# Sternentstehung - Star Formation

## Winter term 2017/2018

Henrik Beuther & Thomas Henning

*17.10 Today: Introduction & Overview (H.B.)*
*24.10 Physical processes I (H.B.)*
*31.10 no lecture – Reformationstag*
*07.11 Physcial processes II (H.B.)*
*14.11 Molecular clouds as birth places of stars (H.L.)*
**21.11 Molecular clouds cont., virial & Jeans Analysis (H.B.)**
28.11 Collapse models I (H.B.)
05.12 Collapse models II (T.H.)
12.12 Protostellar evolution (T.H.)
19.12 Pre-main sequence evolution & outflows/jets (T.H.)
09.01 Accretion disks I (T.H.)
16.01 Accretion disks II (T.H.)
23.01 High-mass star formation, clusters and the IMF (H.B.)
30.01 Planet formation (T.H.)
06.02 Examination week, no star formation lecture

Book: Stahler & Palla: The Formation of Stars, Wileys

More Information and the current lecture files: http://www.mpia.de/homes/beuther/lecture_ws1718.html

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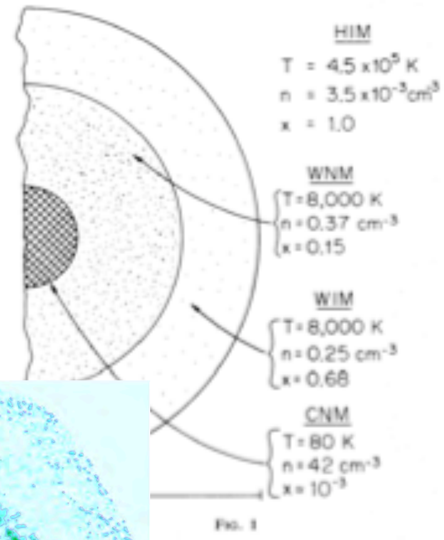
# Last week

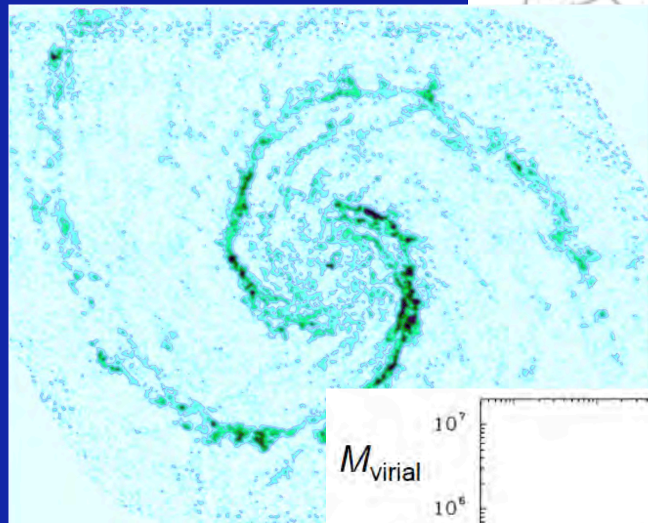
- Different components of ISM, early models

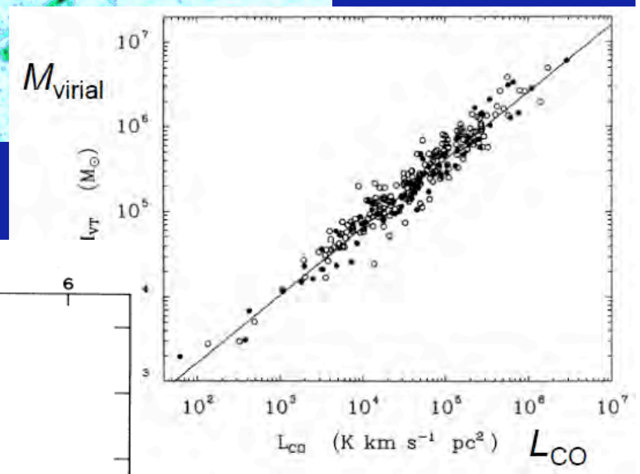
- Basic characteristics

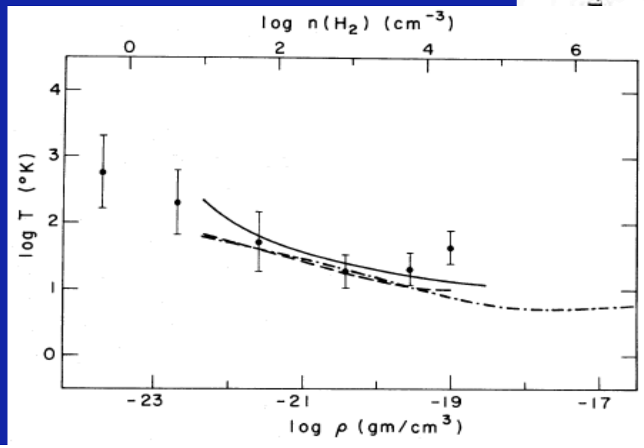
- Important cloud relations

- Cloud fragmentation





# Topics today

- Virial theorem
- Jeans analysis for gravitational instability
- Magnetic fields
- Cloud formation and turbulence

# Virial Analysis

What is the force balance within any structure in hydrostatic equilibrium?

The generalized equation of hydrostatic equlibrium including magnetic fields **B** acting on a current **j** and the full convective fluid velocity **v** is:

$$\rho \, D\mathbf{v}/Dt = -\mathrm{grad}(P) - \rho \, \mathrm{grad}(\Phi_g) + 1/c \, \mathbf{j} \times \mathbf{B}$$

$$D\mathbf{v}/Dt = (\partial \mathbf{v}/\partial t)_x + (\mathbf{v} \, \mathrm{grad})\mathbf{v}$$

$1/2(\partial^2 I/\partial t^2)$ -2T 2U W *M*

(D**v**/D**t** includes the rate of change at fixed spatial position x $(\partial \mathbf{v}/\partial t)_x$ and the change induced by transporting elements to a new location with differing velocity.)

Employing the Poisson equation ($\Delta\Phi_g = 4\Pi G\rho$) and requiring mass conservation, one gets after repeated integrations the **VIRIAL THEOREM**

$$1/2 \, (\delta^2 I/\delta t^2) = 2T + 2U + W + M$$

I: Moment of inertia, this decreases when a core is collapsing ($m*r^2$)
T: Kinetic energy U: Thermal energy W: Gravitational energy *M*: Magnetic energy
All terms except W are positive. To keep the cloud stable, the other forces have to match W.

# Application of the Virial Theorem I

If all forces are too weak to match the gravitational energy, we get

$1/2\ (\delta^2 I/\delta t^2) = W \sim Gm^2/r$

Approximating further $I=mr^2$, the free-fall time is approximately

$t_{ff} \sim \mathrm{sqrt}(r^3/Gm)$

Since the density can be approximated by $\rho=m/r^3$, one can also write

$t_{ff} \sim (G\rho)^{-1/2}$

Or more exactly for a pressure-free 3D homogeneous sphere

$t_{ff} = (3\pi/32G\rho)^{1/2}$

For a giant molecular cloud, this corresponds to

$t_{ff} \sim 7*10^6\ \mathrm{yr}\ (m/10^5 M_{sun})^{-1/2}\ (R/25\mathrm{pc})^{3/2}$

For a dense core with $\rho\sim10^5 \mathrm{cm}^{-3}$ the $t_{ff}$ is approximately $10^5$ yr.

However, no globally collapsing GMCs observed → add support!

# Application of the Virial Theorem II

If the cloud complexes are in approximate force equilibrium, the moment of inertia actually does not change significantly and hence $1/2\ (\delta^2 I/\delta t^2)=0$

$$2T + 2U + W + \mathcal{M} = 0$$

This state is called VIRIAL EQUILIBRIUM. What balances gravitation W best?

Thermal Energy: Approximating U by $U \sim 3/2Nk_BT \sim m\mathcal{R}T/\mu$

$$U/|W| \sim m\mathcal{R}T/\mu\ (Gm^2/R)^{-1}$$

$$= 3*10^{-3}\ (m/10^5M_{sun})^{-1}\ \ (R/25pc)\ \ (T/15K)$$

--> Clouds cannot be supported by thermal pressure alone!

