The Sub-linear and Non-Universal Kennicutt-Schmidt Relationship

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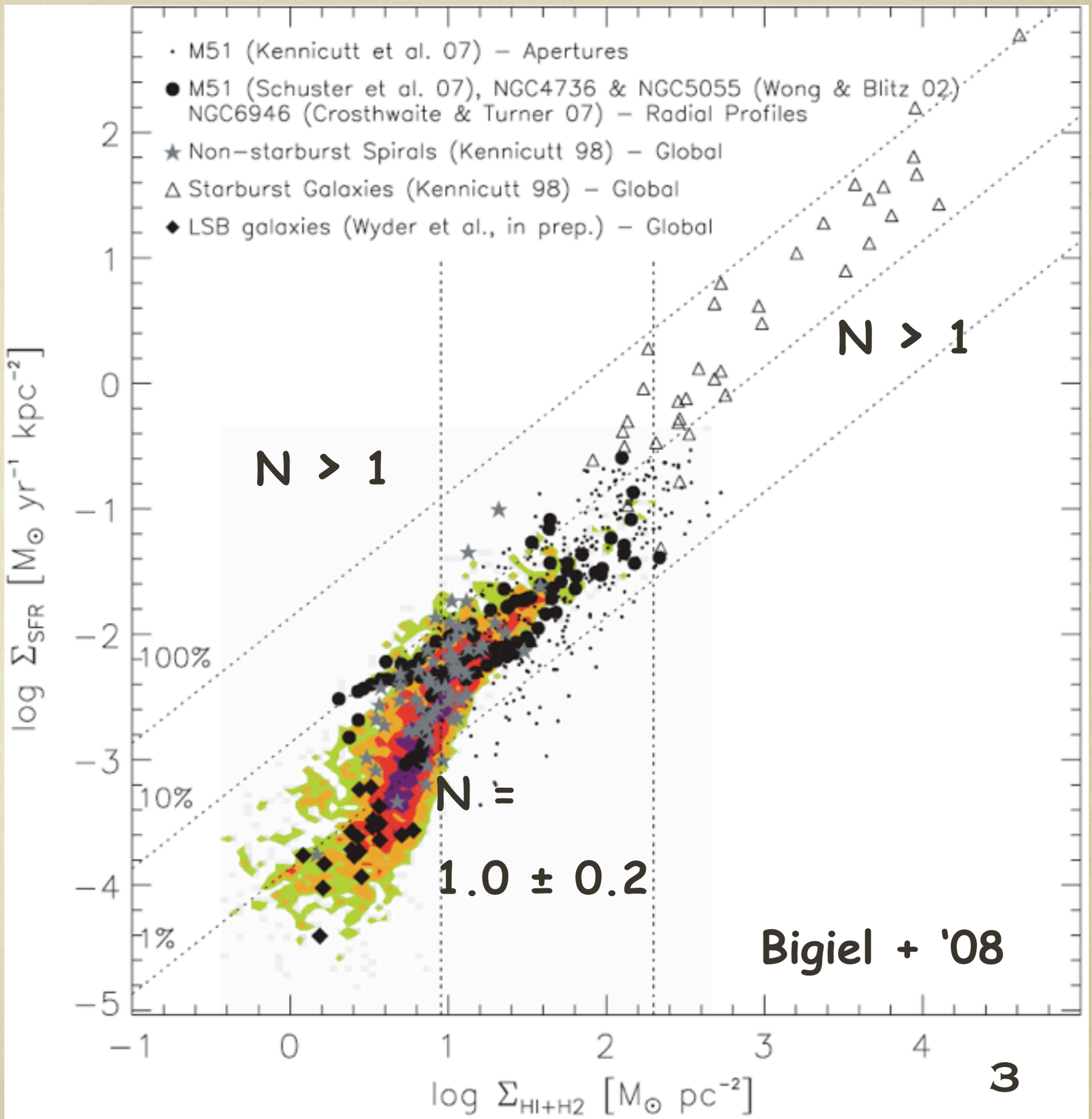
Overview

