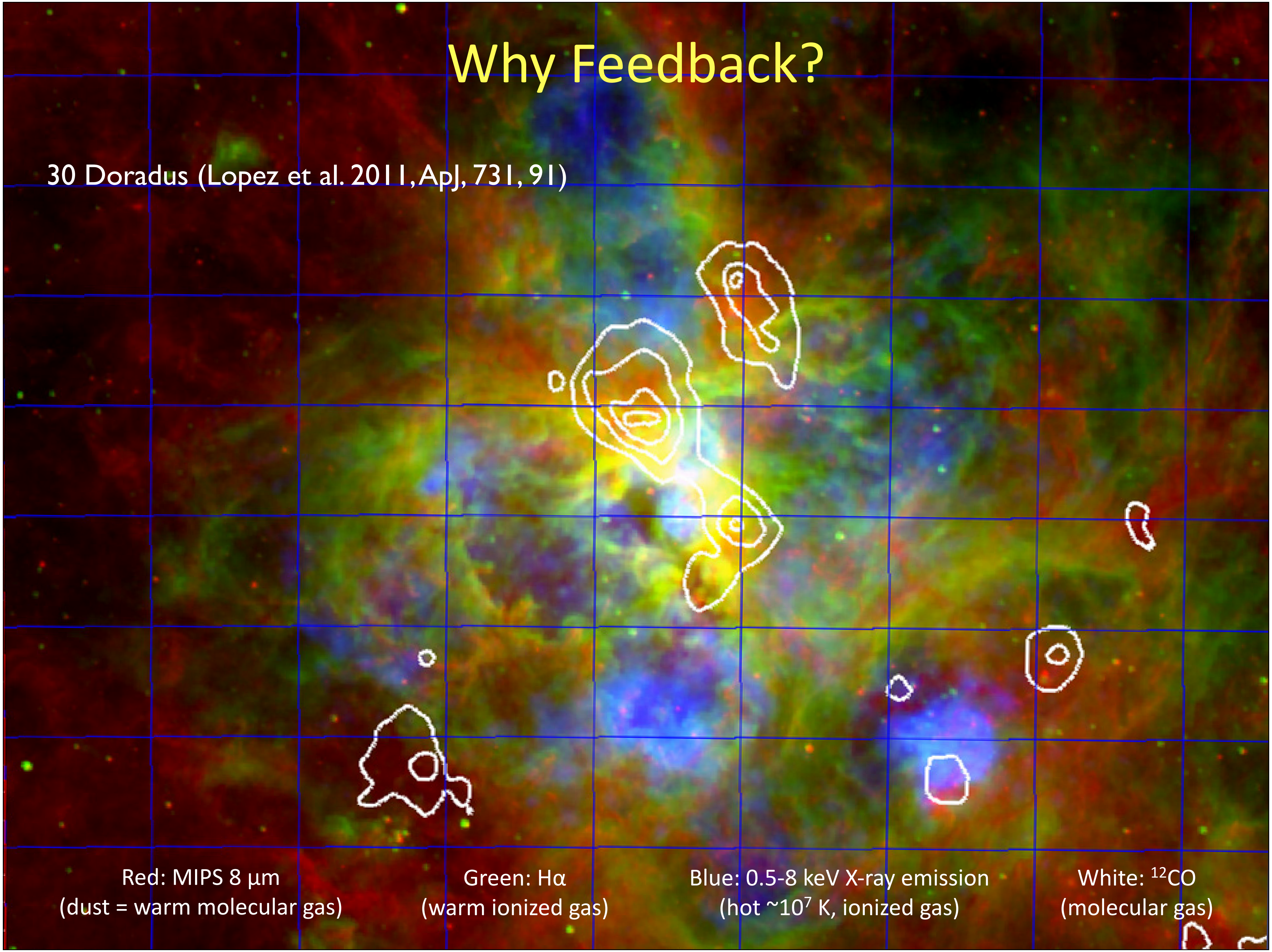
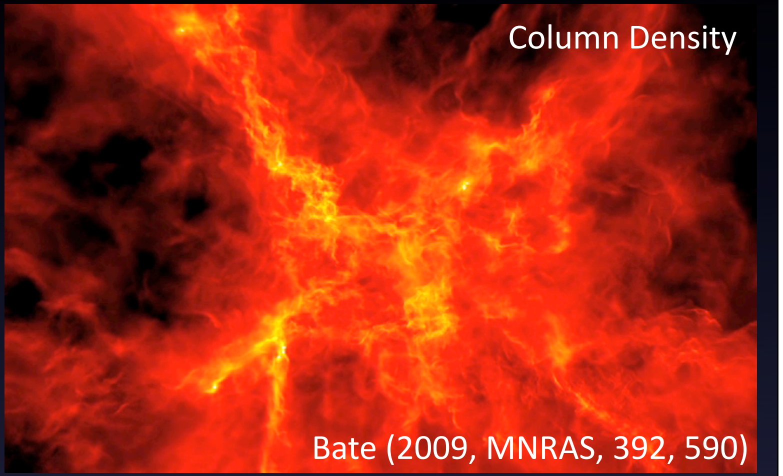
Star Cluster Formation and Feedback

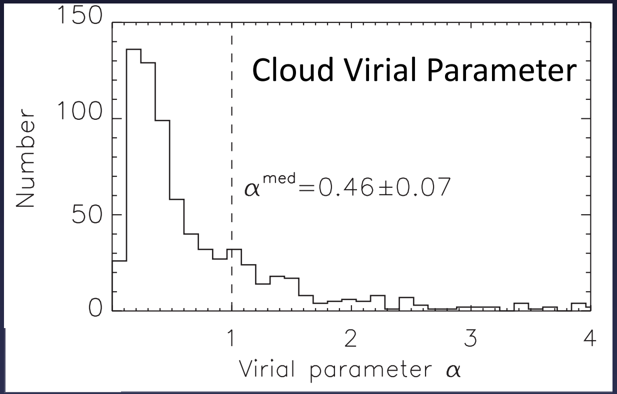
Mark Krumholz (UC Santa Cruz)
Matthew Bate (Exeter)
Hector Arce (Yale)
James Dale (Ludwig-Maximillians-University)
Robert Gutermuth (UM Amherst)
Richard Klein (UC Berkeley/LLNL)
Zhi-Yun Li (Virginia)
Fumitaka Nakamura (NAOJ)
Qizhou Zhang (Harvard-Smithsonian CfA)


Why Feedback?
30 Doradus (Lopez et al. 2011, ApJ, 731, 91)
Red: MIPS 8 μm
(dust = warm molecular gas)
Green: Hα
(warm ionized gas)
Blue: 0.5-8 keV X-ray emission
(hot ~10^7 K, ionized gas)
White: ^12CO
(molecular gas)

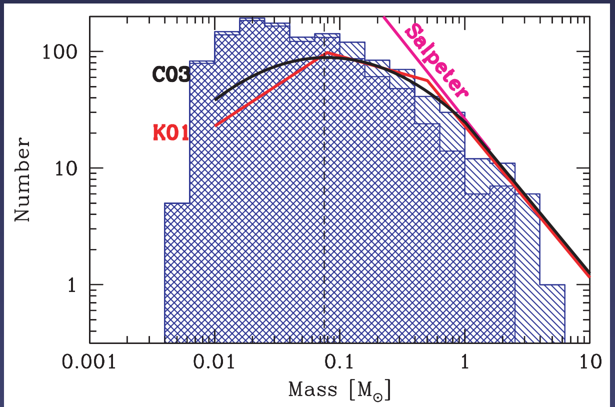
Without feedback

- Star formation is too fast
  - Gas depletion rate observed to be 1-3 orders of magnitude slower than free-fall rate (Zuckerman & Evans 1974; Krumholz & Tan 2007; Evans et al. 2009; Juneau et al. 2009)
  - Without feedback, star formation rate ~ free-fall time (e.g. Klessen & Burkert 2000; Bonnell et al. 2003; Bate 2009)
  - Strong magnetic fields help (e.g. Price & Bate 2008, 2009; Wang et al. 2010), but feedback is a prime candidate
- Star formation is too efficient
  - By 10-100 Myr, only a few percent of stars remain in clusters (Silva-Villa & Larsen 2011; Fall & Chandar 2012)
  - Typical cluster formation simulations produce bound clusters, unless using super-virial initial conditions, which don't appear to be common (Shirley et al. 2003; Roman-Duval et al. 2010)
  - Potential solution: dispersal of gas by feedback
- Star formation produces too many low-mass objects





Roman-Duval et al. (2010, ApJ, 723, 492)



IMF from Bate (2009, MNRAS, 392, 590)

Feedback Taxonomy

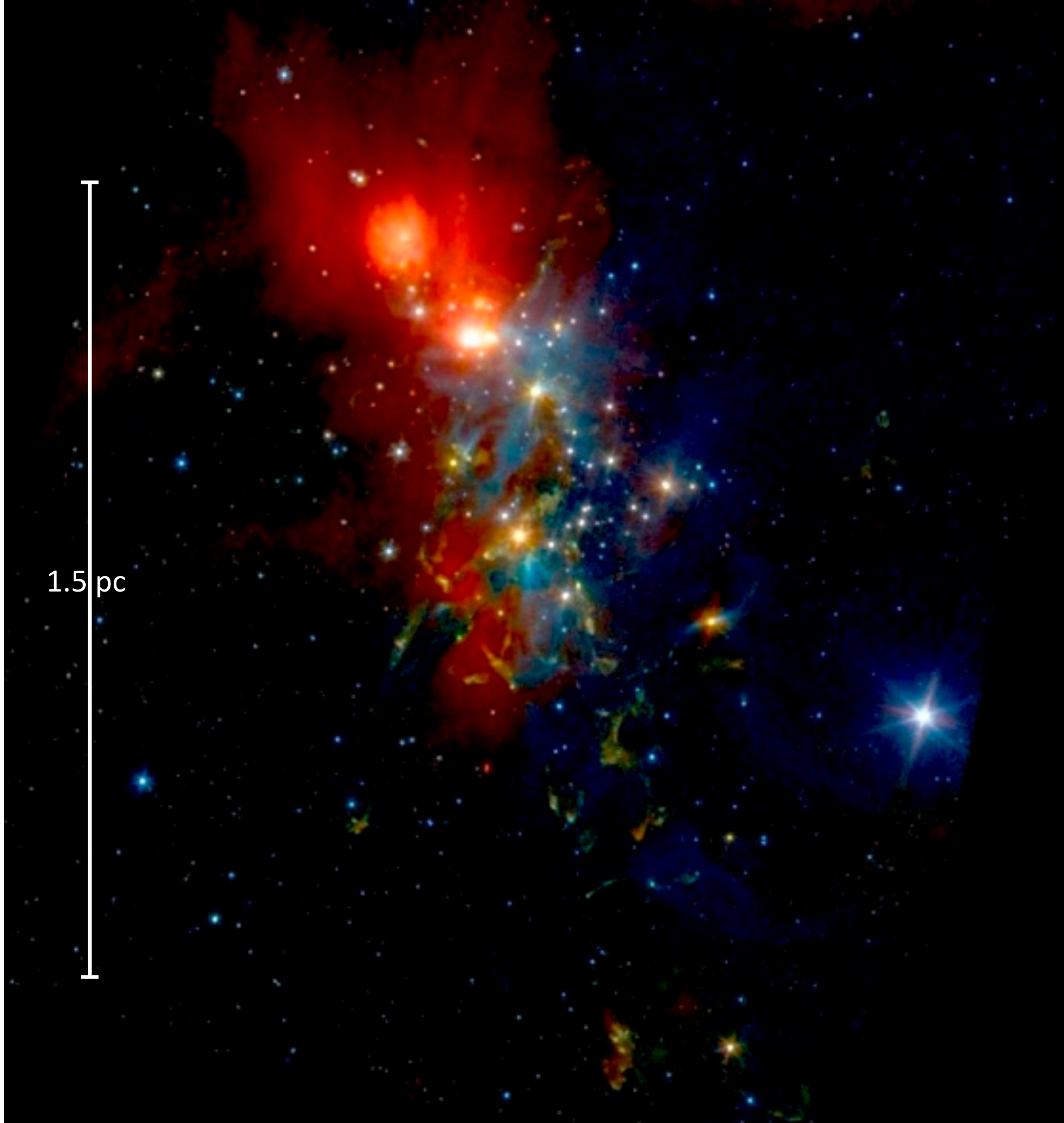
- Momentum feedback
  - Protostellar jets/outflows and radiation pressure add momentum directly to the gas
  - Cooling in the jets/outflows is efficient (< dynamical time of cloud) so thermal energy associated with feedback is unimportant - only the momentum matters
- Hot gas feedback
  - Winds from hot stars, shocks, ionization produce hot gas (>10^4 K) that dynamically expands
  - Cooling inefficient (> dynamical time of cloud) so during adiabatic expansion the hot, overpressure gas does work on the surrounding cloud
- Thermal feedback
  - (Proto)stars radiatively heat molecular gas, altering how it fragments

Momentum Feedback: Protostellar Outflows

- Expected to be important where large number of stars form close together in space and time
  - The outflow momentum divided by the cloud mass can in principle be larger than velocity dispersion of molecular gas in a star forming region
- Possible outcomes
  - If all momentum injected simultaneously, may unbind cloud
  - More gradually, may maintain turbulence
  - If coupling low, may simply punch holes in the cloud

Outflows in protostellar clusters

Example: NGC 1333



Spitzer IRAC composite image (Gutermuth et al. 2008, ApJ, 674, 336)