Magnetic energy: Approximating $\mathcal{M}$ by $M \sim B^2r^3/6$ (cloud approximated as sphere)

$$\mathcal{M}/|W| \sim B^2r^3/6\ (Gm^2/R)^{-1}$$

$$= 0.3\ (B/20\mu G)^2\ (R/25pc)^4\ \ (m/10^5M_{sun})^{-2}$$

--> Magnetic force is important for large-scale cloud stability!

# Application of the Virial Theorem III

The last term to consider in 2T + 2U + W + *M* = 0 is the kinetic energy T

$$T/|W| \sim 1/2 m\Delta v^2 \, (Gm^2/R)^{-1}$$
$$= 0.5 \, (\Delta v/4\mathrm{km/s}) \;\; (M/10^5 M_{sun})^{-1} \;\; (R/25\mathrm{pc})$$

Since the shortest form of the virial theorem is $2T = -W$, the above numbers imply that a typical cloud with linewidth of a few km/s is in approximate virial equilibrium.

The other way round, one can derive an approximate relation between the observed line-width and the mass of the cloud:

$$2T = 2^* \, (1/2 m\Delta v^2) = -W = Gm^2/r$$

→ virial velocity: $v_{vir} = (Gm/r)^{1/2}$

→ or virial mass: $m_{vir} = v^2 r/G$

# Application of the Virial Theorem III

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/25pc)

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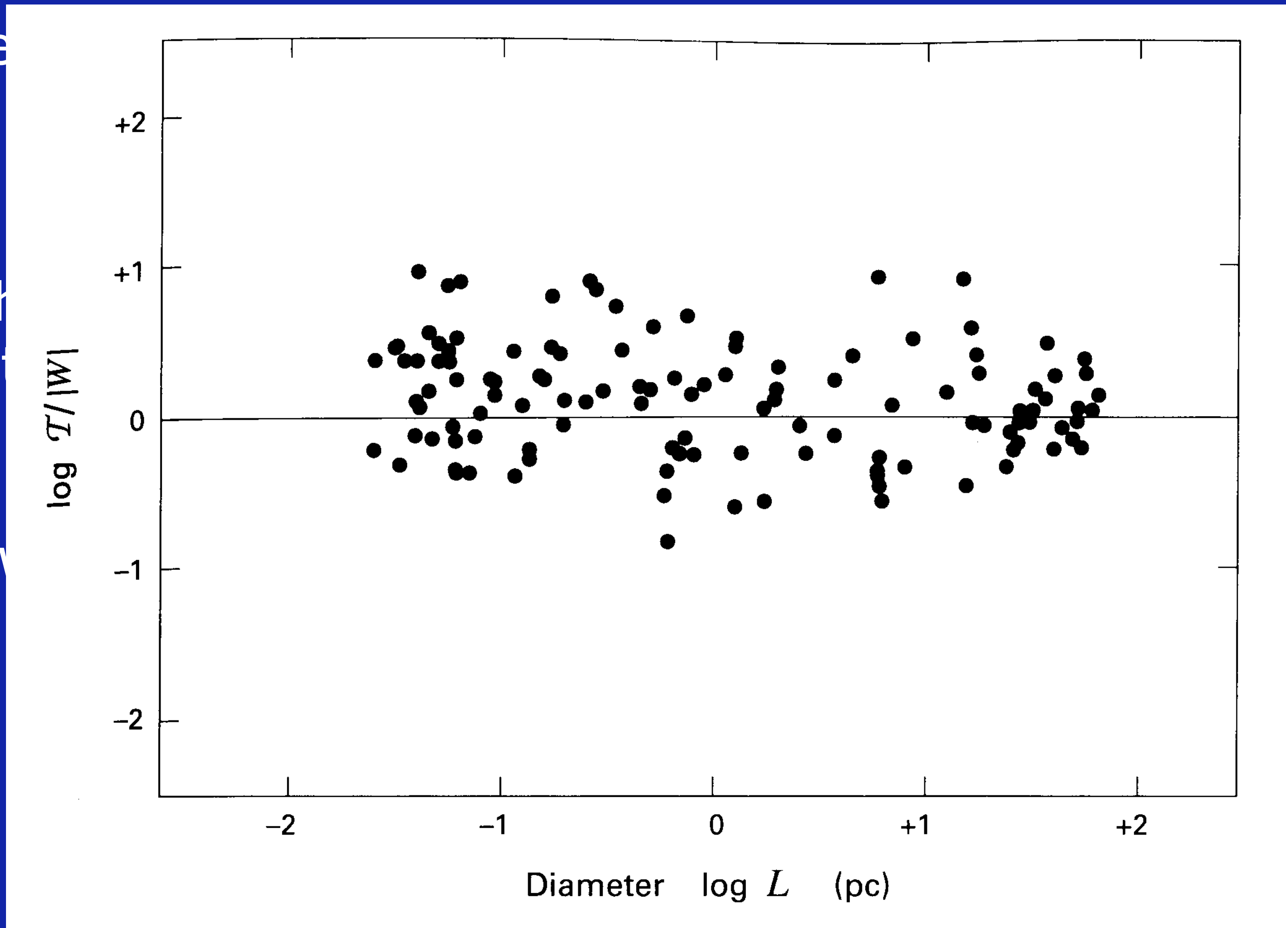
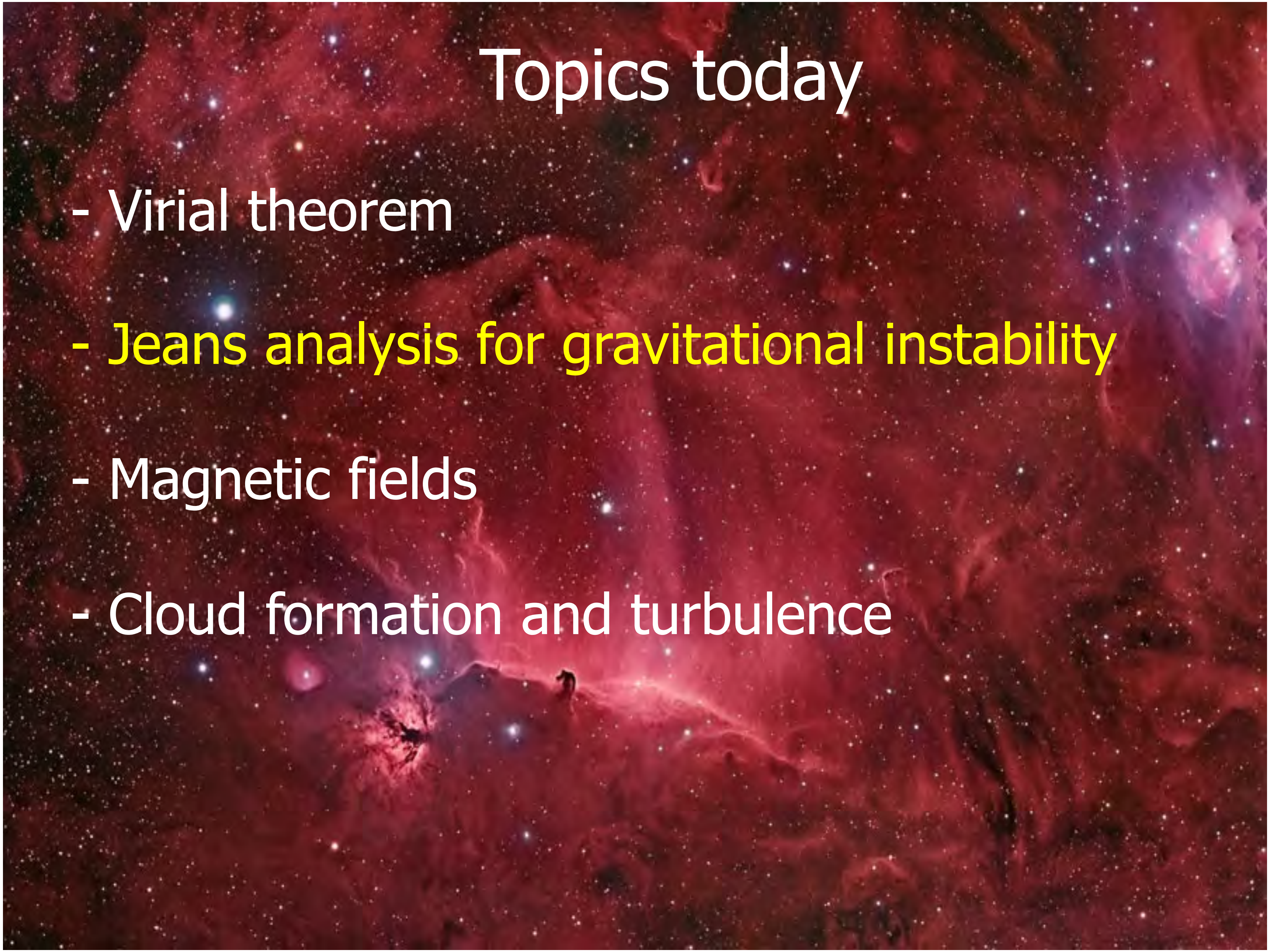