- The Kennicutt-Schmidt (KS) Relationship
- Results
- Fitting the KS relationship
  - OLS Methods (y|x, x|y, bisector)
  - Hierarchical Data
    - Monte Carlo & Bayesian Methods
  - Assessing fits, including "by eye"
- Results and Interpretation of KS parameters
- Caveats, Future Work, and Summary

The KS Relationship

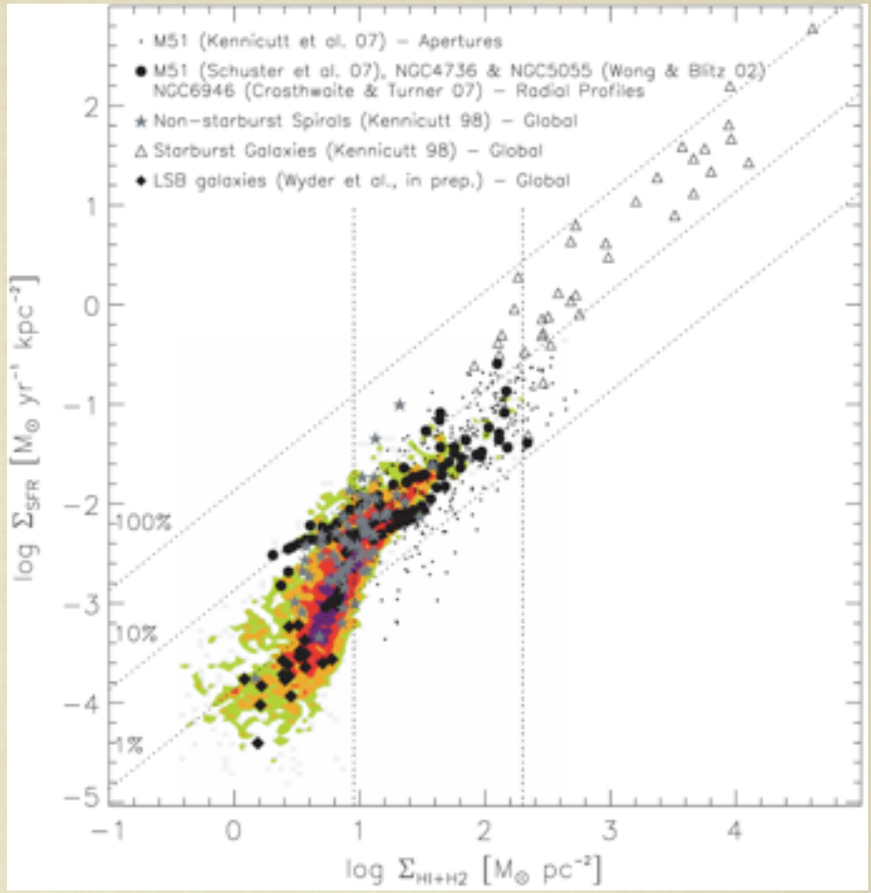
$$\Sigma_{SFR} \propto \Sigma_{gas}^{N}$$

Schmidt ‘59
Buat + ‘89
Kennicutt ‘89
Kennicutt ‘98
Hunter + ‘98
Boselli + ‘02
Wong & Blitz ‘02
Boissier + ‘03
Heyer + ‘04
Leroy + ‘05
Kennicutt + ‘07
Leroy + ‘08
Bigiel + ‘08
Blanc + ‘09
Verley + ‘10
Daddi + ‘10
Genzel + ’10
Liu + ‘11
Schruba + ’11
Rahman + ’12
Leroy + ’13
Momose + ’13
...