Outflows in protostellar clusters

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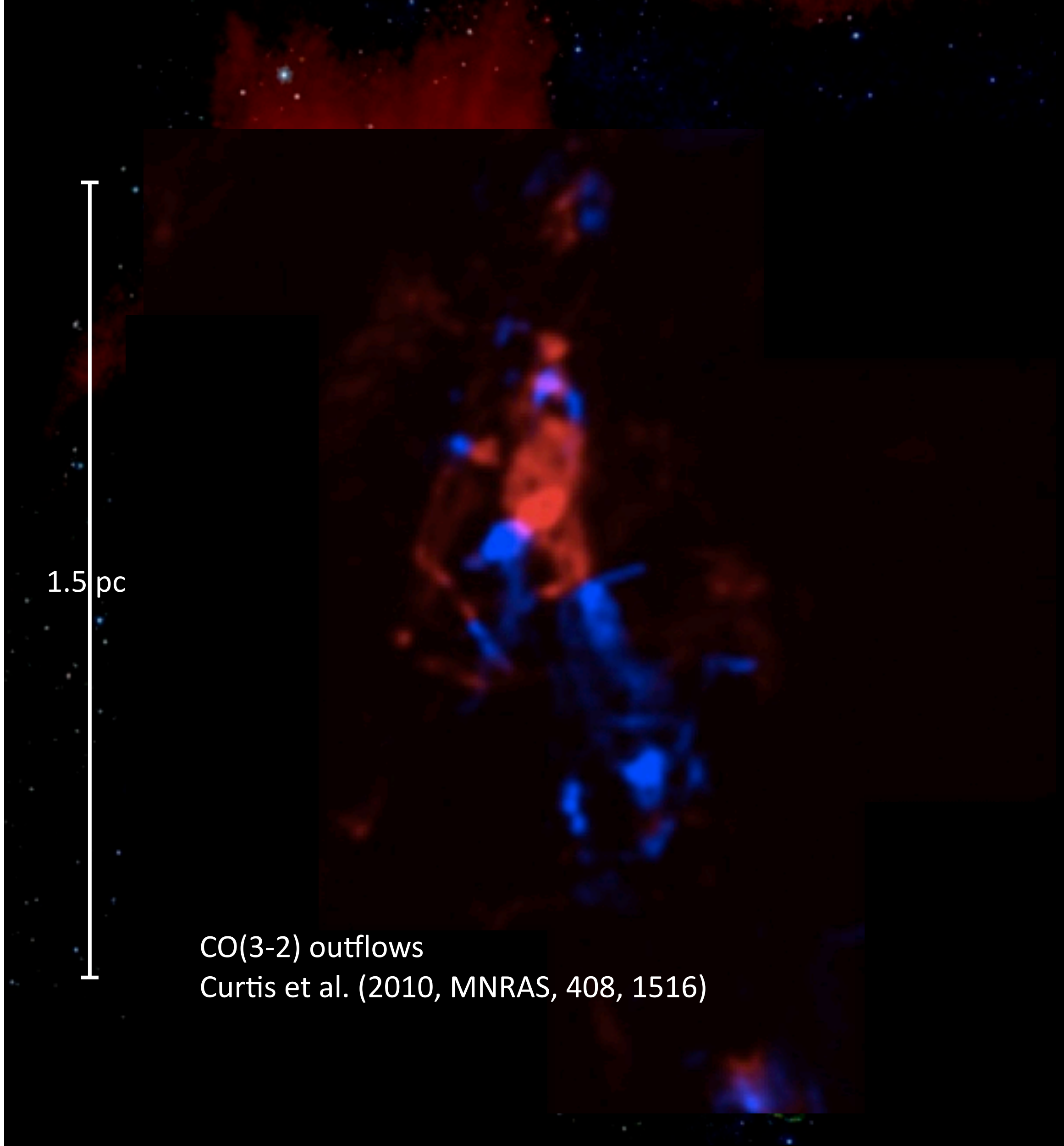

Outflows in protostellar clusters

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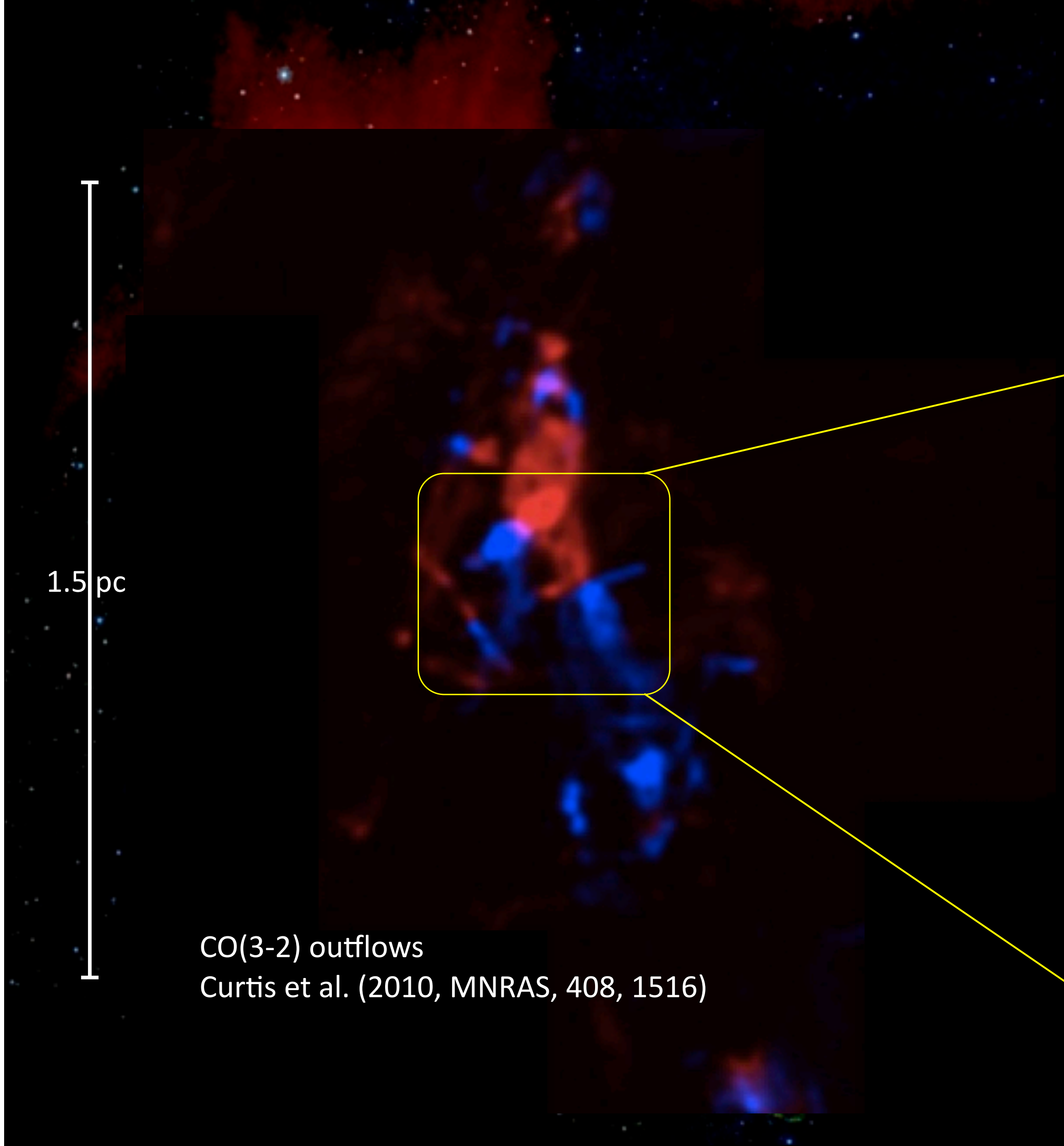
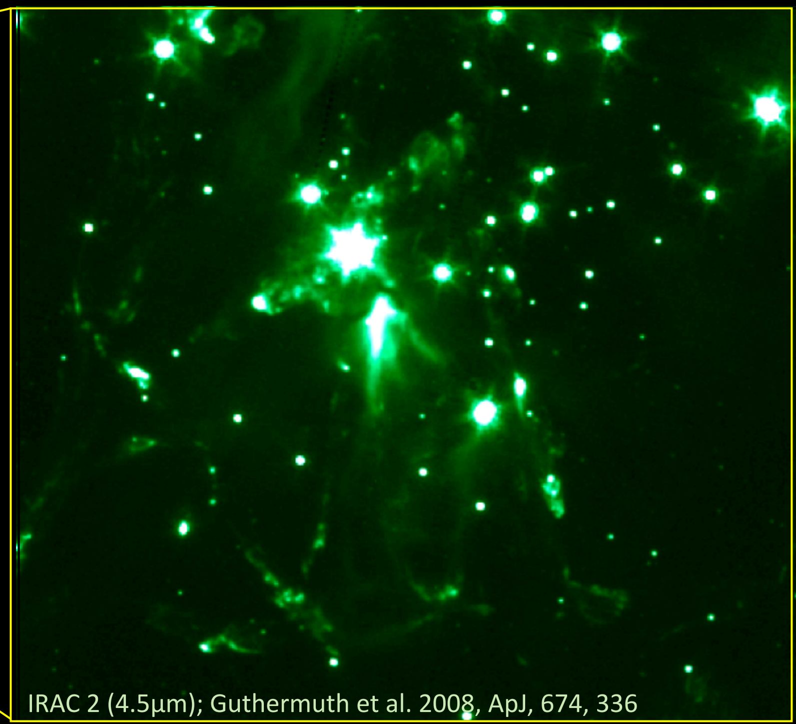


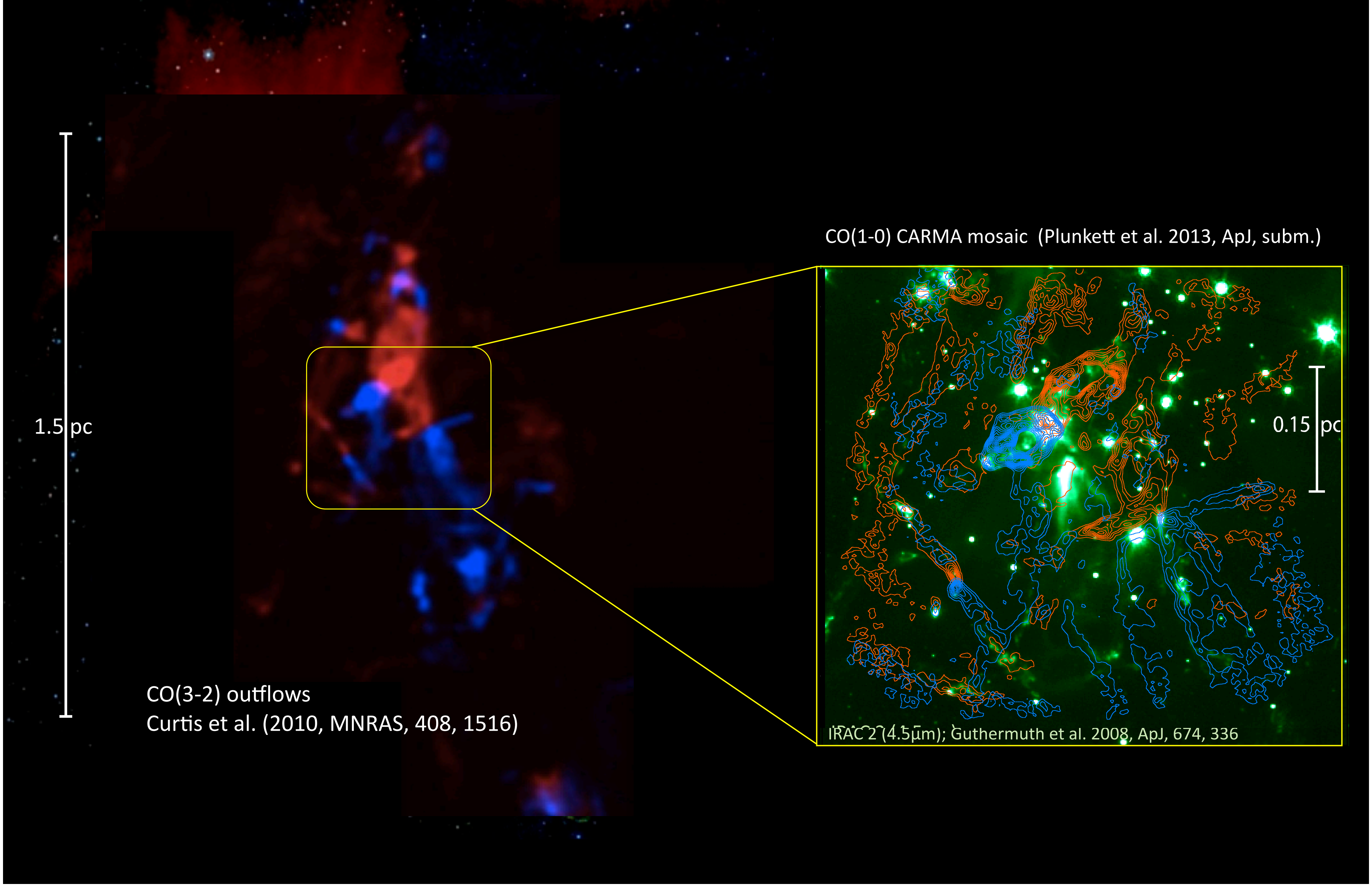
CO(3-2) outflows
Curtis et al. (2010, MNRAS, 408, 1516)



