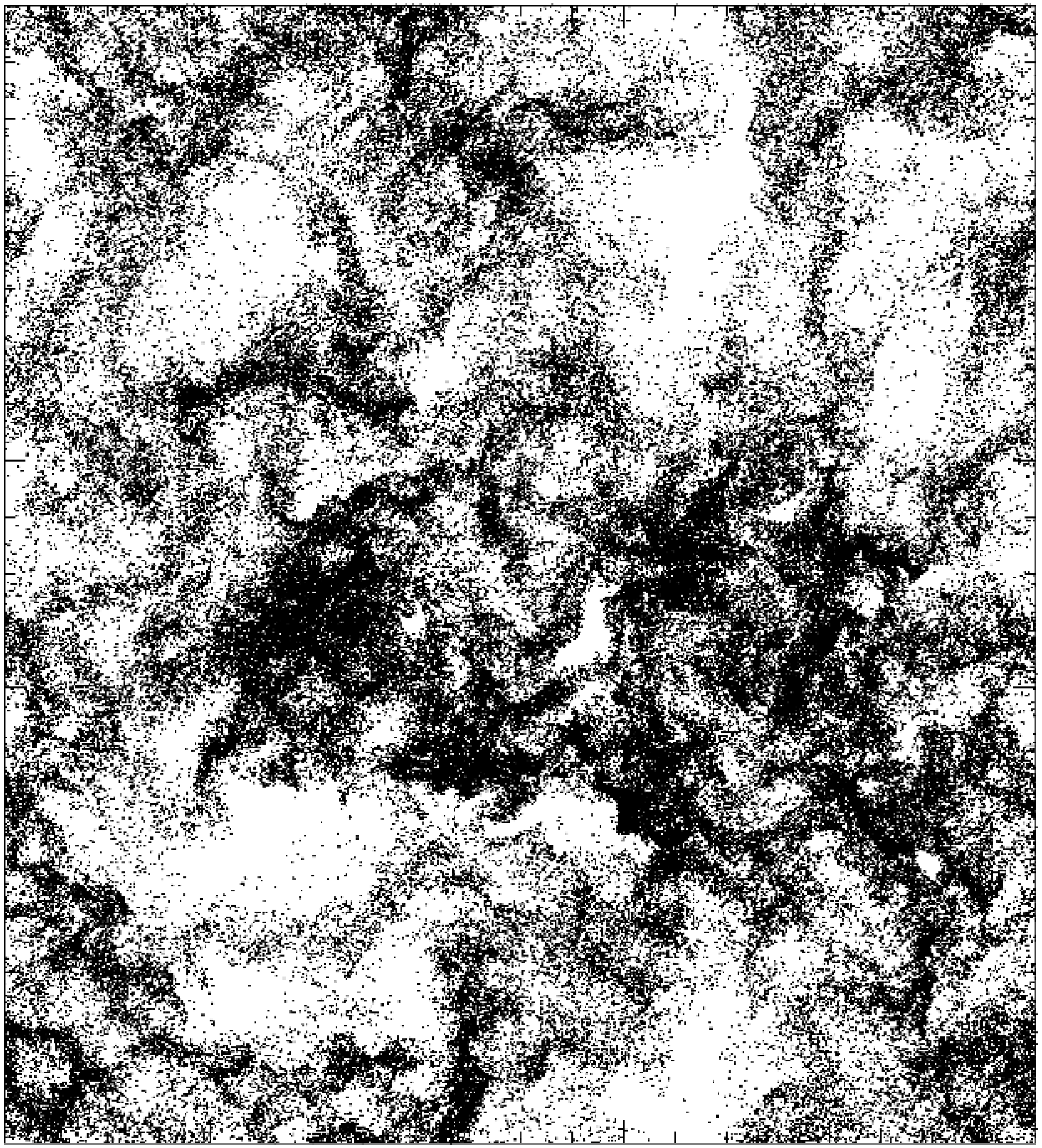
$$S = \frac{\tau_{\mathrm{fr}}}{\tau_{\mathrm{K}}}$$