The KS Slope

- At intermediate 10 $M_\odot$ pc^{-2} < $\sum_{gas}$ < 100 $M_\odot$ pc^{-2}, Bigiel +'08 find N~1

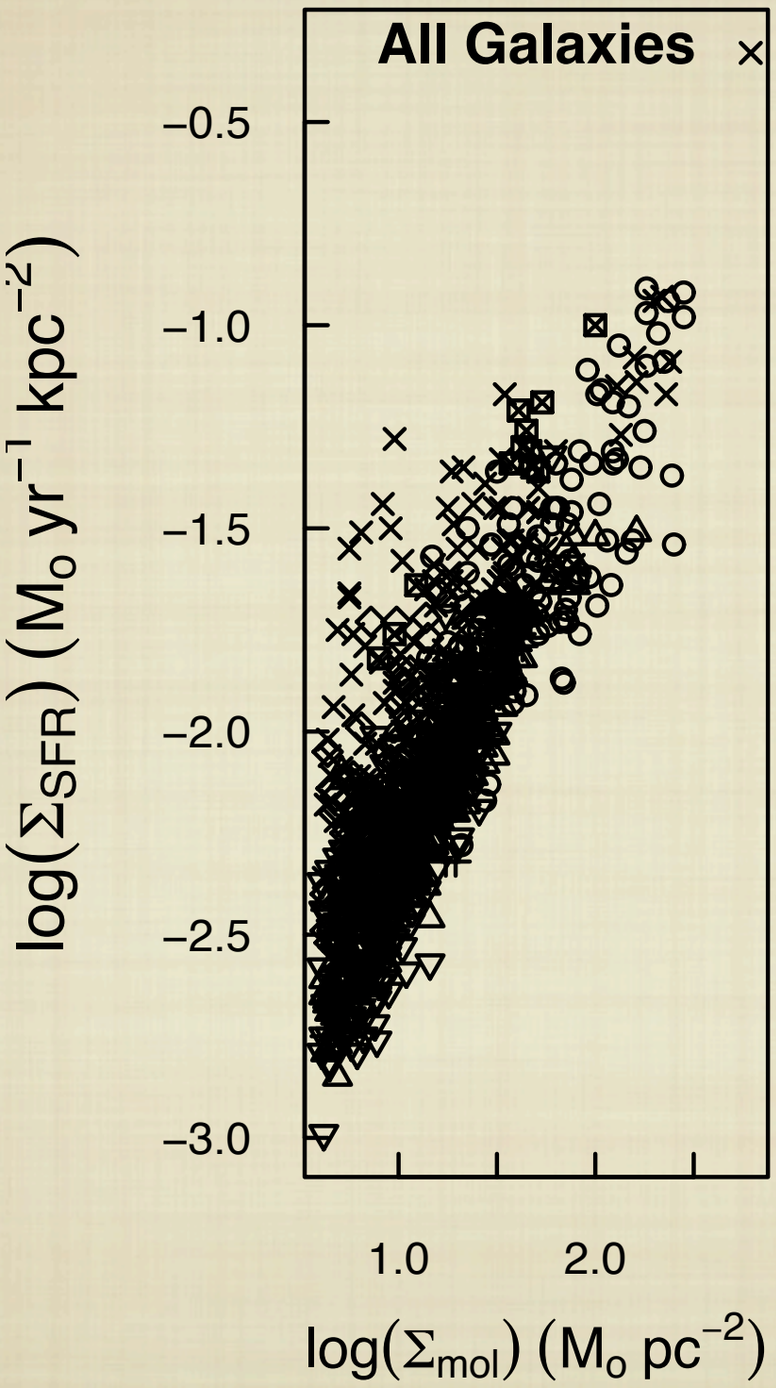


- A linear relationship suggests a star formation efficiency, or constant depletion time t_{dep} ~ 2 Gyr, with significant scatter

  (e.g. Bigiel + '08, Rahman + '12, Schruba + '12 Leroy + '13, etc...)

- Super-linear N~1.5 KS relationship indicative of t_{dep} decreasing with surface density (e.g. Kennicutt 98, + '07, Liu + '11, Momose + '13)