IRAC 2 (4.5μm); Guthermuth et al. 2008, ApJ, 674, 336

Outflows in protostellar clusters

Example: NGC 1333

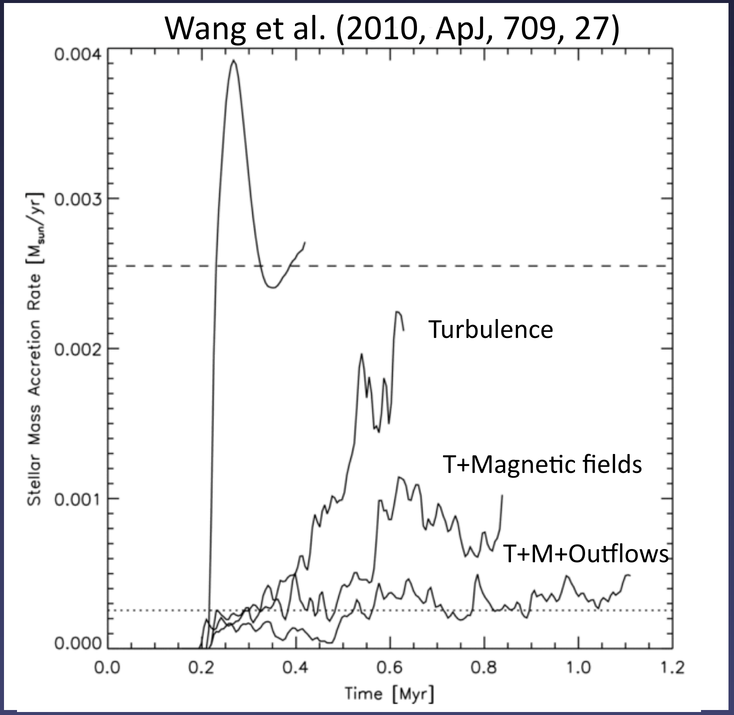


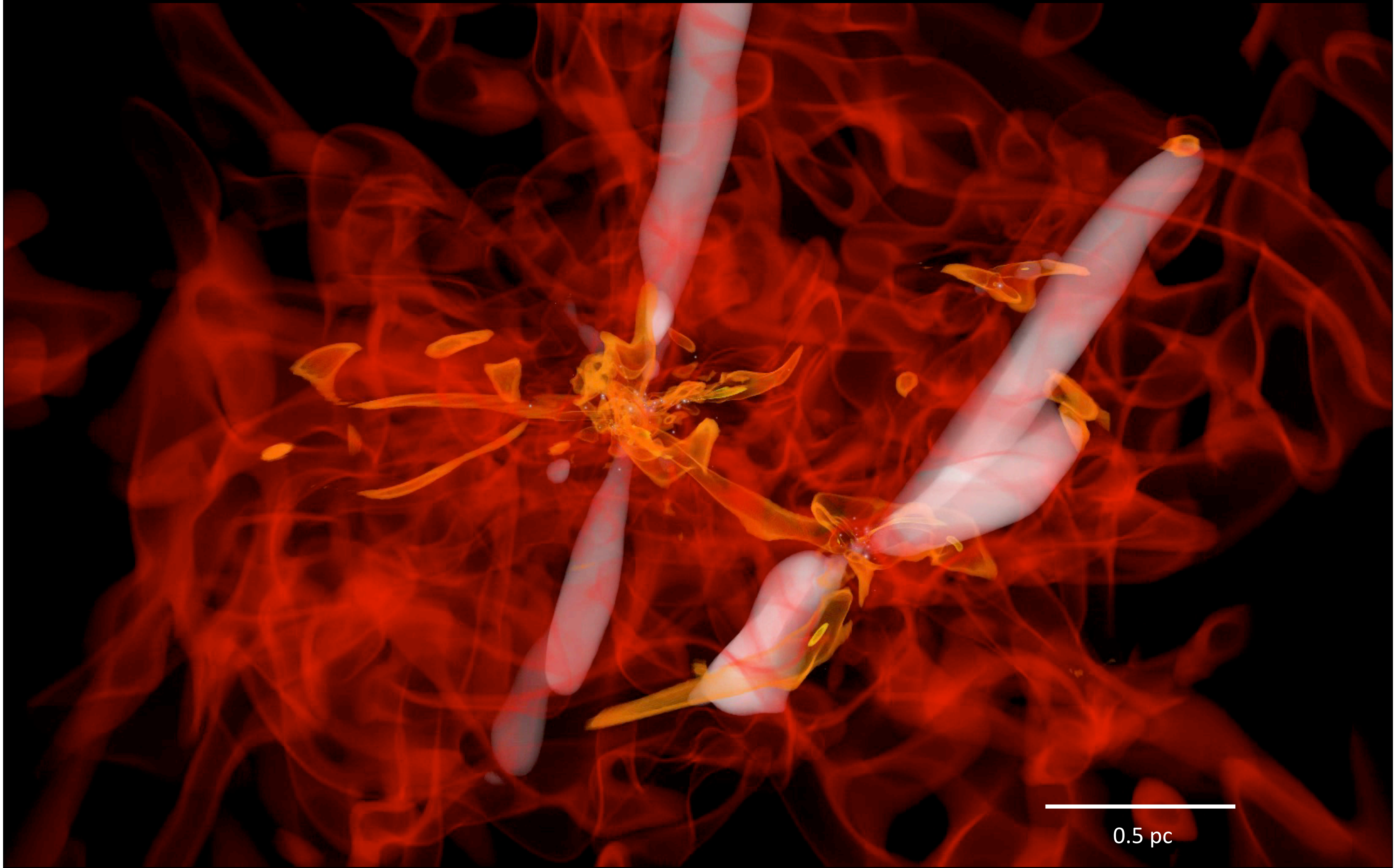
Observations of Outflows

- Combined outflows often seem to be enough to drive cluster turbulence locally
- Compare total outflow kinetic energy with turbulent energy
  - For a number of clusters, the ratio is ~30% (Arce et al. 2010; Curtis et al. 2010; Graves et al. 2010; Duarte-Cabral et al. 2012)
  - But for some just 1-20% (Arce et al. 2010; Narayanan et al. 2012)
  - Energy insufficient to unbind cloud, but observations only show current outflow energy, which may only be a fraction of that injected in the lifetime of the cloud
- Compare total outflow power (rate of kinetic energy injection) with the turbulent energy dissipation rate
  - More difficult to estimate, but it seems that for most protoclusters $L_{outflow} \sim L_{turb}$ (Williams et al. 2003; Stanke & Williams 2007; Swift & Welch 2008; Maury et al. 2009; Acre et al. 2010; Nakamura et al. 2011a,b)
- Large-scale surveys show outflows lack the power to sustain turbulence in molecular cloud complexes and giant molecular clouds (Walawender et al. 2005; Dent et al. 2009; Acre et al. 2010; Ginsburg et al. 2011; Narayanan et al. 2012)
  - Large regions of clouds with few outflows
  - Implies additional energy source is responsible for large-scale turbulence

Protostellar Outflows: Simulations

- Outflow feedback in magnetised clouds can keep cluster-forming clumps close to virial equilibrium (Li & Nakamura 2006)
- Collimated flows are more efficient at turbulent driving than spherical (Nakamura & Li 2007; Matzner 2007)
- Fast jets into smooth ambient mediums do not excite significant supersonic motions (Banerjee et al. 2007)
- But jets propagating into a turbulent medium
  - are more efficient at driving turbulence (Cunningham et al. 2009)
  - and can maintain turbulence even without magnetic fields (Carroll et al. 2009)
- Magnetic fields allow coupling between different of the clump and more efficient momentum (Nakamura & Li 2007; Wang et al. 2010)
  - Magnetic fields, turbulence, and outflows all contribute to lowering star formation rate (Wang et al. 2010)

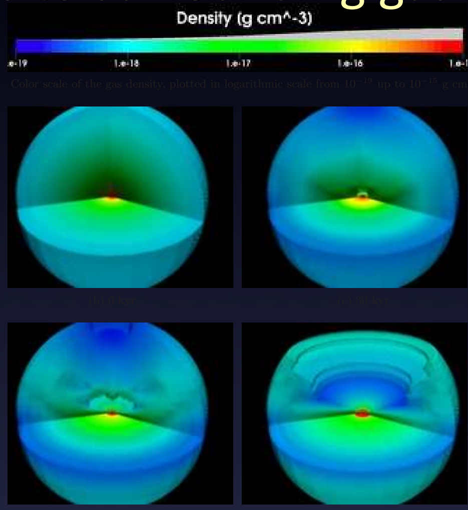




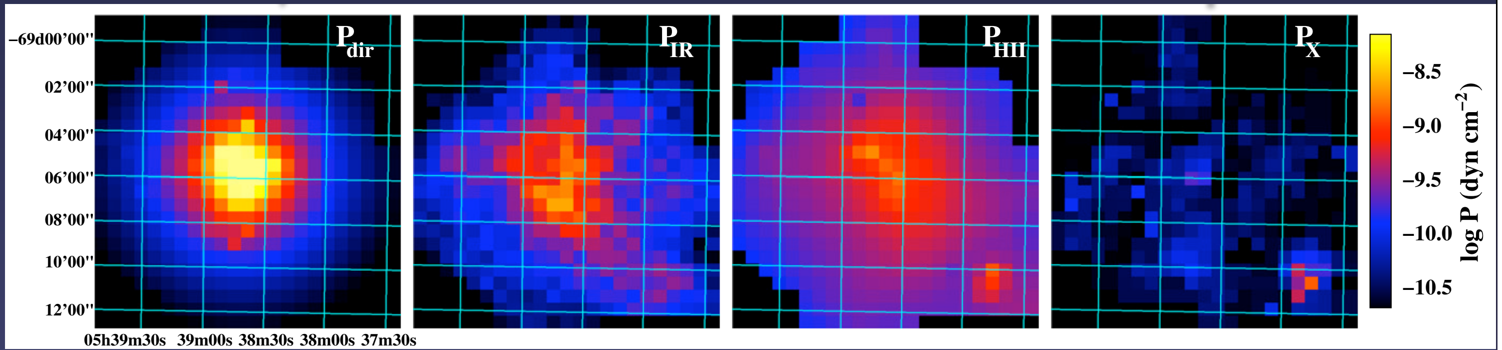
Wang et al. (2010, ApJ, 709, 27)

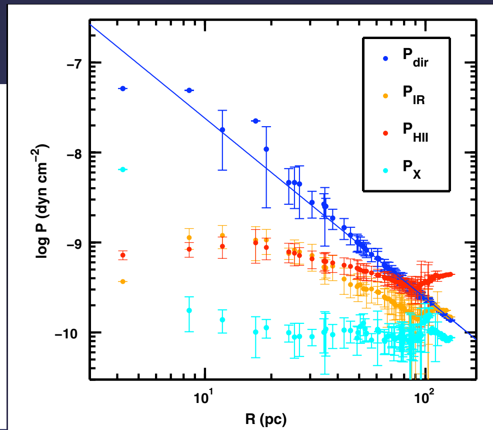
Momentum Feedback: Radiation Pressure

- Transfer of energy and momentum from stellar radiation fields to surrounding gas
  - Photons >13.6 eV directly interact with the gas
  - For lower energy photons it is mainly mediated by dust grains
- The very steep mass-luminosity relation of stars ensures radiation pressure feedback is dominated by the most massive stars
  - Very important near a massive star, in massive clusters, and possibly on galactic scales
  - Radiation pressure less important than outflows in clusters with masses less than ~10^4 $M_\odot$ because they do not fully sample the IMF (Cervino & Luridiana 2004; de Silva et al. 2012) whereas outflow energy essentially scales with cluster mass



Massive star formation
Kuiper et al. (2010, ApJ, 722, 1556)





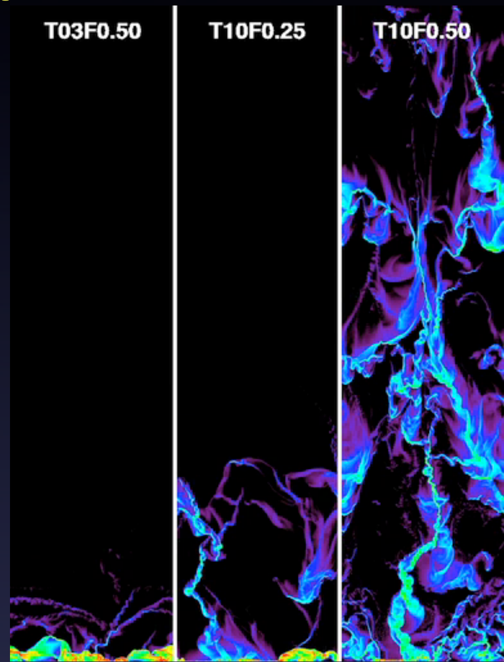
Pressures of direct starlight, reprocessed IR, warm gas, hot gas in 30 Dor (Lopez et al. 2011, ApJ, 731, 91)

When is Radiation Pressure Important?