Numerical Simulations of Compressible Turbulence

$N_{\mathrm{mesh}} = 512^3$; $N_{\mathrm{particles}} = 512^3$ (16 Stokes numbers), $M_S = 1$

S=1.2 (0 < z < 10)

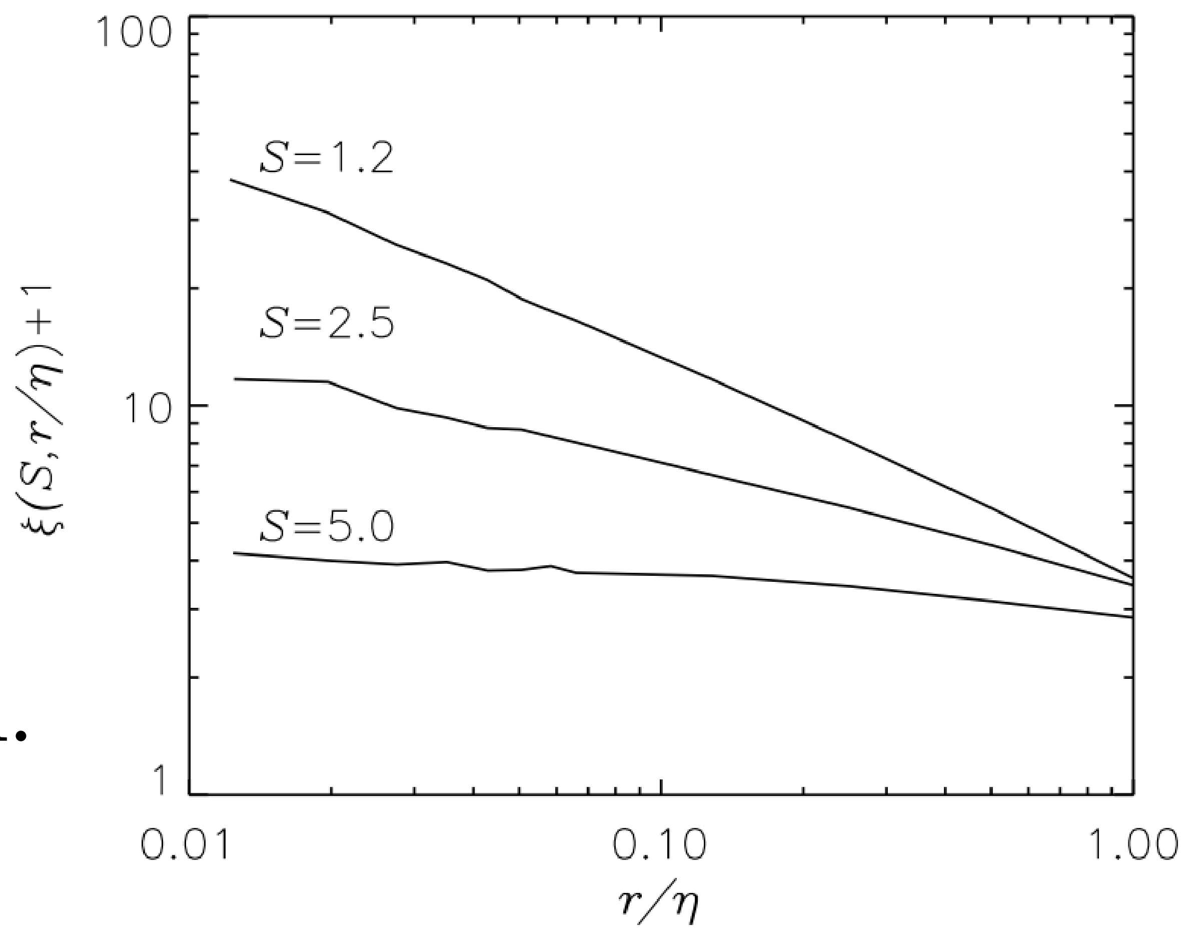
S=5 (0 < z < 10)



Statistics of clustering in the simulations

Limited number of particles: Counts in cells can define particle density only within a finite density range and a finite spatial resolution.