Topics today
- Virial theorem
- Jeans analysis for gravitational instability
- Magnetic fields
- Cloud formation and turbulence

# Jeans analysis I

Start again with equation of hydrodynamic equilibrium (without magn. Field):

Equation of motion: $\rho\, D\mathbf{v}/Dt = -\mathrm{grad}(P) - \rho\, \mathrm{grad}(\Phi_g)$

$$\rho\, (\partial\mathbf{v}/\partial t) + (\mathbf{v}\ \mathrm{grad})\mathbf{v} = -\mathrm{grad}(P) - \rho\, \mathrm{grad}(\Phi_g)$$

Continuity equation: $(\partial\rho/\partial t) = -\mathrm{grad}(\rho\mathbf{v})$

Poisson equation: $\Delta\Phi_g = 4\pi G\rho$

Static solution: $\rho = \rho_0 = \mathrm{const}$; $P = P_0 = \mathrm{const}$; $\mathbf{v} = \mathbf{v_0} = \mathrm{const}$; $\Phi_g = \Phi_0 = \mathrm{const}$

Little pertubation: linear stability analysis:

$$\rho = \rho_0 + \rho_1;\ P = P_0 + P_1;\ \mathbf{v} = \mathbf{v_0} + \mathbf{v_1};\ \Phi_g = \Phi_0 + \Phi_1$$

(with $|\rho_1| << \rho_0$ etc.)

# Jeans analysis II

Considering only expressions of first order:

$$\partial \mathbf{v_1}/\partial t = - 1/\rho_0 \, \mathrm{grad}(P_1) - \mathrm{grad}(\Phi_1) \qquad \text{(Eq. 1)}$$

$$\partial \rho_1/\partial t = -\rho_0 \, \mathrm{grad}(\mathbf{v_1}) \qquad \text{(Eq. 2)}$$

$$\Delta\Phi_1 = 4\pi G \rho_1 \qquad \text{(Eq. 3)}$$

Using furthermore: $P_1 = a_t^2 \rho_1$ and $a_t = kT/(\mu \, m_H)$

*($a_t$ sound speed; μ mean mass of particle; $m_H$ mass of hydrogen)*

Apply grad to Eq. 1: $\mathrm{grad}(\partial \mathbf{v_1}/\partial t) = -\Delta \, (a_t^2 \rho_1/\rho_0 + \Phi_1)$

Time derivative for Eq. 2: $\partial^2 \rho_1/\partial t^2 = -\rho_0 \, \mathrm{grad}(\partial \mathbf{v_1}/\partial t)$

→ wave equation: $\partial^2 \rho_1/\partial t^2 = a_t^2 \Delta \rho_1 + 4\pi G \rho_0 \rho_1$

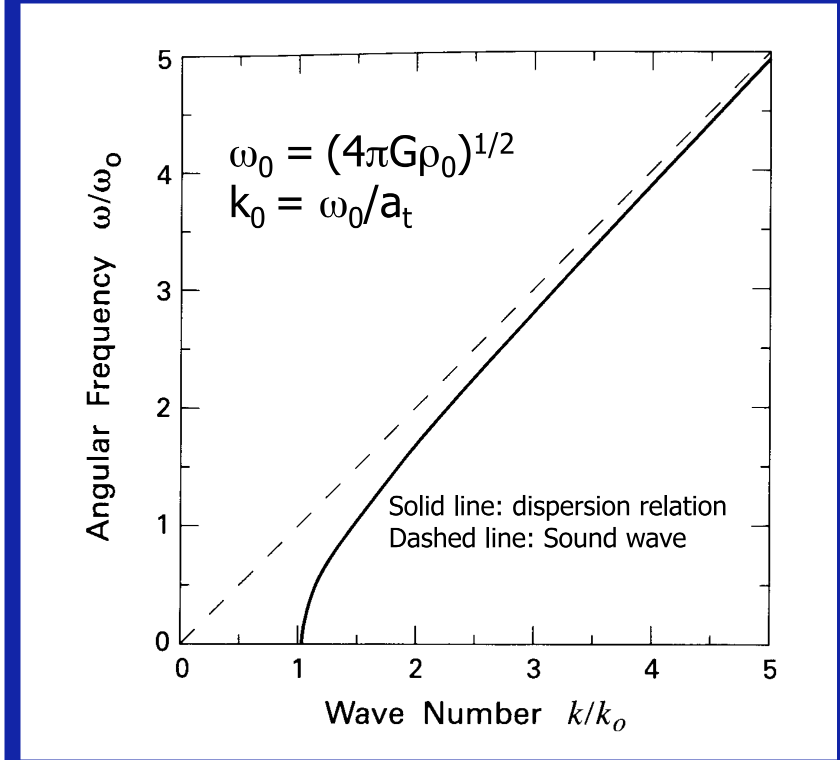
# Jeans analysis III

- A travelling wave in an isothermal gas can be described as:

$\rho(x,t) = \rho_1 \exp[i(kx - \omega t)]$ wave number $k=2\pi/\lambda$ and frequency $\omega$

Then: $\partial^2\rho_1/\partial t^2 = -\omega^2\rho_1$ and $\Delta\rho_1 = -k^2\rho_1$

- This results in *dispersion equation:* $\omega^2 = k^2 a_t^2 - 4\pi G\rho_0$




Large k high-frequency disturbances
→ sound wave $\omega=ka_t$
→ isothermal sound speed of background

Low k $(k<=k_0)$ $\omega^2 \rightarrow 0$.
→ Jeans-length: $\lambda_J = 2\pi/k_0 = (\pi a_t^2/G\rho_0)^{1/2}$

Perturbations larger $\lambda_J$ have exponentially growing amplitudes → instable

# Jeans analysis IV

This corresponds in physical units to Jeans-lengths of

$$\lambda_J = (\pi a_t^2/G\rho_0)^{1/2} = 0.19\text{pc}\,(T/(10K))^{1/2}\,(n_{H2}/(10^4\text{cm}^{-3})^{-1/2}$$

and Jeans-mass

$$M_J = m_1 a_t^3/(\rho_0^{1/2} G^{3/2}) = 1.0M_{sun}\,(T/(10K))^{3/2}\,(n_{H2}/(10^4\text{cm}^{-3})^{-1/2}$$

→ Clouds larger $\lambda_J$ or more massive than $M_J$ may be prone to fragmentation.

→ Conversely, small or low-mass cloudlets could be stable if there is sufficient external pressure. Otherwise only transient objects.