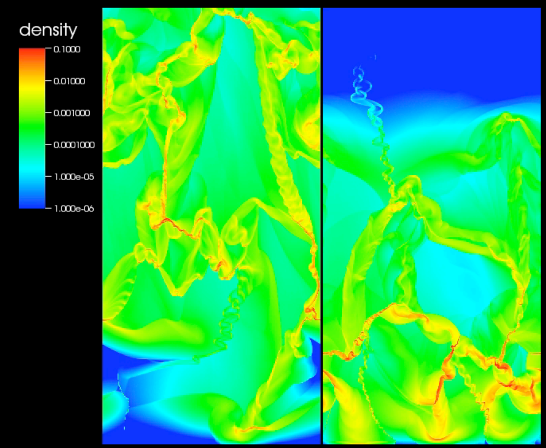
- Feedback significant if $\dot{p} t_{\rm ff} \gtrsim M v_{\rm virial}$
- Radiation pressure $\dot{p} \approx f_{\rm trap} L/c$
- Combine conditions

$$\Sigma < f_{\rm trap} \langle L/M_* \rangle / (Gc) \approx f_{\rm trap} \cdot 1 \text{ g cm}^{-2}$$

- Depends on $f_{\rm trap}$ which is poorly known
    - Photons absorbed, re-radiated, absorbed again...
    - Gas can be ejected if direct radiation pressure is insufficient if $f_{\rm trap}$ is high
    - If $f_{\rm trap} \sim \tau_{\rm IR}$, radiation pressure disrupts clusters, launches galactic winds (e.g. Murray et al. 2010, 2011; Hopkins et al. 2012a,b,c,d)
    - If $f_{\rm trap} \sim 1$ radiation pressure only important in massive clusters, no galactic winds



Radiation pressure-driven winds using FLD
Krumholz & Thompson (2012, ApJ, 760, 155)



Radiation pressure-driven winds using VET
Shane Davis, in prep.

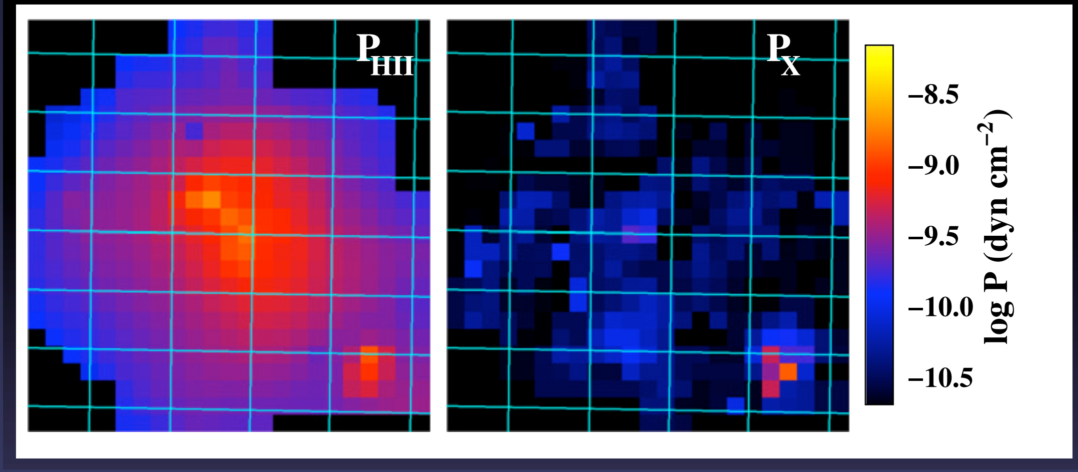
Hot Gas Feedback: Winds from Hot Stars

- Strong winds produced by stars with $T > 4 \times 10^4$ K (Vink et al. 2000)
  - Main sequence stars > 40 $M_\odot$, which join main sequence as they are forming (Hosokawa & Omukai 2009)
  - Momentum flux less than stellar radiation field (Kudritzki et al. 1999), so if their energy was quickly radiated they should be less important than radiation pressure
- However, terminal velocities can exceed 10^3 km/s
  - Shocks can produce $T > 10^7$ K, for which cooling times are long (Castor et al. 1975)
  - They may build up an energy-driven adiabatic flow far more effective than radiation
  - Or, they may leak out of the shells of interstellar matter that they sweep up, reducing the pressure build-up, reducing to the momentum-limited case
  - Wind parameter can be used to determine the relative importance of hot winds and photoionized gas (Yeh & Matzner 2012):

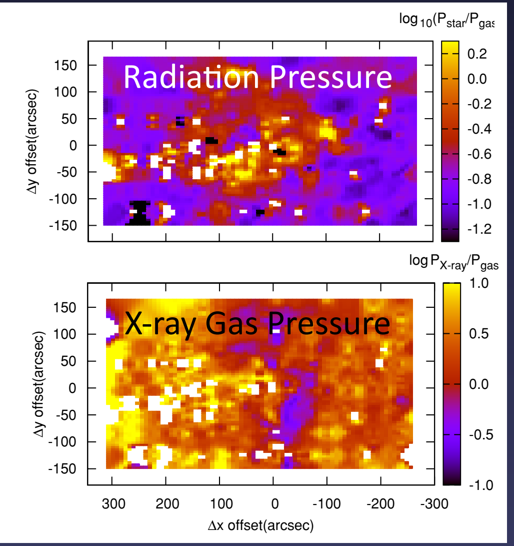
$$\Omega \equiv \frac{P_{\rm w} V_{\rm w}}{P_{\rm HII} V_{\rm HII} - P_{\rm w} V_{\rm w}}$$

Winds: Observations

- Chandra has made X-ray emission from HII regions possible (Townsley et al. 2003, 2006, 2011; Lopez et al. 2011, 2013; Pellegrini et al. 2011)
    - Rule out large values of Ω from confined wind models where shocked gas is trapped (Castor et al. 1975; Weaver et al. 1977)
    - But larger than expected from freely expanding wind (Chevalier & Clegg 1985)
    - Depends on region - X-ray emitting gas dominate in most of 30 Dor, but radiation pressure dominates in M17 (Pellegrini et al. 2011)



P_{HII} vs. P_{wind} in 30 Dor, Lopez + (2011) (ApJ, 731, 91)

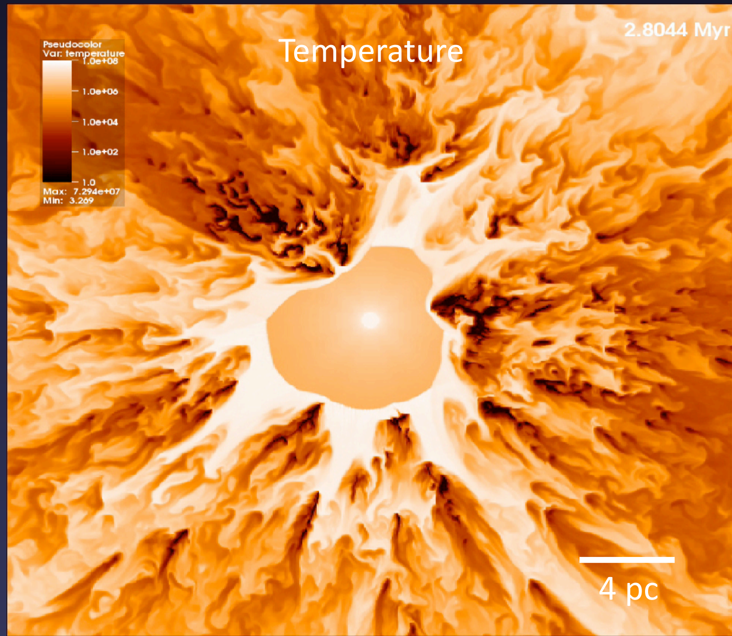


Pellegrini et al. (2011, ApJ, 738, 34)

- Optical and infrared line ratios can also be used to constrain Ω
    - Current data favours Ω<1 (Yeh & Matzner 2012; Yeh et al. 2013; Verdolini et al. 2013)

Winds: Theory

- Classical hot gas models (Castor et al. 1975; Weaver et al. 1977)
- Spherically-symmetric models
  - Better treatments of conduction and radiative cooling (Capriotti & Kozminski 2001; Tenorio-Tagle et al. 2007, 2013; Arthur 2012)
- Analytic models for cold shells
  - Driven by ionized gas, radiation pressure, wind pressure with parameterised leakage (Harper-Clark & Murray 2009; Krumholz & Matzner 2009)
- Simulations generally find that leakage is very significant
  - Majority of wind energy escaping or lost due to radiation rather than doing work on the cold ISM (Tenorio-Tagle 2007; Dale & Bonnell 2008; Rogers & Pittard 2013)

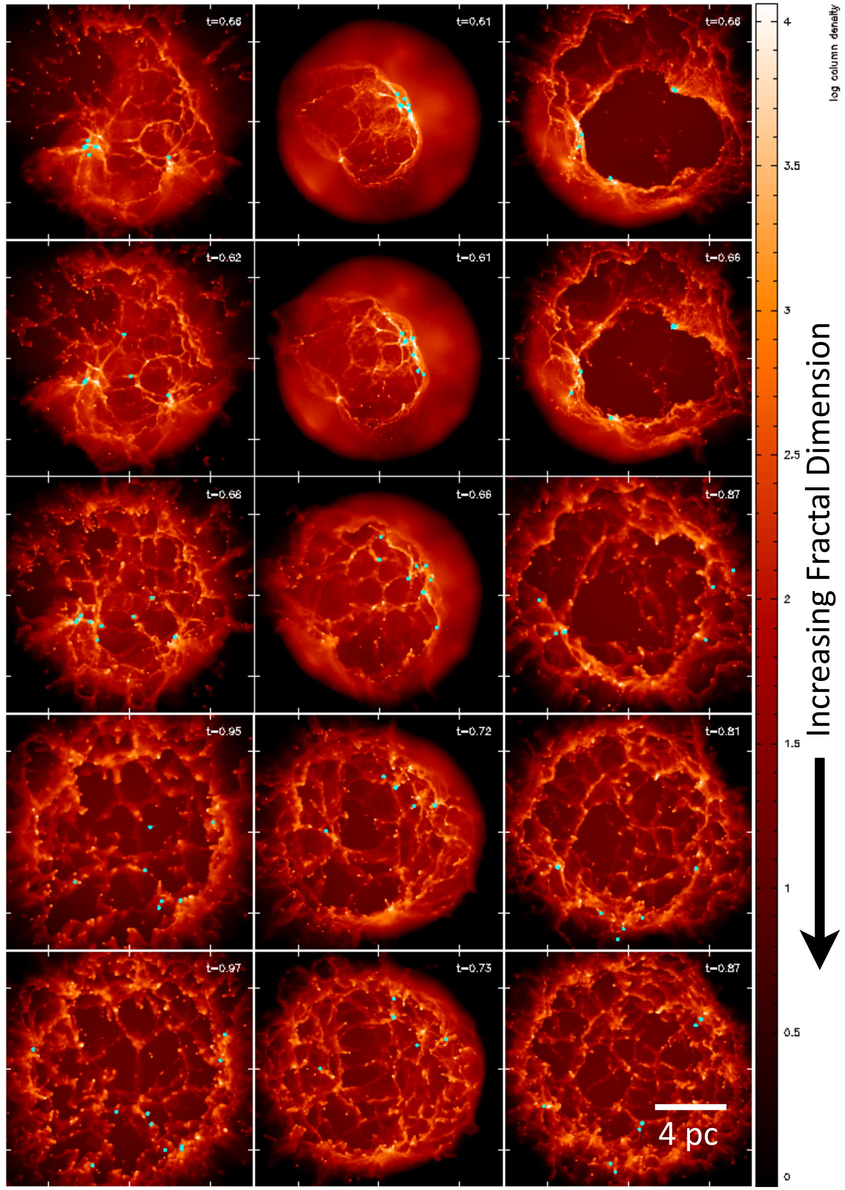


Hot star wind breaking out of a molecular cloud
Rogers & Pittard (2013, MNRAS, 431, 1337)

  - Hot gas can entrain cold ISM via Kelvin-Helmoltz instabilities and eventually remove all the cold gas from a star-forming cluster, but unclear if it is dominant

Hot Gas Feedback: Photoionization

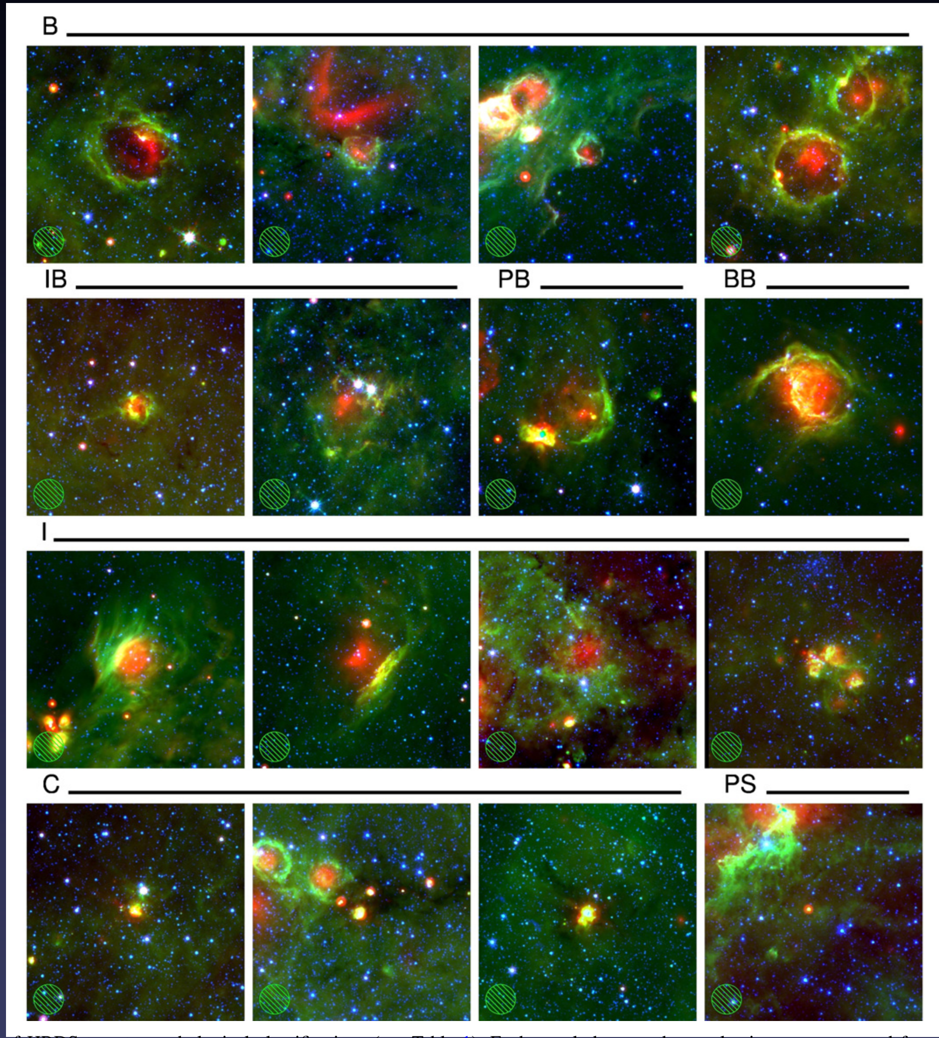
- Massive stars (>10 $M_\odot$) emit large numbers of ionizing photons
  - Create ionized bubbles (HII regions) with $T=10^4$ K, $c_s=10$ km/s
- Over-pressure causes expansion
  - Uniform medium (Spitzer 1978)
  - But interaction between cold ISM and hot gas is complex
- Potential effects
  - Limiting the growth of massive stars?
  - Triggering new star formation?
  - Drive GMC turbulence?
  - Disrupt GMCs?