However, we know the clustering increases toward smaller scales, **below the Kolmogorov dissipation scale**:

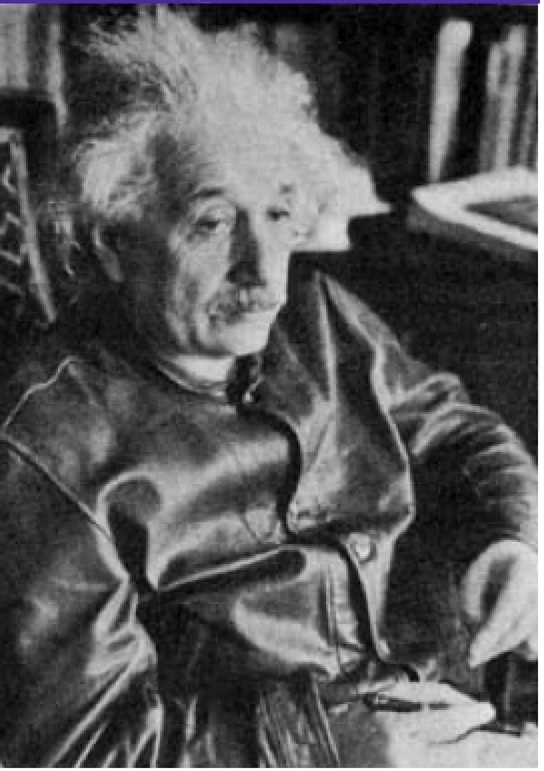


Pair correlation function:

$$P(r) = \bar{n}\, dV\, [1 + \xi(r)]$$

The clustering grows most rapidly for particles with S~1.

The Brownian Diffusion Scale



The particle clustering increases toward smaller scales, until we reach the scale of Brownian diffusion, $\boldsymbol{l}_{\mathbf{B}}$:

$$\frac{l_{\mathrm{B}}}{\eta} \approx \frac{10^{-4}}{a_{\mu\,\mathrm{m}}}$$ (Epstein regime)

$$\eta \approx 4 \times 10^{17}\,\mathrm{cm} \left(\frac{n}{\mathrm{cm}^{-3}} \right)^{-1} \left(\frac{N}{10^{24}\,\mathrm{cm}^{-2}} \right)^{1/4} M_S^{-3/4}$$