# Jeans analysis V

## **Examples:**

Small HI cloud:

$T \sim 100K$; $n_H \sim 20\ cm^{-3}$; $L \sim 5pc$; $M \sim 20M_{sun}$

→ $L_J \sim 13pc$

→ Jeans stable

Giant Molecular Cloud (GMC)

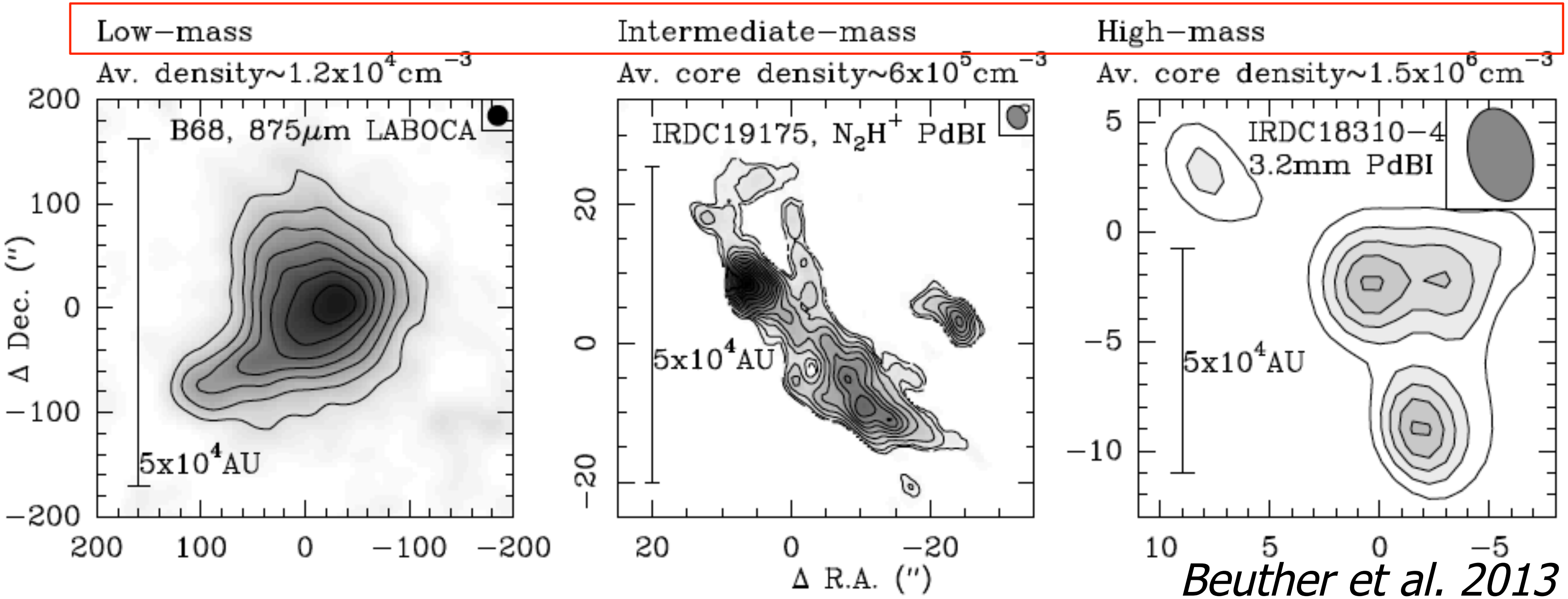
$T=10K$ and $n_{H2}=10^3 cm^{-3}$

→ $M_J = 3.2\ M_{sun}$

Orders of magnitide too low → Jeans instable

→ Additional support necessary, e.g., magnetic field, turbulence …

# Jeans fragmentation in star formation




*Beuther et al. 2013*

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# Magnetic fields I

| Object | Type | Diagnostic | $\|B_{\|\|}\|$ [$\mu$G] |
|---|---|---|---|
| Ursa Major | Diffuse cloud | HI | 10 |
| NGC2024 | GMC clump | OH | 87 |
| S106 | HII region | OH | 200 |
| W75N | Maser | OH | 3000 |

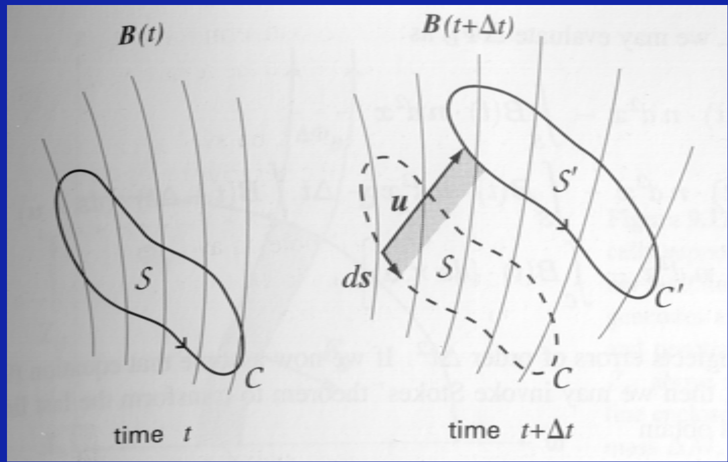
Increasing magnetic field strength with increasing density indicate “field-freezing” between B-field and gas

(B-field couples to ions and electrons, and these via collisons to neutral gas).

# Magnetic fields II

This field freezing can be described by ideal MHD:

$$\frac{d\mathbf{B}}{dt} = \nabla \times (\mathbf{u} \times \mathbf{B})$$



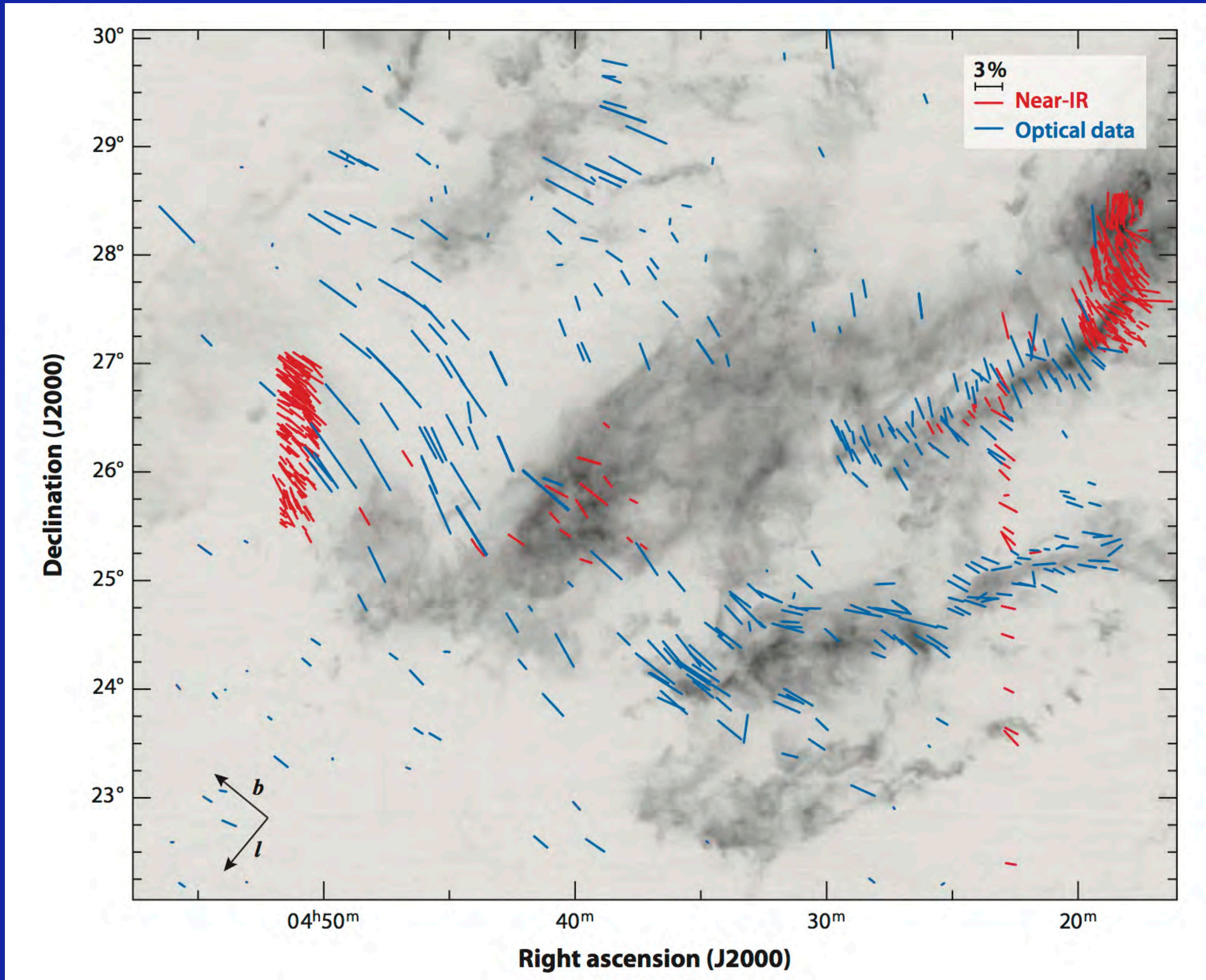

However, ideal MHD must break down at some point.