Walch et al. (2013, arXiv/1306.4317)

Photoionization: Observations

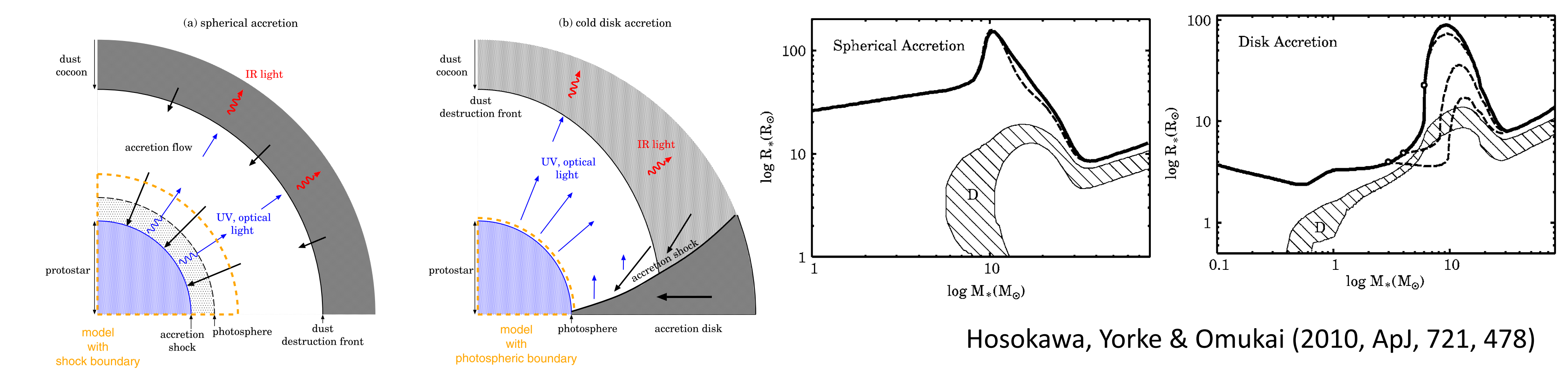
- Can now detect HII regions from single O-stars at 12 kpc (Anderson et al. 2011)
  - Many hundreds of HII regions
- Recent analysis concentrates on interaction of HII regions with
  - IR dust bubbles (e.g. Watson et al. 2008; Deharveng et al. 2010; Anderson et al. 2011)
  - molecular gas (e.g. Anderson et al. 2011)
  - stellar winds (e.g. Townsley et al. 2003)



Green Bank HII Region Discovery Survey
Spitzer RGB images: 24, 8, 3.6 μm
Anderson et al. (2011, ApJS, 194, 32)

Photoionization: Theory

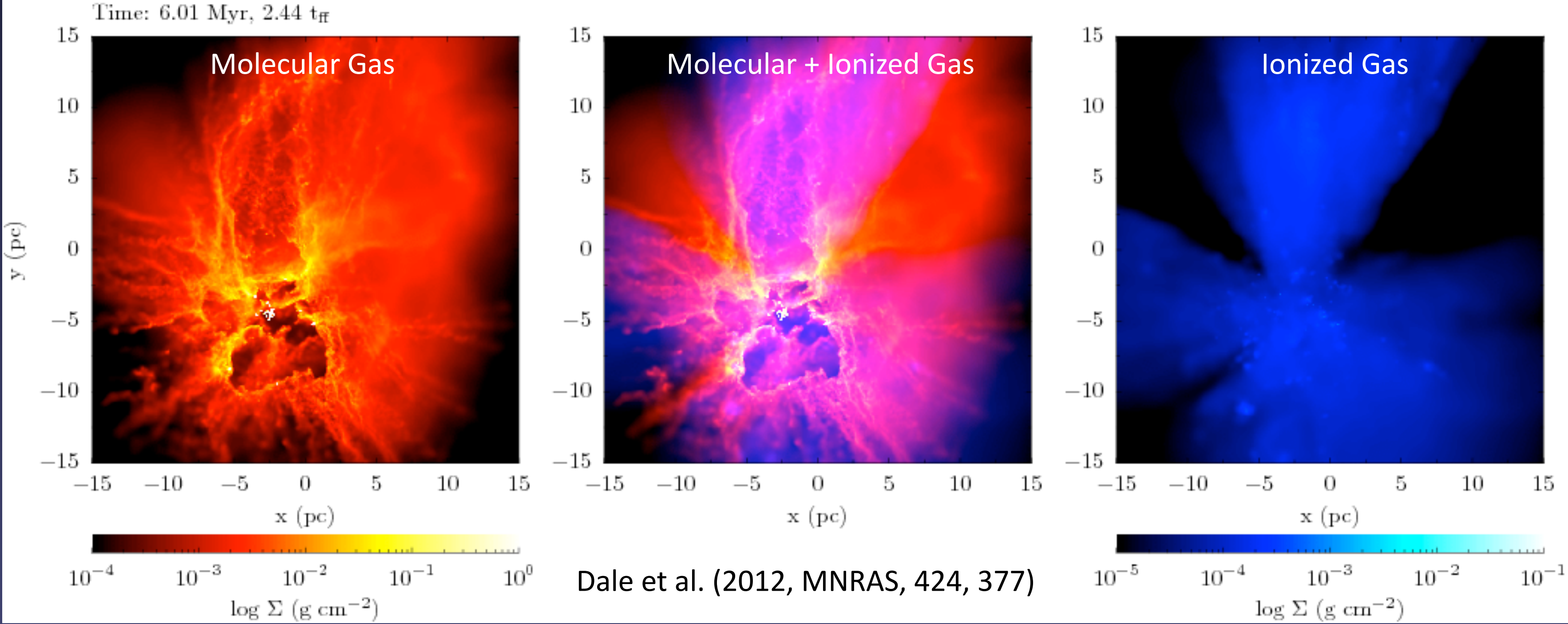
- Often invoked to limit the growth of OB stars
  - However, accretion flows can cause HII expansion to stall or reverse (Walmsley 1995)
  - Or ionised accretion can continue in gravitationally-trapped HII regions (Keto 2003), supported by observations (Keto & Wood 2006; Klaassen & Wilson 2007)
  - Or accretion can continue via disc accretion (e.g. Yorke & Sonnhalter 2002)
  - Rapid accretion can also cause stars to expand, cooling their photospheres and reducing ionizing flux (Hosokawa et al. 2010, 2012)
  - In some simulations ionization is unable to disrupt accretion until cold material is drained by other lower-mass accretors (Peters et al. 2010a,b)



Hosokawa, Yorke & Omukai (2010, ApJ, 721, 478)

Photoionization: Theory

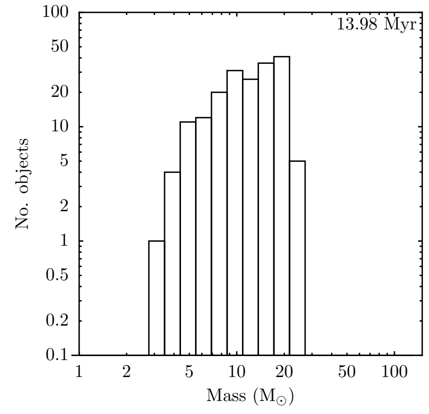
- Simulations of HII region expansion into turbulent clouds have been performed (Melleman et al. 2006; Mac Low et al. 2007; Gritschneder et al. 2009; Dale et al. 2012, 2013)
  - But further study required to determine whether could drive turbulence
- Destruction of low-mass clouds certainly seems possible
  - 10^4 $M_\odot$ fractal clouds can be destroyed ~ 1 Myr by a single O7 star which drives a mass-weighted rms velocity of 6 km/s - only 1% of the O7 star's energy (Walch et al. 2012)
  - Ionization generally destroys clouds with escape velocities well below ~10 km/s but becomes ineffective when the escape speed reaches this value (Dale et al. 2005, 2011, 2012, 2013)



Dale et al. (2012, MNRAS, 424, 377)

Triggering

- Although negative feedback may dominate, positive feedback may occur in some circumstances
  - Dale et al. (2007) make a distinction between
    - Weak triggering - temporarily increase star formation rate by inducing stars to form earlier
    - Strong triggering - increase star formation efficiency by causing the birth of stars that would not otherwise form
- Shells of molecular gas swept-up by HII regions or winds may fragment to form stars (e.g. Whitworth et al. 1994)
  - May produce a top-heavy IMF (Whitworth et al. 1994; Dale et al. 2011)
  - Prospect of star formation as a self-propagating process (Shore 1981, 1983)
  - However, simulations of ionizing feedback in fractal (Walch et al. 2011) or turbulent clouds (Dale et al. 2007, 2012, 2013) find the rate, efficiency, and number of stars can all be changed, but not the IMF



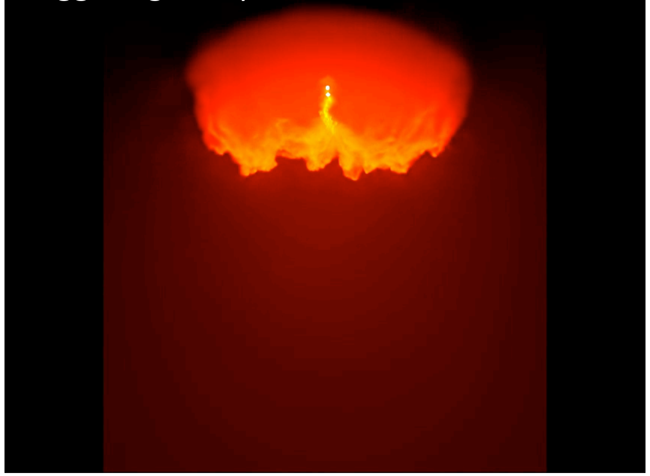
Top-heavy IMF

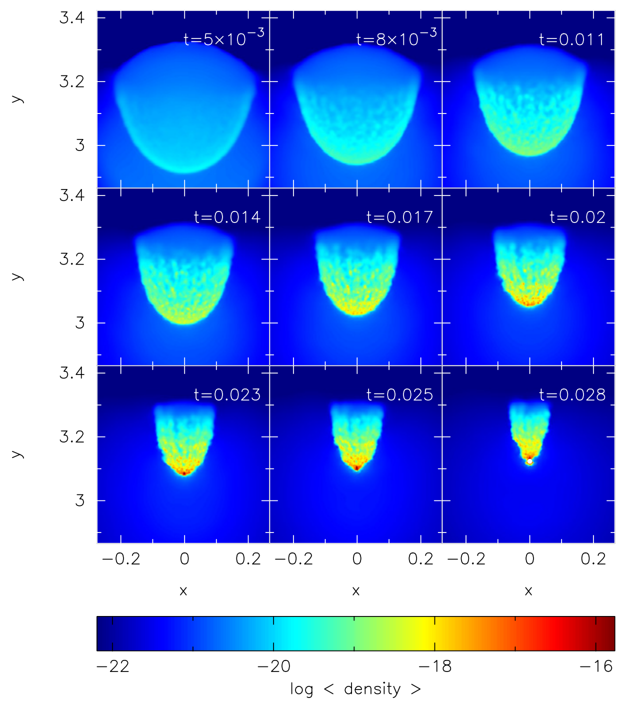
Dale et al. (2011, MNRAS, 411, 2230)

Radiation Driven Implosion

- Stable molecular cloud cores can be induced to collapse by winds or HII regions (e.g. Sandford et al. 1982; Klein et al. 1983; Kessel-Deynet & Burkert 2003; Bisbas et al. 2011; Haworth & Harries 2012)
    - Example of positive feedback and triggering