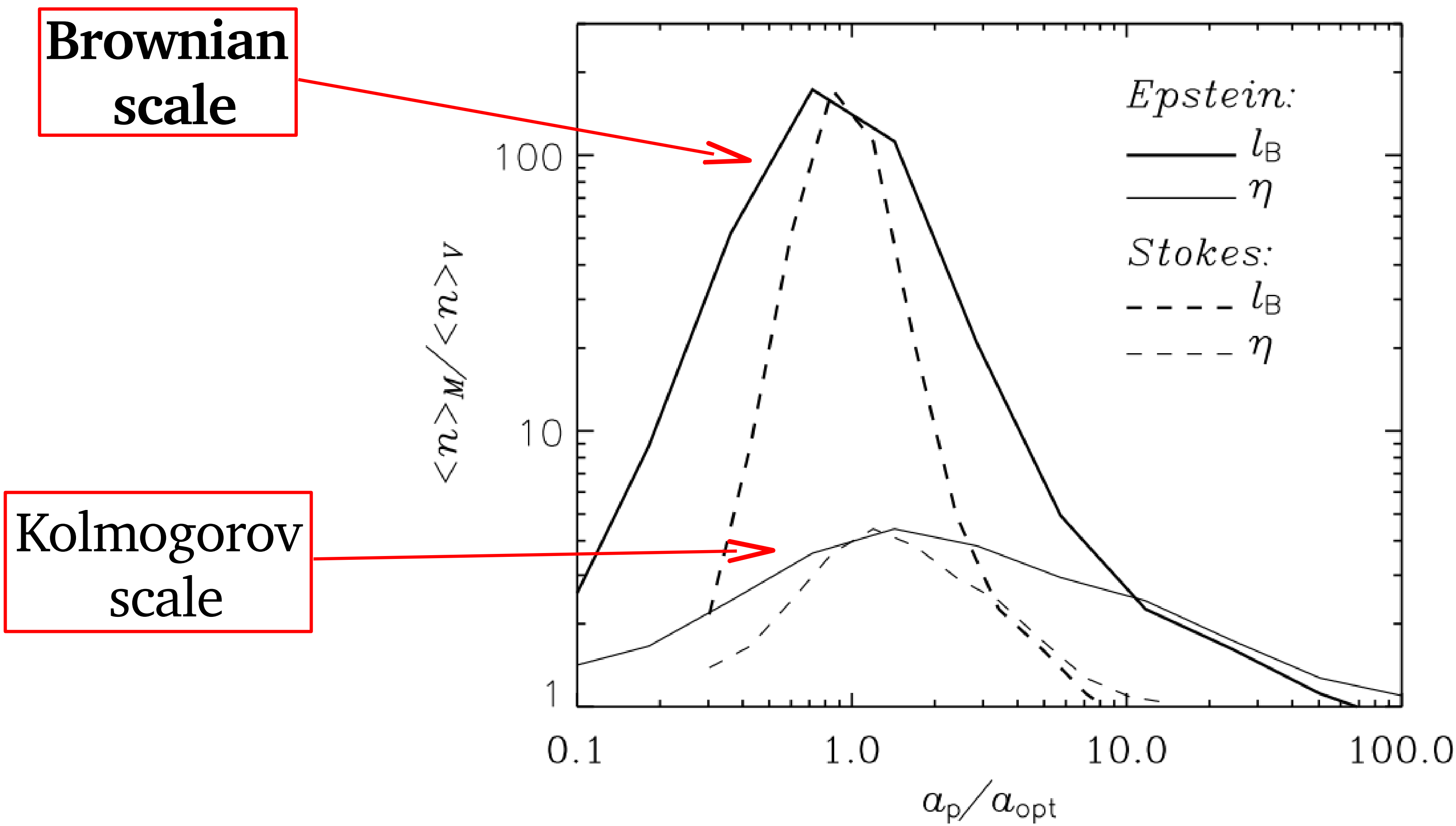
Advection-diffusion equation for particle concentration, Z:

$$\frac{\partial Z}{\partial t} + \boldsymbol{u} \cdot \nabla Z = D_{\mathrm{B}} \nabla^2 Z \qquad \frac{u}{L} \sim \frac{D_{\mathrm{B}}}{L^2} \qquad \frac{1}{\tau_{\mathrm{K}}} = \frac{\nu}{\eta^2} \sim \frac{D_{\mathrm{B}}}{L_{\mathrm{B}}^2}$$

$$L_{\mathrm{B}} \sim \sqrt{D_{\mathrm{B}} \tau_{\mathrm{K}}} \qquad D_{\mathrm{B}} \sim \tau_{\mathrm{f}} u_{\mathrm{B}}^2$$

Correlation-fluctuation theorem: The pair correlation function gives the standard deviation of the particle density on the scale r:

$$\sigma(r)^2 = \frac{\overline{(N(r) - \bar{N}(r))^2}}{\bar{N}(r)^2} = \frac{1}{\bar{N}(r)} + \frac{1}{V}\int_0^V \xi(V')\,dV'$$