Dense core: $1M_{sun}$, $R_0$=0.07pc, $B_0$=30μG *<u>versus</u>* T Tauri star: $R_1=5R_{sun}$

If flux-freezing → magnetic flux $\Phi_M = \pi B R^2$ should remain constant:
→ $B_1 = 2\times10^7$ G, which exceeds observed values by orders of magnitude

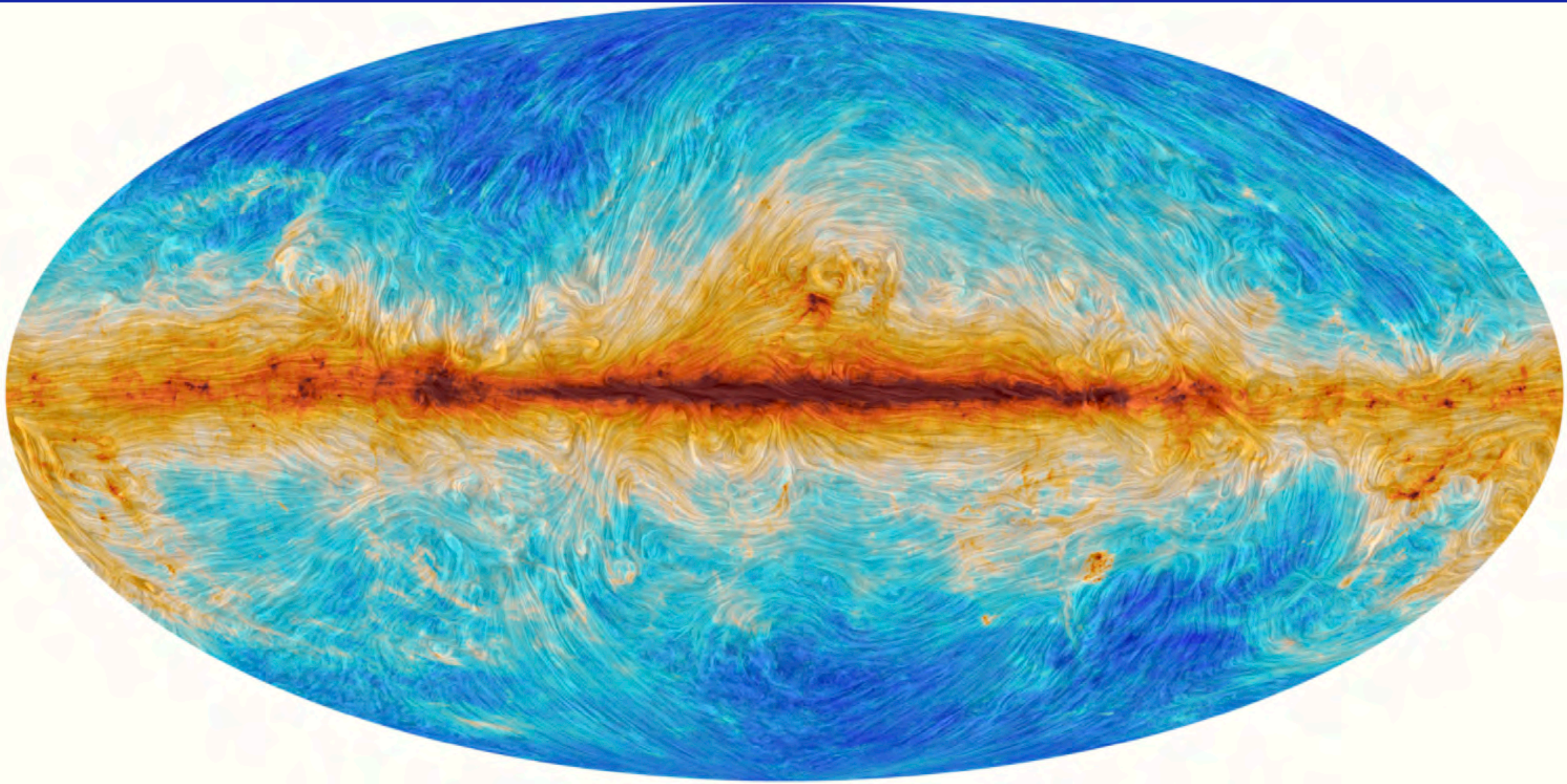
Ambipolar diffusion: neutral and ionized medium decouple, and neutral gas can sweep through during the gravitational collapse.