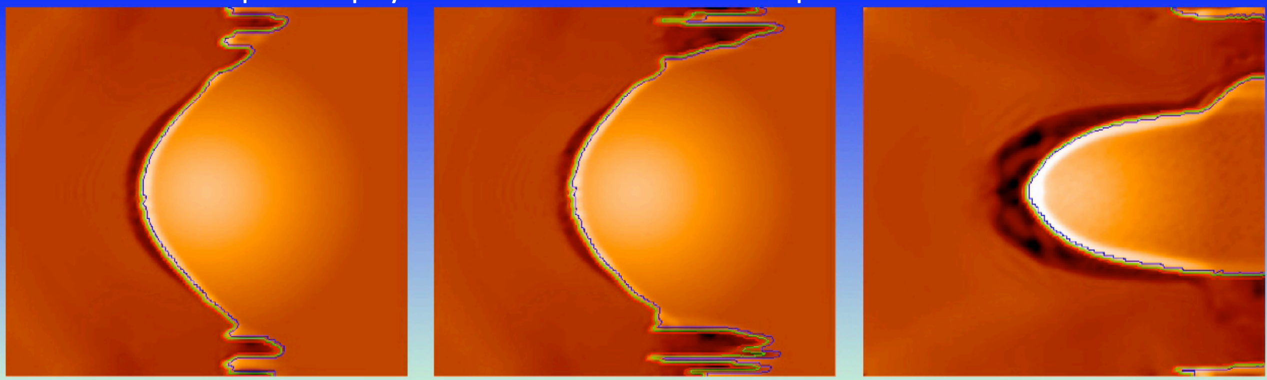
Triggering collapse of a stable core





Bisbas et al. (2011, ApJ, 736, 142)

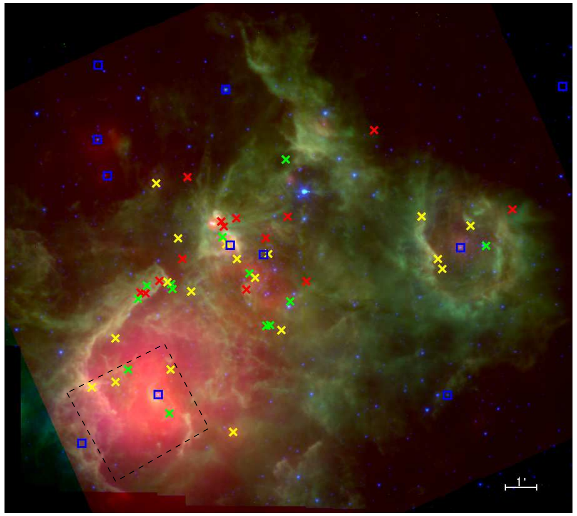
The effect of improved physics on Radiation Driven Implosion



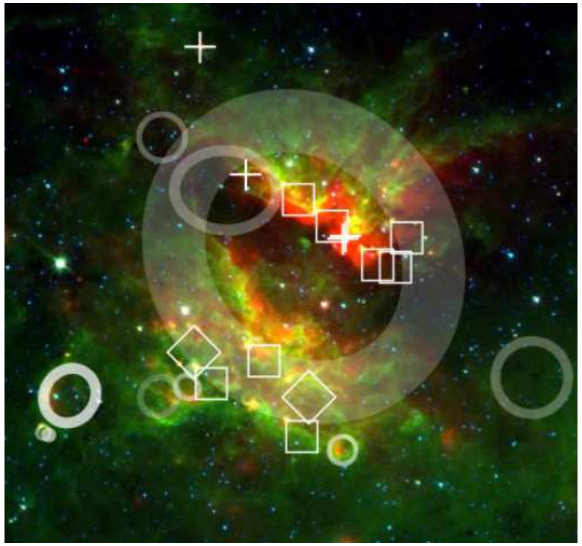
Haworth & Harries (2012, MNRAS, 420, 562)

Triggering: Observations

- Evidence for triggering difficult to establish observationally
  - Searching for young stars near bubbles, ionization fronts, bright-rimmed clouds or pillars (e.g. Urquhart et al. 2007; Deharveng et al 2008; Snider et al. 2009; Smith et al. 2010)
  - Looking for age gradients or associations with feedback sources (e.g. Sugitani et al 1995; Chauhan et al. 2009; Getman et al. 2012)
  - Casual links claimed when age difference is less than the crossing time (e.g. CepOB2 bubble; Patel et al. 1998; MonR2 GMC; Xie & Goldsmith 1994)
  - Higher star-formation efficiencies (Getman et al. 2009, 2012)
- Contribution of triggering to overall star formation rate
  - Correlation between bubbles and YSOs imply ~10-20% level (Kendrew et al. 2012; Thompson et al. 2012)
  - Negative feedback dominates on GMC scales (Dale et al. 2012, 2013)



Young stars near ionization fronts: NGC2467
Snider et al. (2009, ApJ, 700, 506)



Bubble with most near young stars
Kendrew et al. (2012, ApJ, 755, 71)

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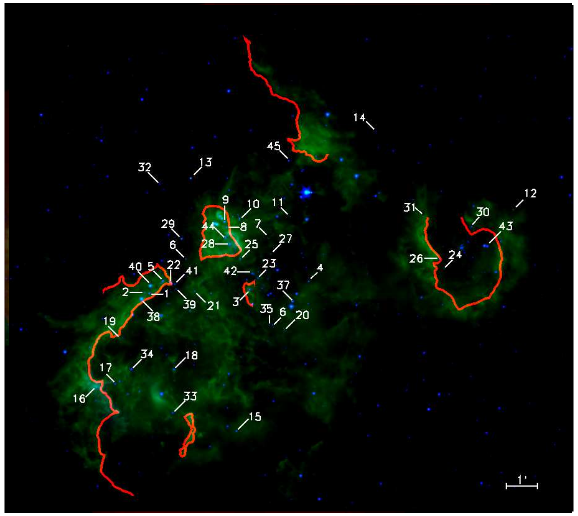

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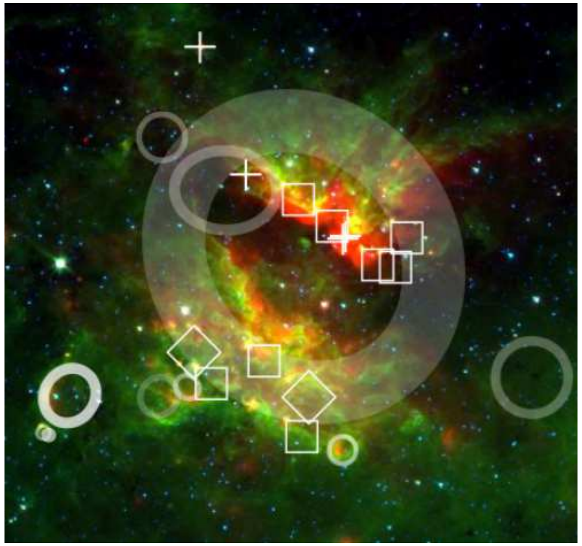


Bubble with most near young stars
Kendrew et al. (2012, ApJ, 755, 71)

Hot Gas Feedback: Supernovae

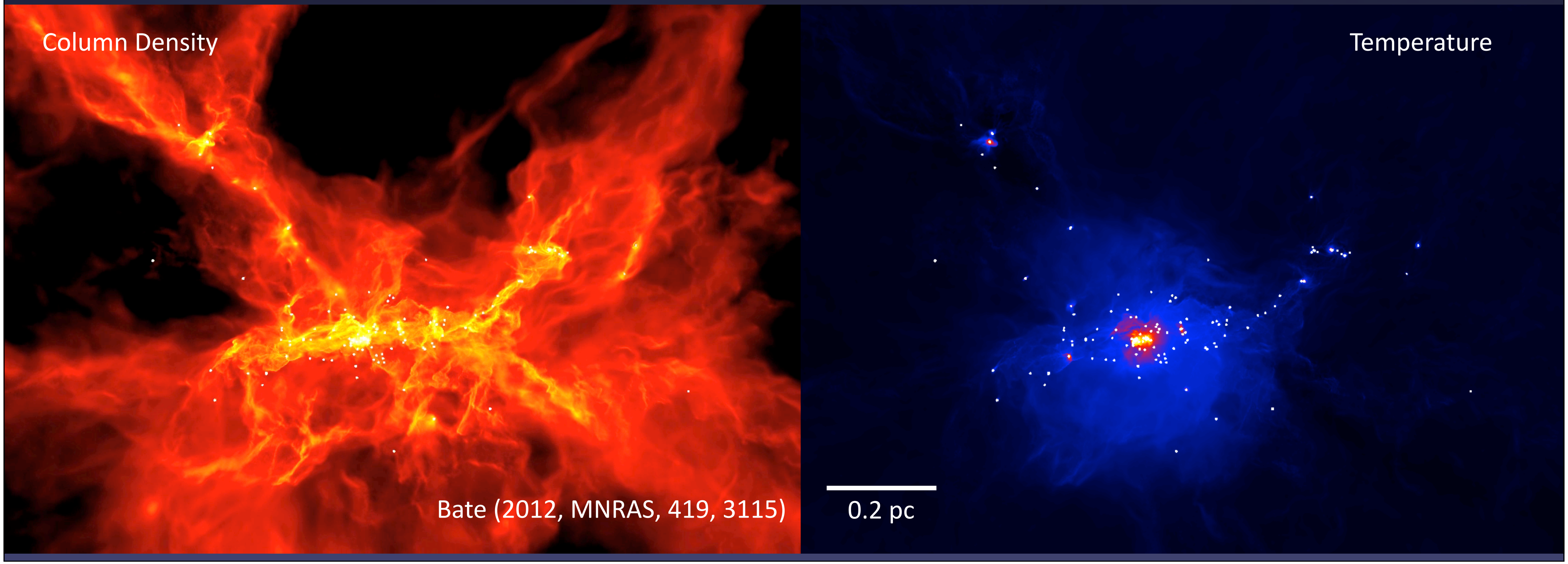
- Occur at a rate of ~1 per 100 $M_\odot$ of stars formed (e.g. Dekel & Krumholz 2013)
  - Much thermal energy stored in hot gas with long cooling time
- But occur >4 Myr after onset of star formation
- 30 Doradus only one detectable supernova remnant, radius much smaller than the evacuated bubble (e.g. Lopez et al. 2011)
- Westerland 1, no supernova remnant detected (Muno et al. 2006)
- Both clusters have managed to eject gas, implying other forms of feedback dominate
- However, supernovae are one of the prime candidates for feedback on GMC and galactic scales (1st talk on Monday)
  - Dynamical timescales comparable or much greater than massive star lifetime

The Origins of Thermal Feedback

- Following stellar core formation: 3 sources of thermal feedback
  - Intrinsic luminosity from the stellar core itself
  - Luminosity from accretion onto the stellar core: $L_{\rm acc} \approx \frac{G M_* \dot{M}_*}{R_*}$
  - Luminosity from the continued collapse of the cloud and disc accretion
- Which dominates?
  - Accretion luminosity for low-mass protostars (<3 $M_\odot$) accreting at rates >10^{-6} $M_\odot$/yr
    - e.g. 1 $M_\odot$ star with radius 2 $R_\odot$ accreting at 10^{-6} $M_\odot$/yr has an accretion luminosity of 15 $L_\odot$ and an intrinsic luminosity of 1 $L_\odot$ (Hosokawa & Omukai 2009)
  - Intrinsic stellar luminosity dominates for high-mass protostars (>9 $M_\odot$) and realistic accretion rates (<10^{-3} $M_\odot$/yr)
- Details depend on
  - Energy released in other forms (e.g. outflows)
  - Whether accretion is steady or episodic, and the entropy of the material accreted
    - Alters stellar structure (e.g. Baraffe et al. 2009; Hosokawa et al. 2011)
    - Alters thermal feedback through variable accretion luminosity (e.g. Stamatellos et al. 2011)

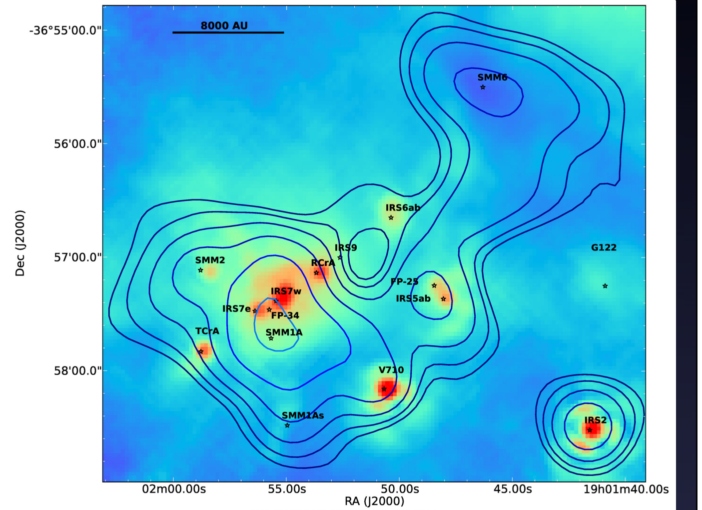
Magnitude of the Heating by Thermal Feedback

- Krumholz (2006) analytically estimated the effect of accretion luminosity on the temperature structure of a variety of massive molecular cloud cores
  - Sub-solar-mass protostars can heat interiors to 100 K at 100s AU or 30 K at 1000s of AU
- This is confirmed in 3-D radiation hydrodynamical calculations
  - e.g. Whitehouse & Bate (2006); Krumholz et al. (2007); Bate (2009); Offner et al. (2009)