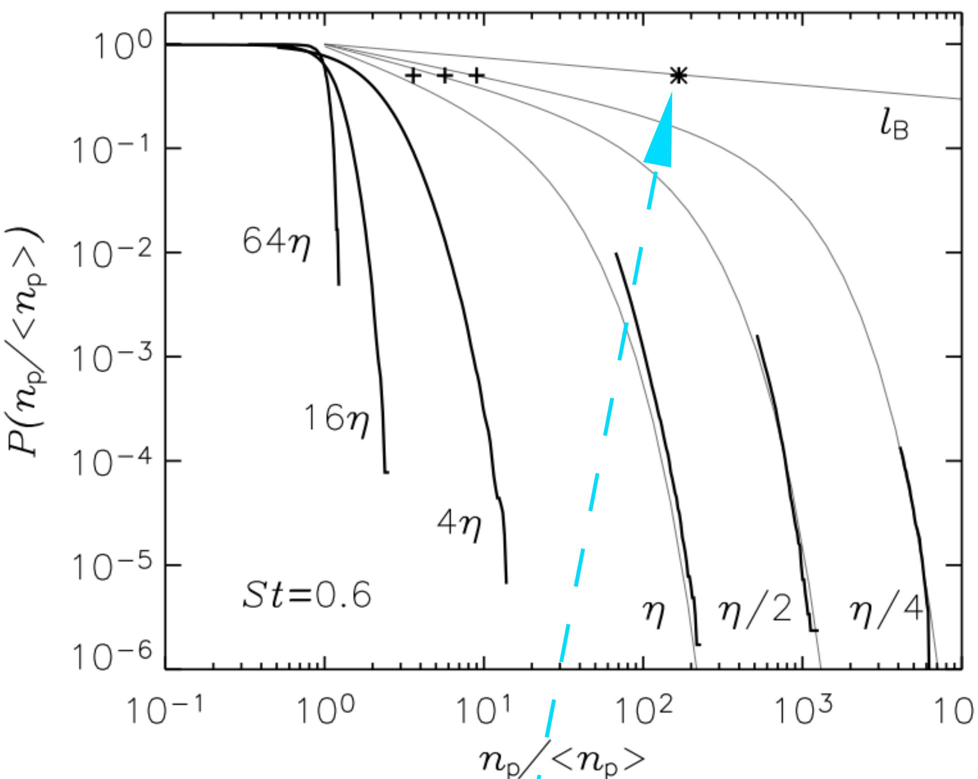
Cumulative PDFs

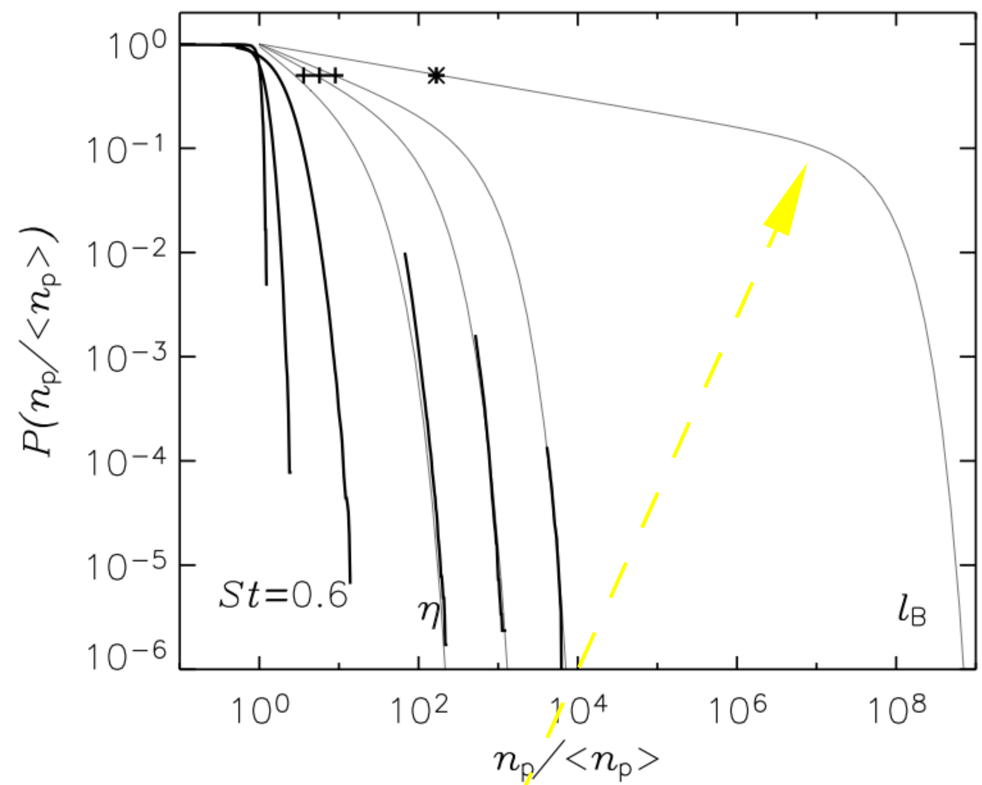
Counts in cell, then extrapolation to the Brownian scale:

$$F(n,l) = n^{-B(l)} e^{-n/A(l)}$$

$$B(l) = -0.54/\left[1 - 1.18\log_{10}(l/\eta)\right]$$

$$A(l) = 20(l/\eta)^{-2.5}$$





50% of optimal size particles at $n_p > 100\langle n_p \rangle$

10% of optimal size particles at $n_p > 10^7\langle n_p \rangle$

Stokes Number and Optimal Particle Size

$$S = \frac{\tau_{\mathrm{fr}}}{\tau_{\mathrm{K}}} = \left(\frac{2}{9}\right)^{i-1} \left(\frac{a}{\lambda}\right)^{i} \left(\frac{\lambda}{L_0}\right)^{1/2} \left(\frac{u_0}{C_{\mathrm{S}}}\right)^{3/2} \frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{g}}}$$

(*i*=1: *Epstein regime*; *i*=2: *Stokes regime*)

Example for **Molecular Clouds**:

$$S \approx 1 \left(\frac{a}{0.3\mu\mathrm{m}}\right) \left(\frac{\rho_{\mathrm{p}}}{1\,\mathrm{g/cm^3}}\right) \left(\frac{N_{\mathrm{gas}}}{10^{22}\,\mathrm{cm^{-2}}}\right)^{-1/2} M_{\mathrm{S}}^{3/2}$$

So clustering occurs in the approximate range $0.01\,\mu\mathrm{m} \le a \le 1\,\mu\mathrm{m}$,
with an optimal grain size $a_{\mathrm{opt}} \sim 0.1\,\mu\mathrm{m}$

--> The clustering is very important for molecular clouds.

Other astrophysical environments?