# Magnetic fields morphology in Taurus



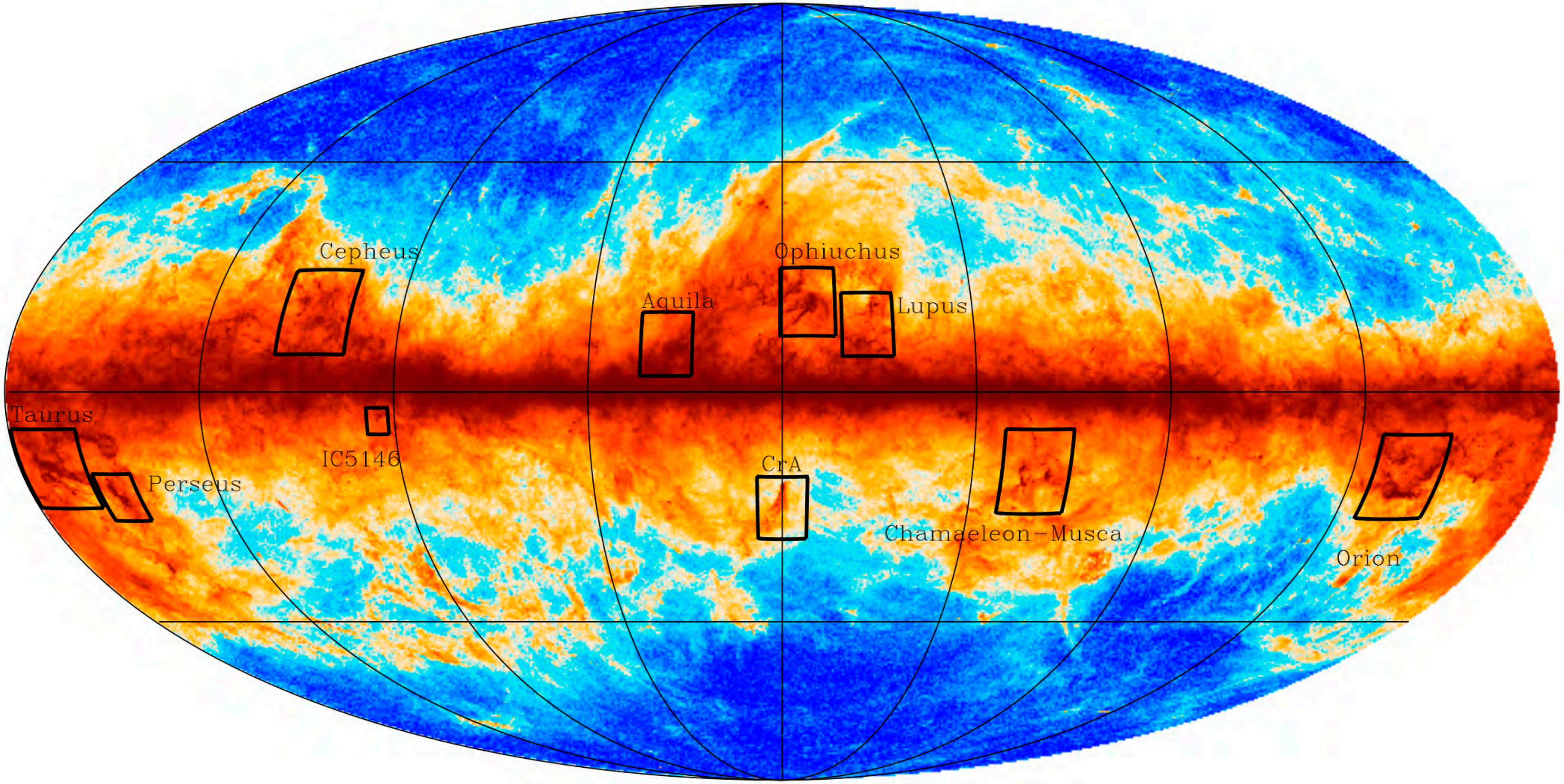

Grey: $^{13}CO$; line segments: optical polarization *Chapman et al. 2011*

# Planck and the magnetic field



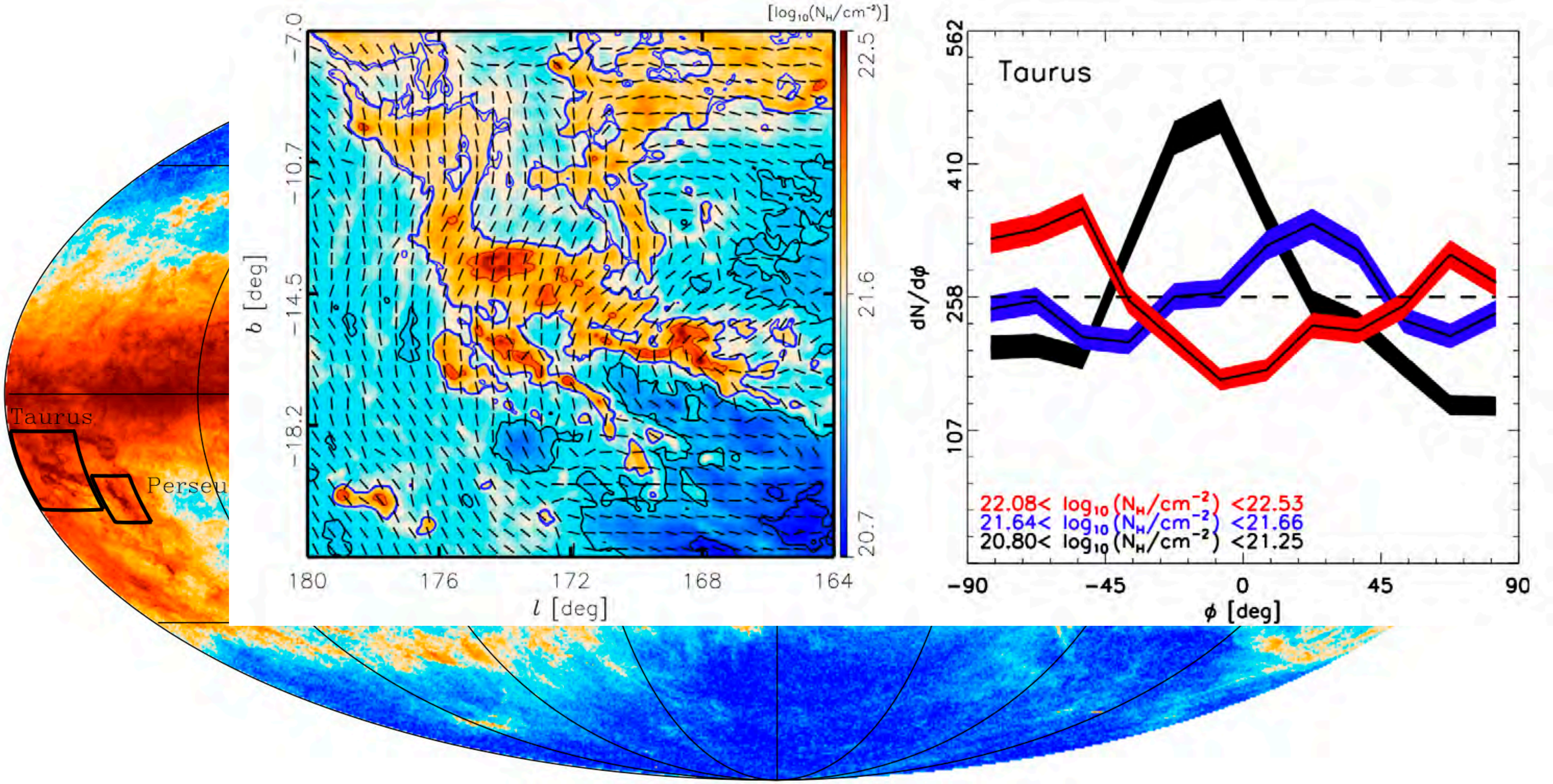
*Soler et al. 2015*

# Planck and the magnetic field



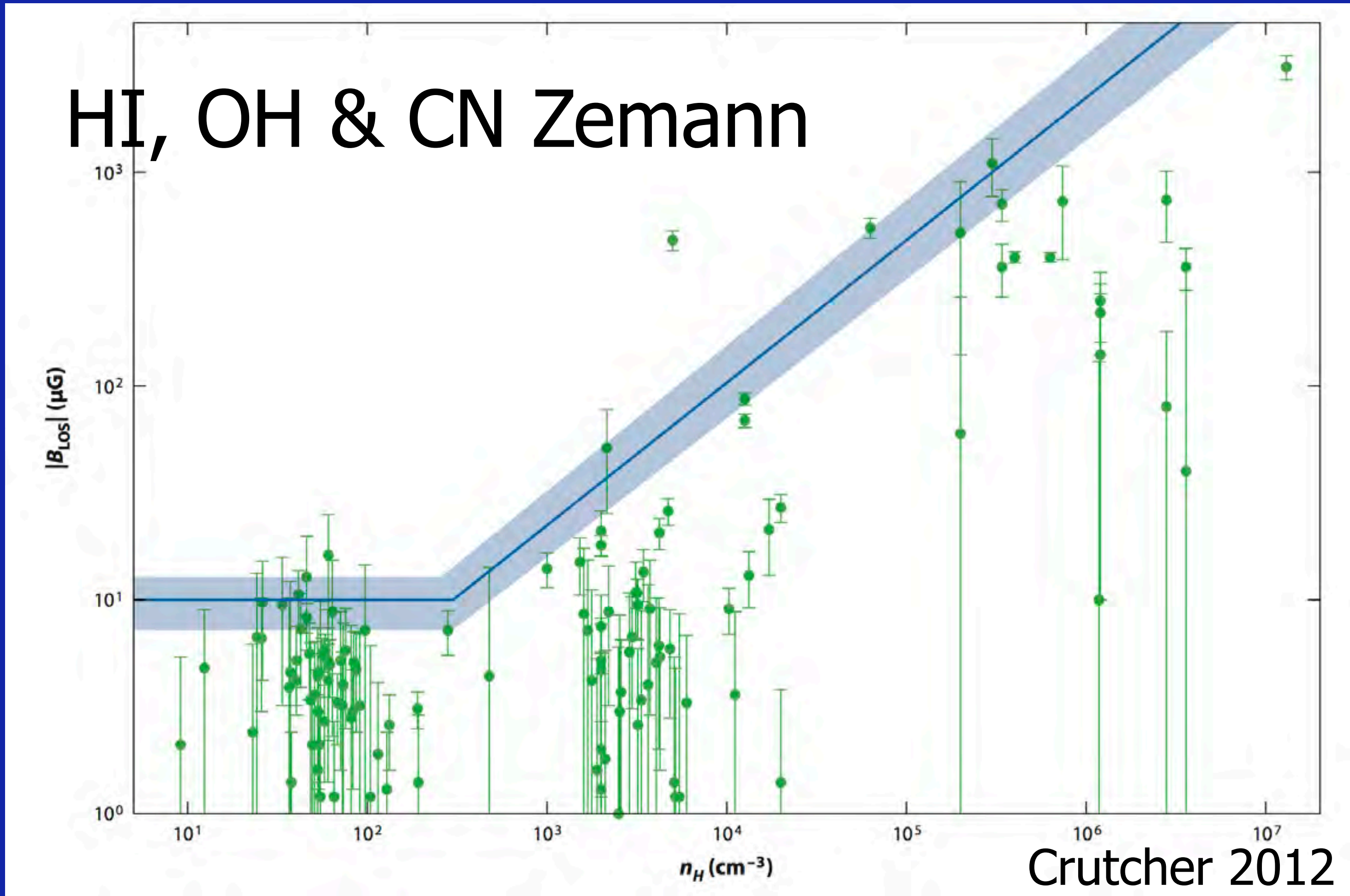

*Soler et al. 2015*

# Planck and the magnetic field




*Soler et al. 2015*

# Magnetic fields strength




Jeans-like analysis: $M_{cr} = 1000 M_{sun} (B/(30\mu G)) (R/(2pc)^2$

$M<M_{cr}$ magnetically subcritical; $M>M_{cr}$ magnetically supercritical

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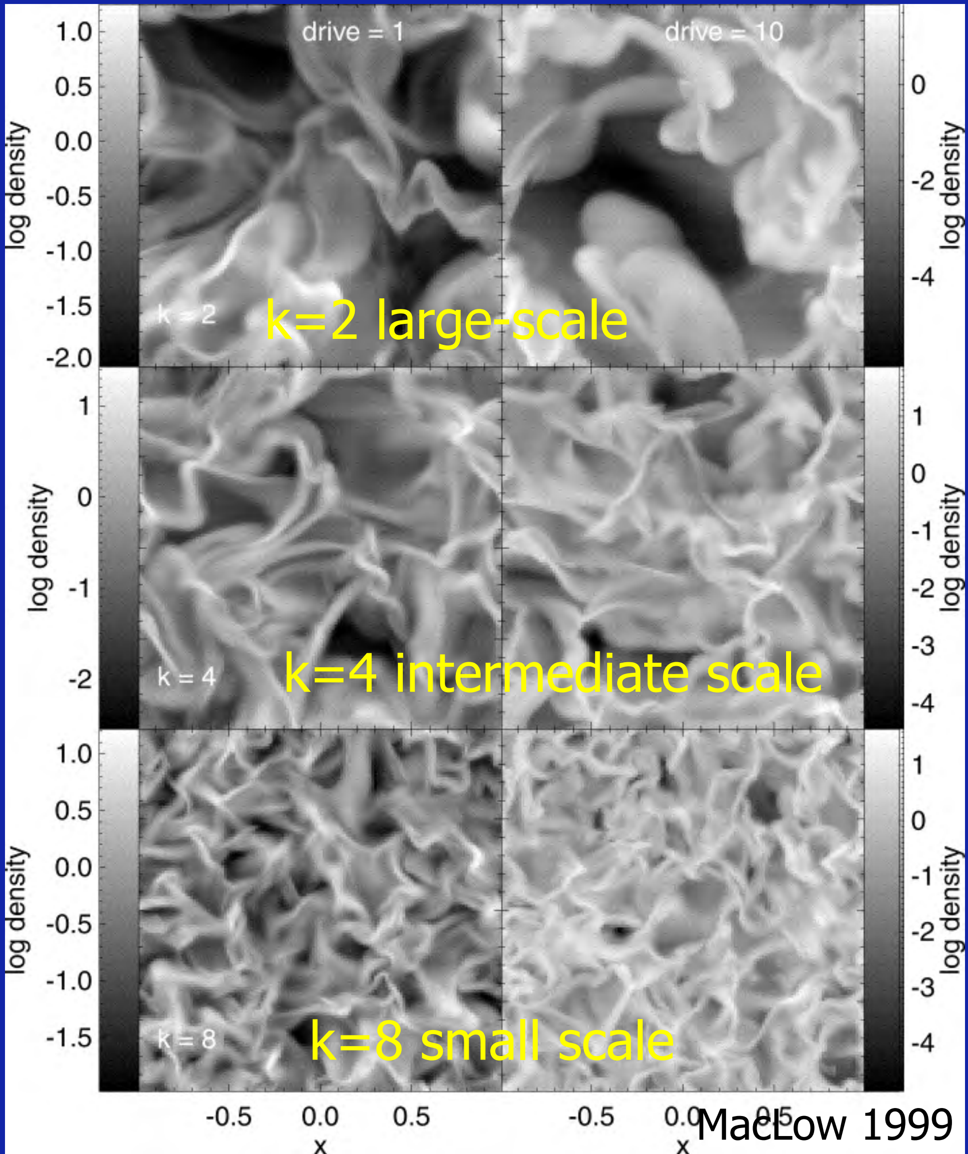