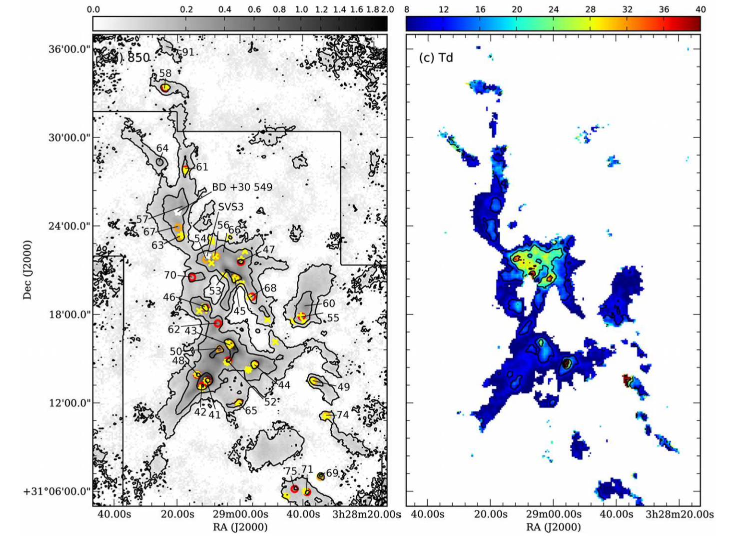


Observations of Thermal Feedback

- Recently, new telescopes have allowed us to start directly measuring the effects of thermal feedback on sub-parsec scales
  - Longmore et al. 2011: ATCA+SMA
  - Zhang et al. 2009, Wang et al. 2012: JVLA
  - Hatchell et al. 2013: JCMT/SCUBA-2
  - Sicilia-Aguilar et al. 2013: Herschel

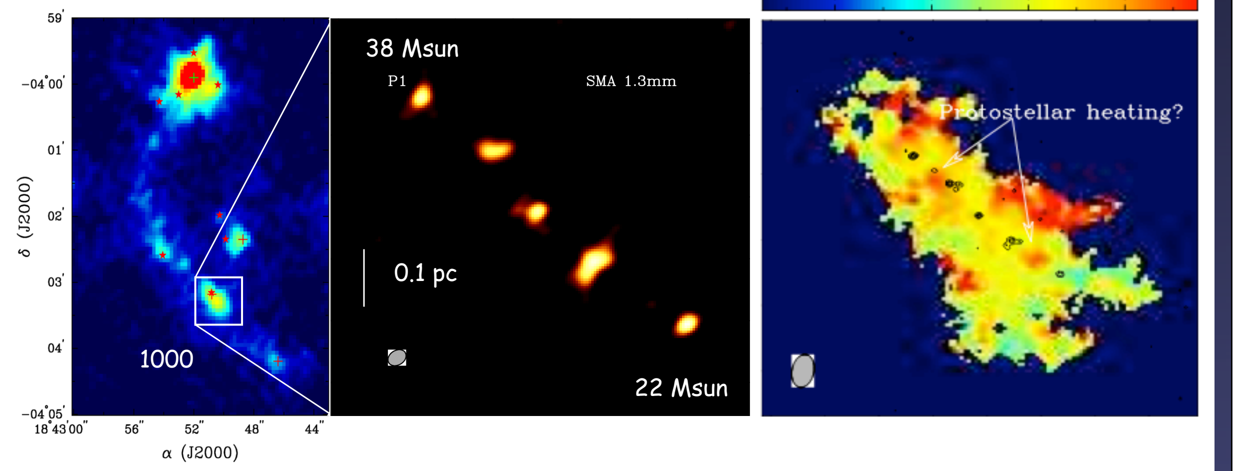


Corona Australis with Herschel
Sicilia-Aguilar et al. (2013, A&A, 511, 34)



NGC 1333 / SCUBA-2; Hatchell et al. (2013, MNRAS, 429, L10)

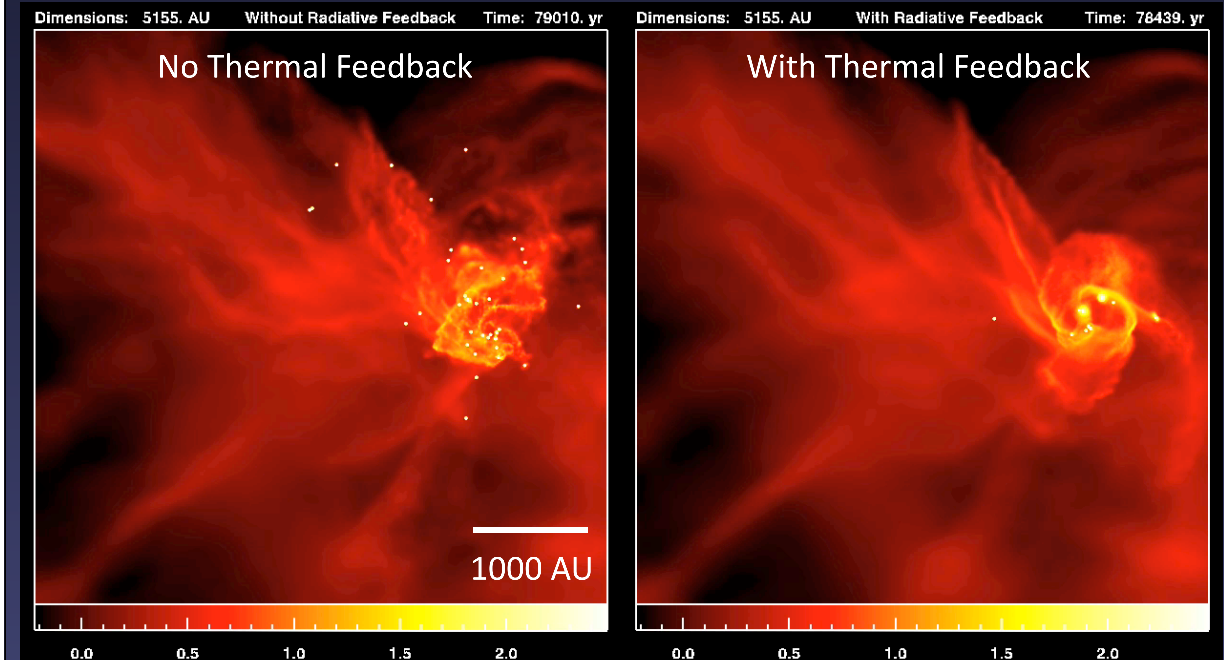
Formation of massive cores: IRDC G28.34



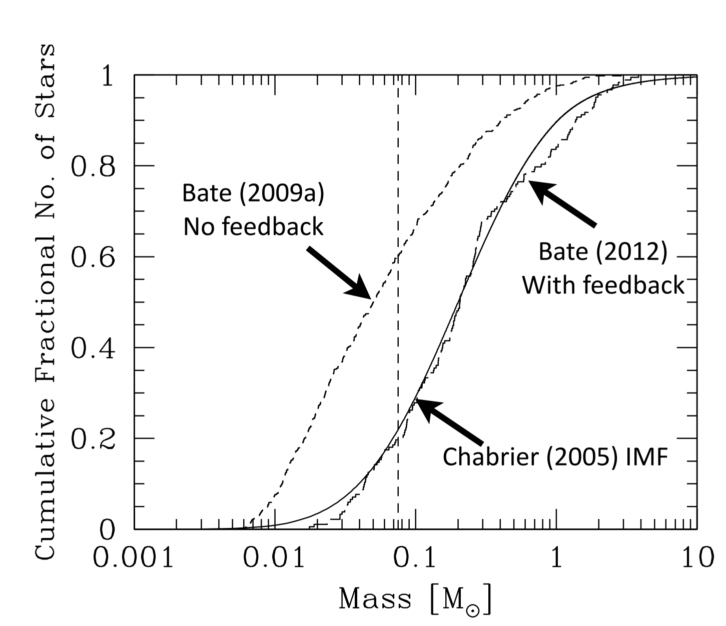
Rathborne et al. (2006, ApJ, 641, 389); Zhang et al. (2009, ApJ, 696, 268); Wang et al. (2012, ApJ, 745, L30)

Influence on IMF: low-mass end

- Thermal feedback has huge effect (Bate 2009, 2012; Offner et al. 2009; Urban et al. 2010)
  - Reduces number of objects by factors of 4-5, particularly inhibiting disc fragmentation
  - Reduces the proportion of brown dwarfs, bringing it into agreement with observations
  - Weakens the dependence of the IMF on the initial Jeans mass in clouds, helping to produce a `universal' IMF (Bate 2009; Krumholz 2011)
- Large-scale RHD simulations produce good agreement with the observed IMF



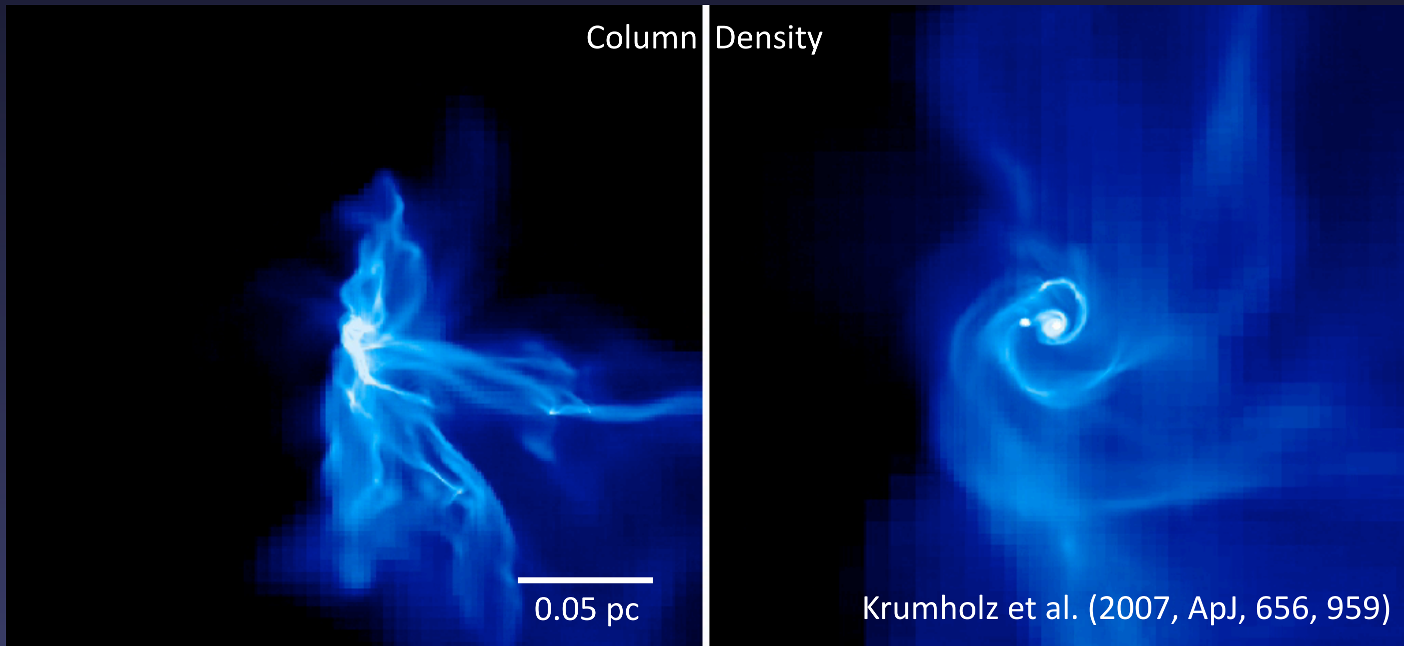
Bate (2009, MNRAS, 392, 1363)



Bate (2012, MNRAS, 419, 3115)