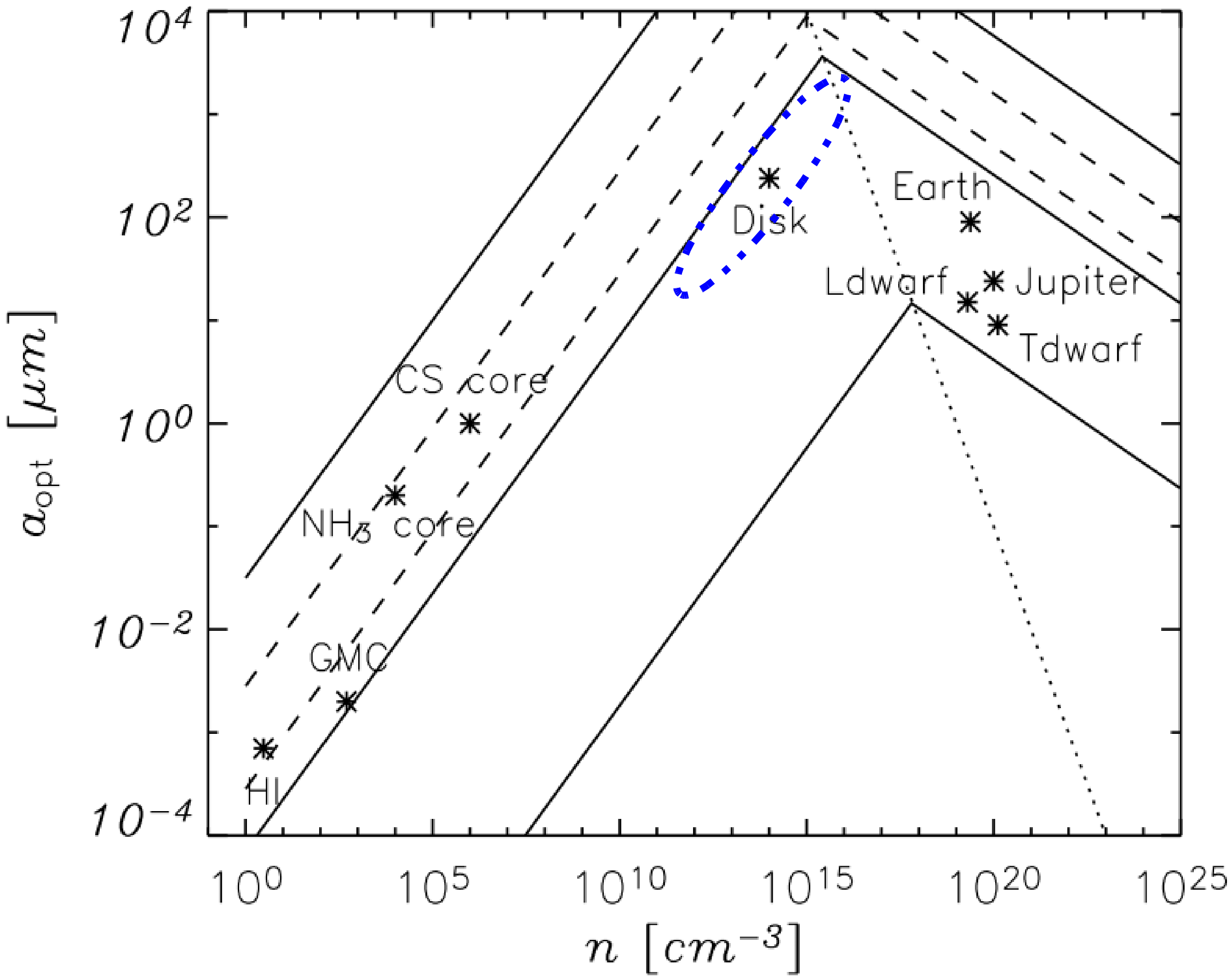
Optimal Particle Size (S=1)

Epstein regime

$$a_{\rm opt}^{\rm Ep} = (\pi^{1/2} 15/8)^{1/2} \sigma^{1/2} \gamma^{-3/4} \rho_{\rm p}^{-1} m_{\rm g} n_{\rm g}^{1/2} M_{\rm s}^{-3/2} L_0^{1/2}$$

Stokes regime

$$a_{\rm opt}^{\rm St} = [(9/2)\lambda_{\rm g} a_{\rm opt}^{\rm Ep}]^{1/2} \propto n_{\rm g}^{-1/4}$$



Artificial Clustering

Potential problem for all simulations generating turbulence:

Turbulence simulations of clouds or disks never resolve the Kolmogorov scale; their numerical dissipation scale is much larger (~mesh size).

--> Numerical Stokes number << physical Stokes number

Numerical Stokes numbers in simulations may often be small enough to cause artificial clustering. This is a numerical artifact, it does NOT occur in Nature.

You should estimate your numerical particle Stokes numbers before venturing into physical interpretations or applications of the clustering.....

Conclusions

- Dust grains with a range of sizes are strongly clustered.

- Grains with S~1, both in cloud cores and disks:
50% with Dust/Gas >1
10% with Dust/Gas > 10^5

- The clustering may affect many astrophysical processes involving dust grains.

- The clustering of large particles may be a numerical artifact in simulations that don't resolve the Kolmogorov scale.