- Jeans analysis for gravitational instability

- Magnetic fields

- Cloud formation and turbulence

# Interstellar Turbulence




- Supersonic → network of shocks

→ Density fluctuations $\delta\rho \propto M^2$

→ Molecular $H_2$ can form.

- $t_{form}$ = $1.5 \times 10^9$yr / (n/1cm$^{-3}$)
  (Hollenbach et al. 1971)

→ either molecular clouds form slowly in low-density gas or rapidly in ~$10^5$yr in n=$10^4$cm$^{-3}$

- Decays on time-scales of order the free-fall time-scale
→ Needs continuous driving

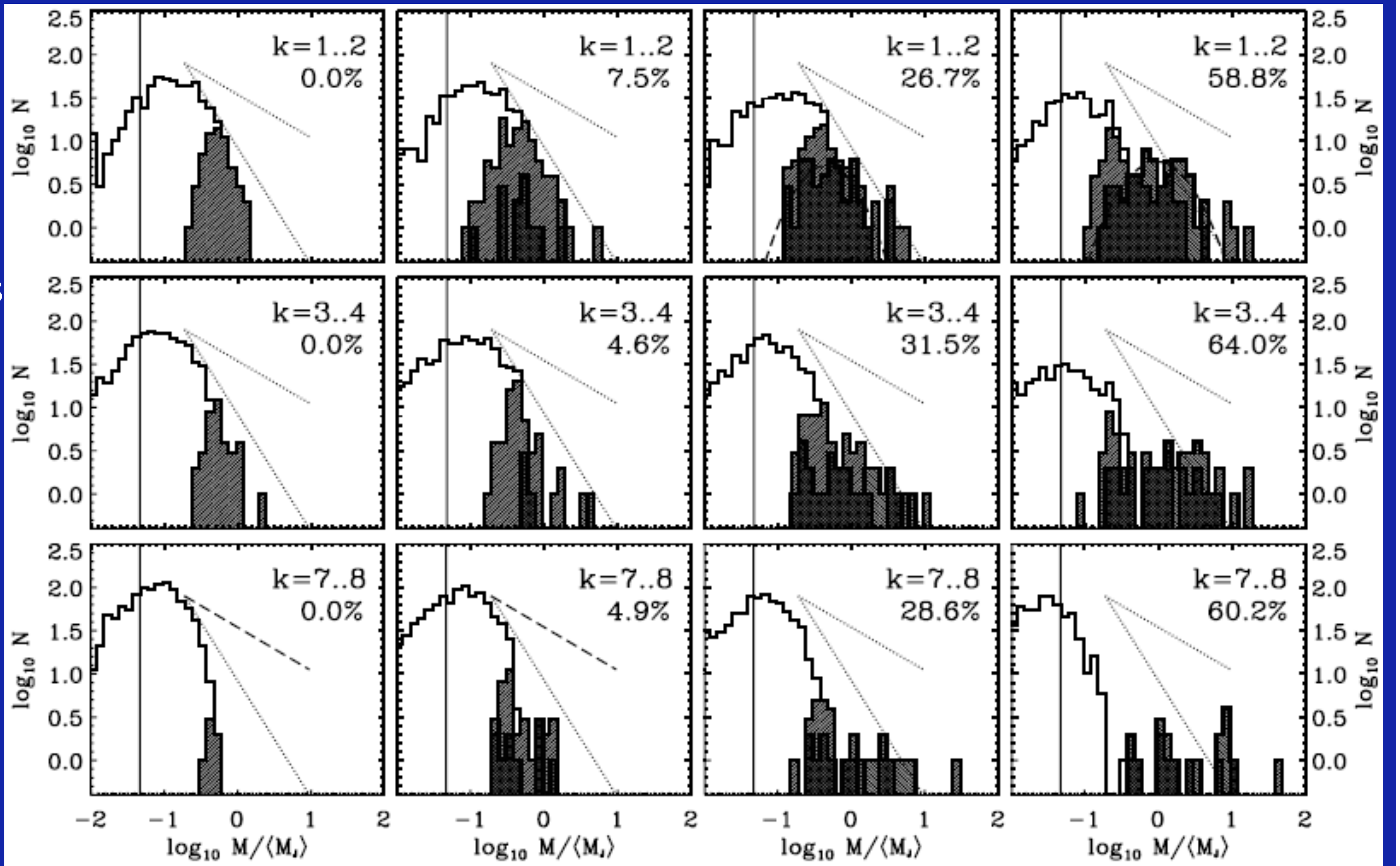
Candidates: Protostellar outflows, radiation from massive stars, supernovae explosions