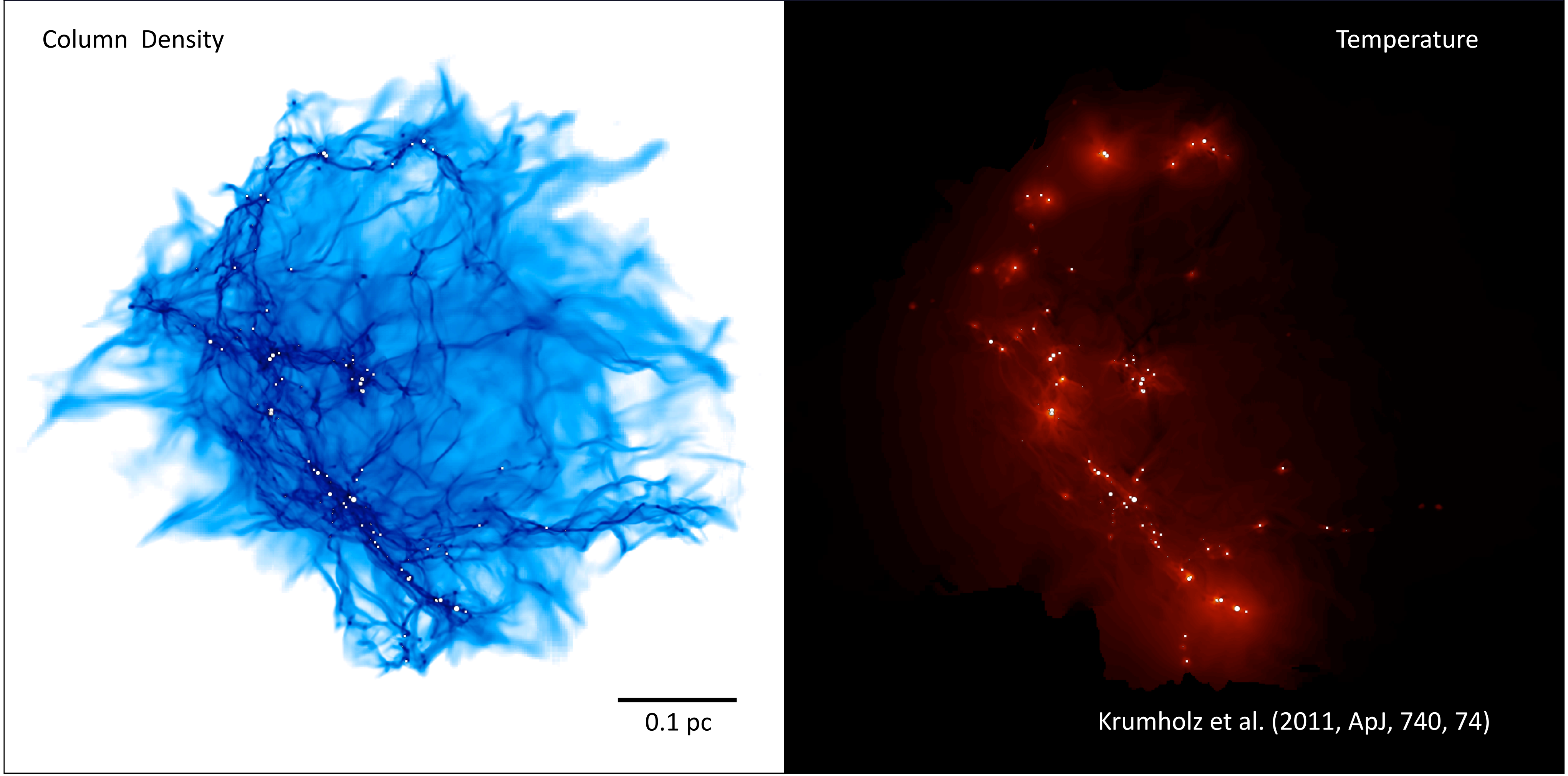
Influence on IMF: high-mass end

- Without thermal feedback a massive dense cloud fragments into many stars
  - Typically leads to a dense protostellar cluster evolving according to competitive accretion (e.g. Bonnell et al. 2004)
- Thermal feedback can substantially alter this picture
  - May be strong enough to exclude most fragmentation and allow only a few massive stars to form (e.g. Krumholz et al. 2007)



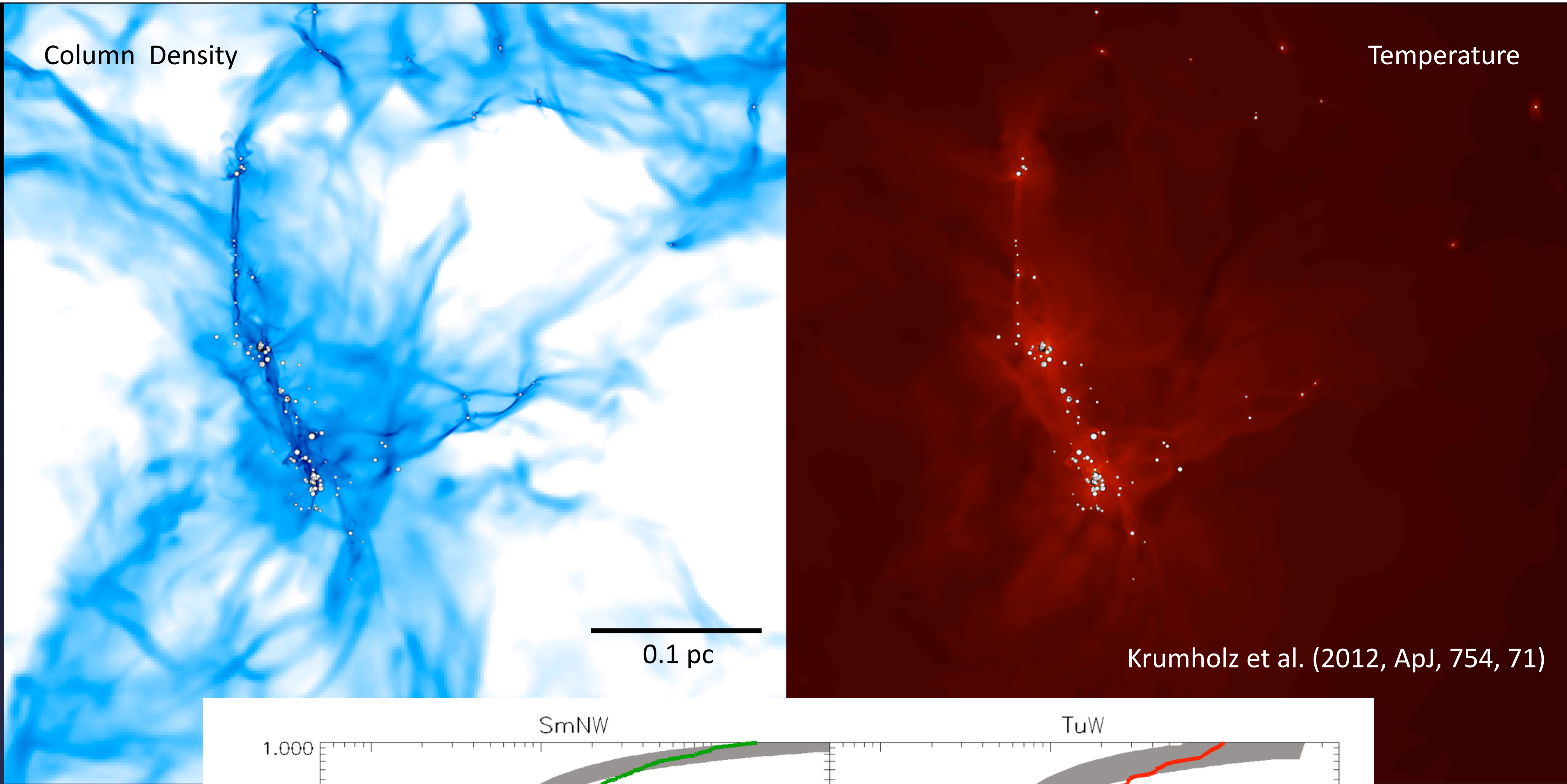
  - An implication is that massive stars may preferentially form in regions with high densities (Krumholz & McKee 2008; Krumholz et al. 2010)

- However, thermal feedback can also be too effective (Krumholz et al. 2011)
  - An `overheating problem' can occur whereby as a cluster forms the heating grows, inhibiting further fragmentation
  - Characteristic mass that increases with time, eventually producing a top-heavy IMF

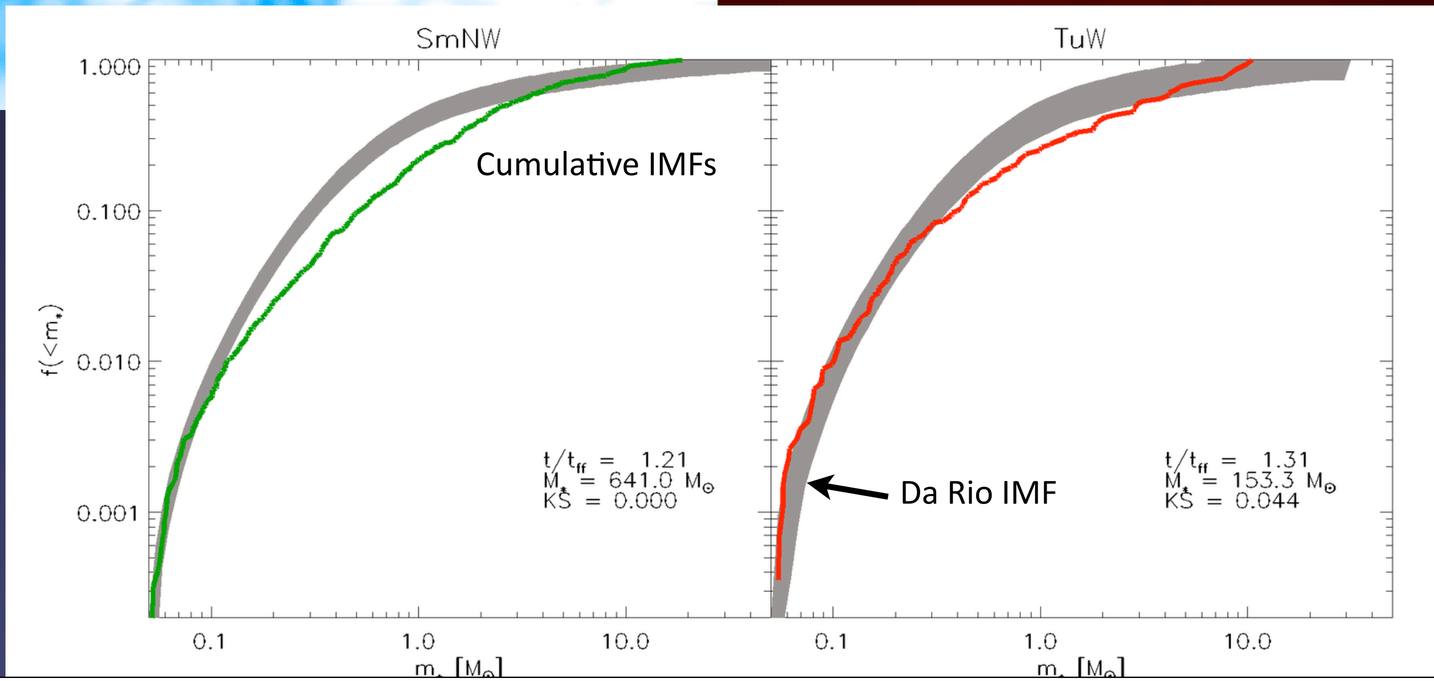


Combinations of Feedback Mechanisms

- Feedback mechanisms frequently act simultaneously
  - Low-mass star formation: outflows, thermal feedback (and photoionization for discs, e.g. Hollenbach et al. 1994; Clarke et al. 2001)
  - High-mass star formation: thermal, radiation pressure, photoionization, outflows are inseparable early on; later hot stars winds and supernovae
- Few simulations with multiple mechanisms to date
  - Cunningham et al (2011) include thermal feedback, radiation pressure, and outflows for massive star formation calculations, finding outflow cavities create an escape route for radiation, reducing effects of radiation pressure
  - Hansen et al. (2012) include thermal feedback and outflows for low-mass clustered star formation: outflows reduce accretion rates, thermal feedback and stellar masses
  - Krumholz et al. (2012) show that outflows and turbulence driven on large scales reduce the `overheating problem' caused by thermal feedback in massive dense star-forming clouds



Includes decaying turbulence, excludes outflows

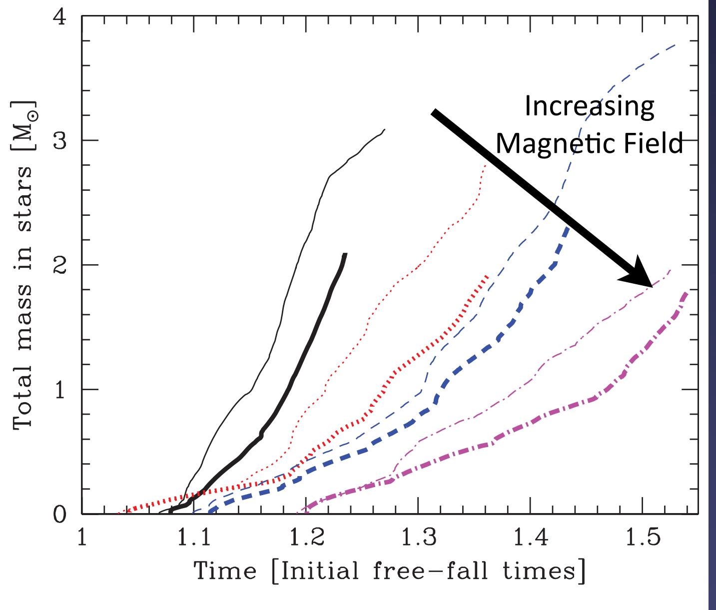


Includes driven large-scale turbulence, and includes outflows

Interactions of Thermal and Magnetic Effects

- Thermal feedback from low-mass stars primarily depends on accretion rate
- Magnetic fields can be highly effective at transporting angular momentum, potentially increasing accretion rates
  - Commercon et al. (2010, 2011) and Myers et al. (2013) showed that magnetised star formation can produce higher accretion rates and enhance thermal feedback
- On larger-scales, magnetic fields and thermal feedback can act together to inhibit fragmentation
  - Thermal feedback inhibits small-scale fragmentation, while magnetic support inhibits large-scale fragmentation (Price & Bate 2009; Myers et al. 2013)
  - Thermal feedback alone does not substantially affect the rate at which gas is converted into (Price & Bate 2009; Krumholz et al. 2010; Bate 2012)
  - But combining them can lead to much lower star formation rates than without either

Price & Bate (2009, MNRAS, 398, 33)



Future Prospects

- Observations
  - Need to improve our understanding in nearby low-mass environments
  - Need larger samples in more extreme (distant) environments
  - ALMA will be key in
    - Studying protoclusters with high angular resolution and sensitivity
- Simulation and theory
  - No simulation currently includes all feedback mechanisms
    - Further code development will continue, both in terms of physics and the use of large-scale parallel computers
    - By PPVII it is probable that there will be multiple codes that include magnetic fields and model a wide range of feedback, more accurately than today
  - Progress in initial conditions should also be tackled, investigating cloud formation and star formation together
  - Finally, current simulations explore a very limited range of parameter space, making it difficult to draw general conclusions