# (Gravo)-turbulent fragmentation

<u>Histogram:</u>
Gas clumps
<u>Grey:</u>
Jeans un-stable clumps
<u>Dark:</u>
Collapsed core

Slopes: -1.5 & -2.3

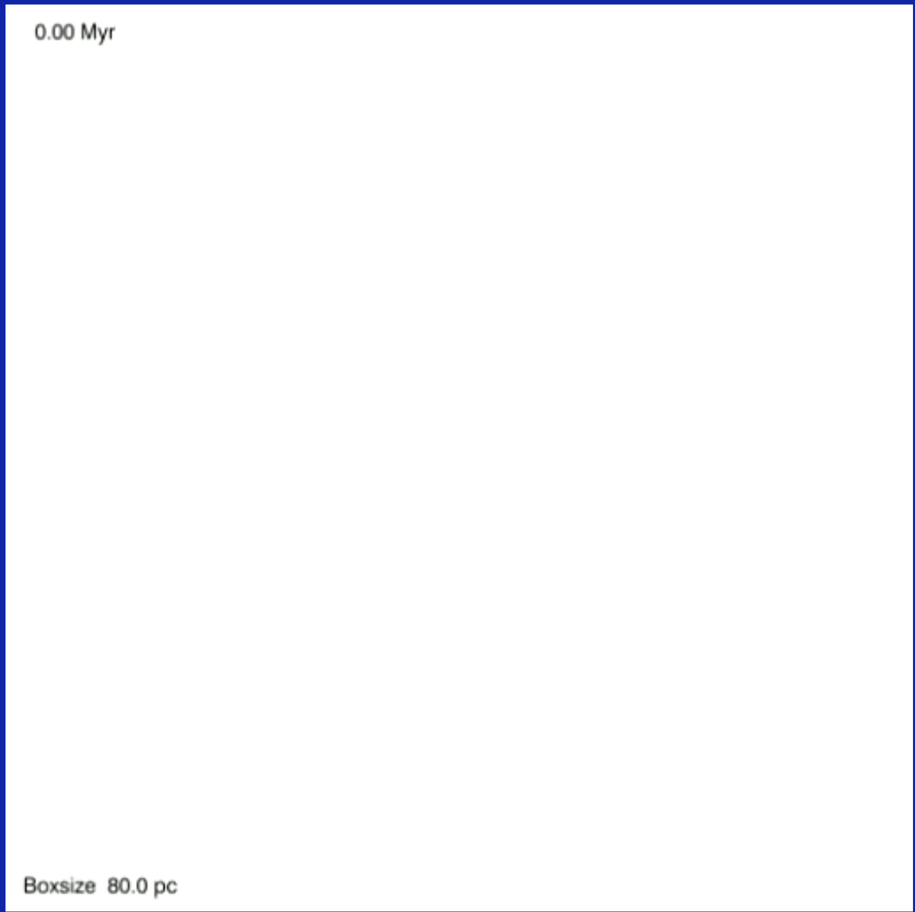
*Klessen 2001*




2 steps: 1.) Turbulent fragmentation → 2.) Collapse of individual core

- Large-scale driving reproduces shape of IMF.
- Discussion whether largest fragments remain stable or fragment further …

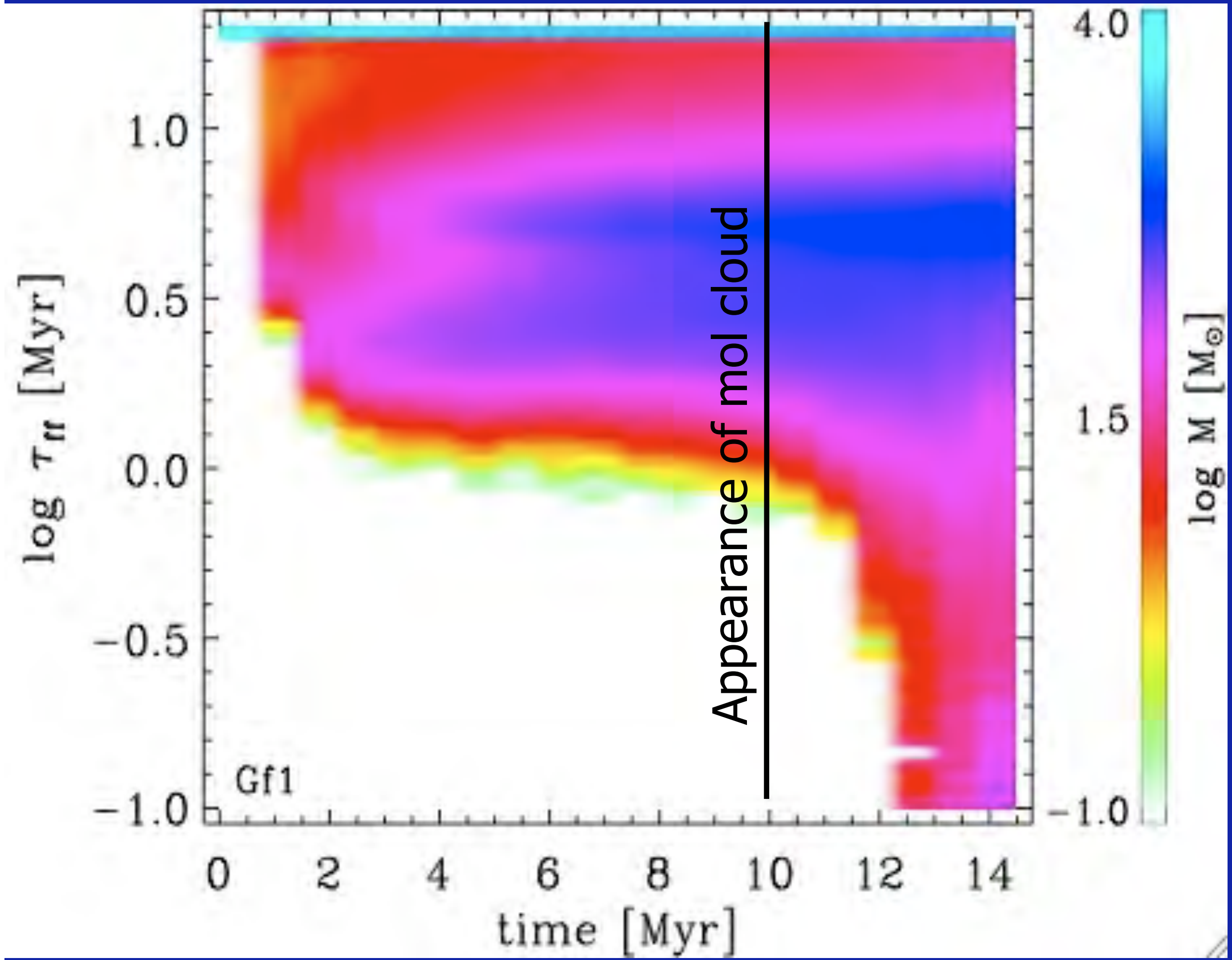
# Simulations of colliding flows





*Banerjee et al. 2009*

# Time scales




Densest regions form stars while the envelope (blue) is not participating.

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\rho}}$$

→ Densest region have shortest free-fall time.

*Heitsch & Hartmann 2008*

# Cloud and star formation with different physics

*Federrath 2015*

# Summary

- Virial theorem and its application

- Jeans analysis and applications

- Magnetic fields in the interstellar medium

- Turbulence, cloud formation and time scales

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