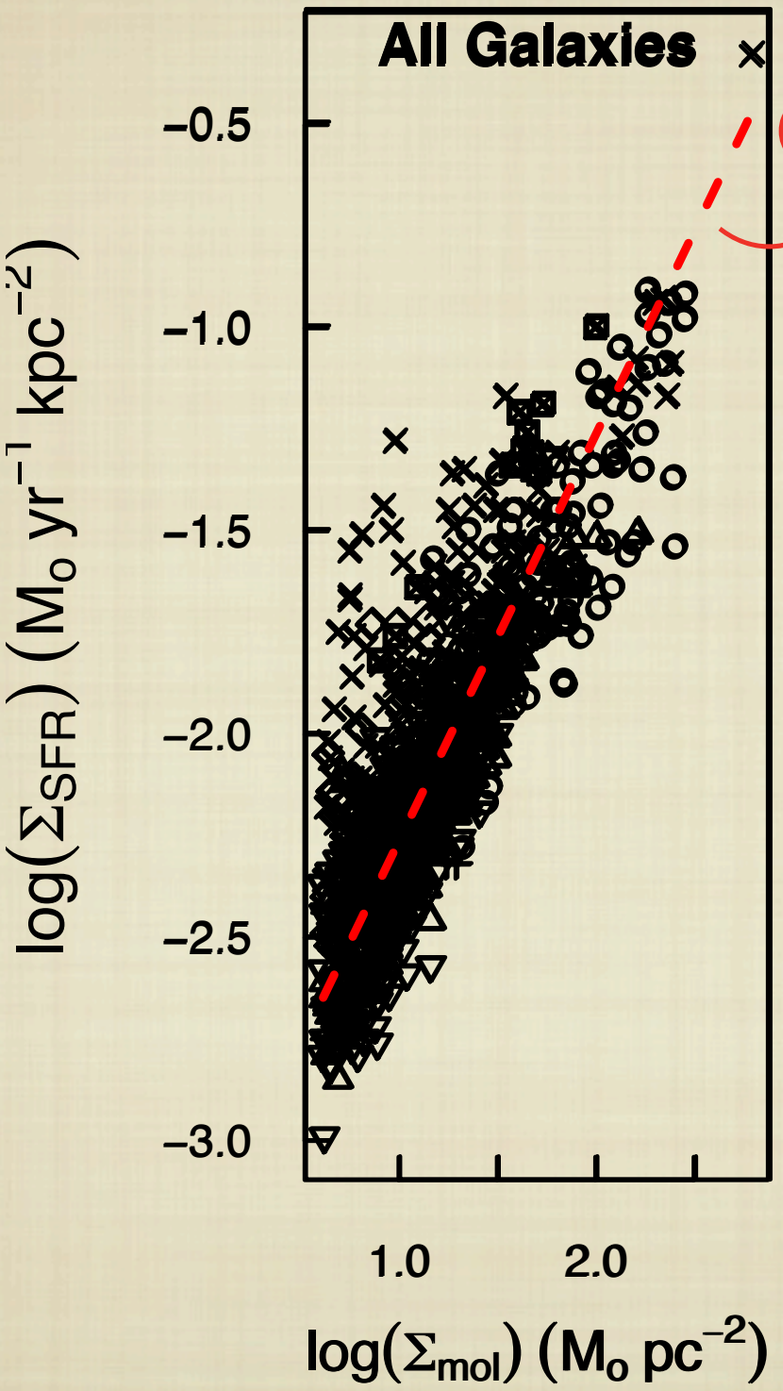
The KS Slope



Bigiel + ’08

Shetty, Kelly, Bigiel ’13

The KS Slope

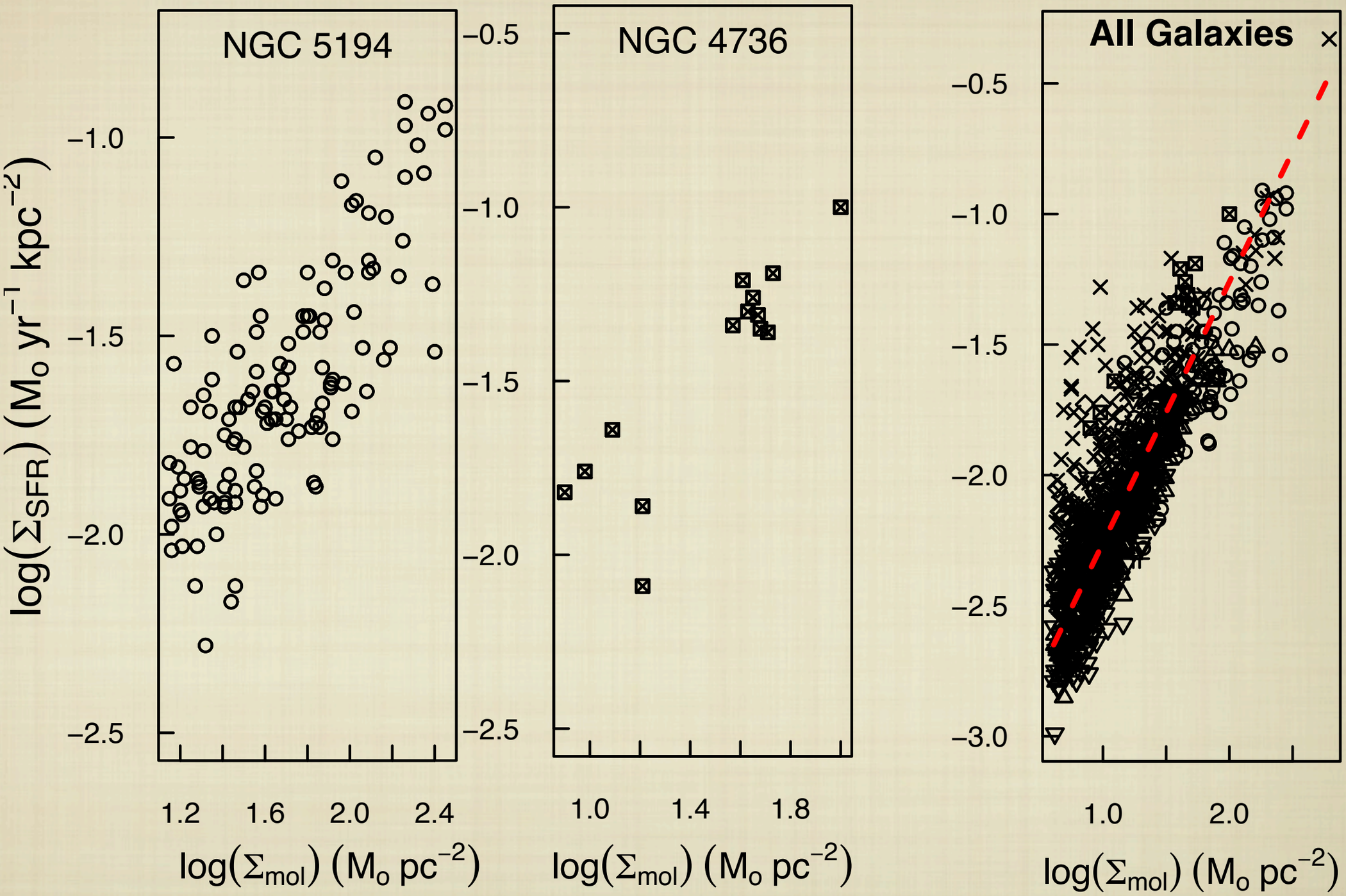


OLS(bisector): slope = 1.0 +/- 0.2

Bigiel + ’08

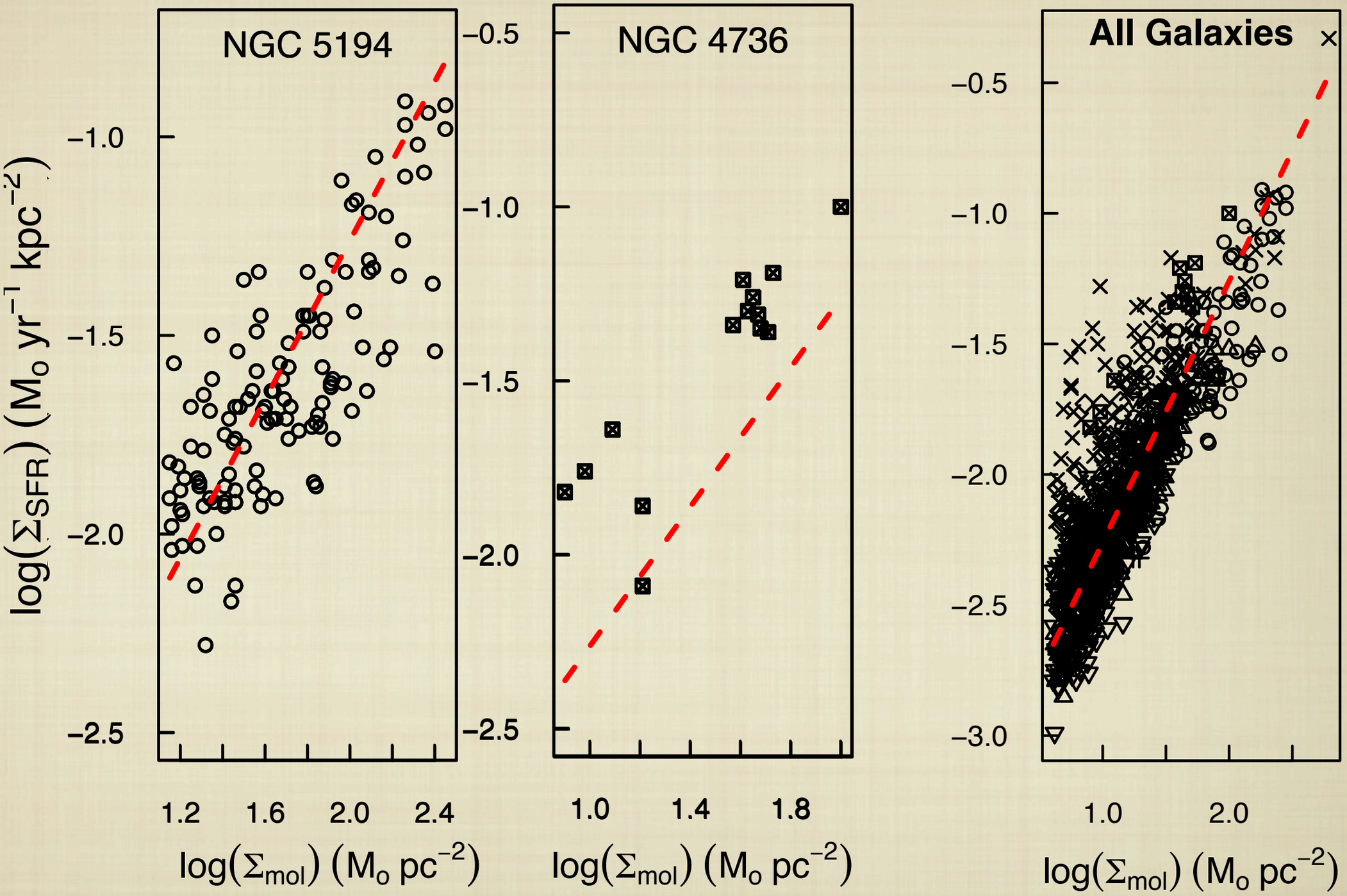
Shetty, Kelly, Bigiel ’13

The KS Slope



Shetty, Kelly, Bigiel '13

The KS Slope



Shetty, Kelly, Bigiel '13

Results:

- Analysis of resolved observations using 7 HERACLES (Leroy + '09, Bigiel + '08) galaxies and 13 STING galaxies (Rahman + '12)

- Significant galaxy-to-galaxy variation: Non-universal KS relationship

- Most galaxies portray a sub-linear KS relationship

- Mean KS relationship also sub-linear

Shetty, Kelly, Bigiel '13 & Shetty + '13b

Fitting a Model to Data

- Measurement uncertainties and intrinsic scatter may lead to biases in fitting a model to data (e.g. Weiner + '06, Kelly '07, Leroy + '12).

- Hierarchical models allow for estimating the model parameters of individual galaxies and for the ensemble

- Monte Carlo methods can account for statistical uncertainties. Bayesian inference is well suited for hierarchical problems, through MCMC methods (Gelman + '04, Gelman & Hill '07, Kruschke '11)

Shetty + '13a,b

Assessing Fitting Methods

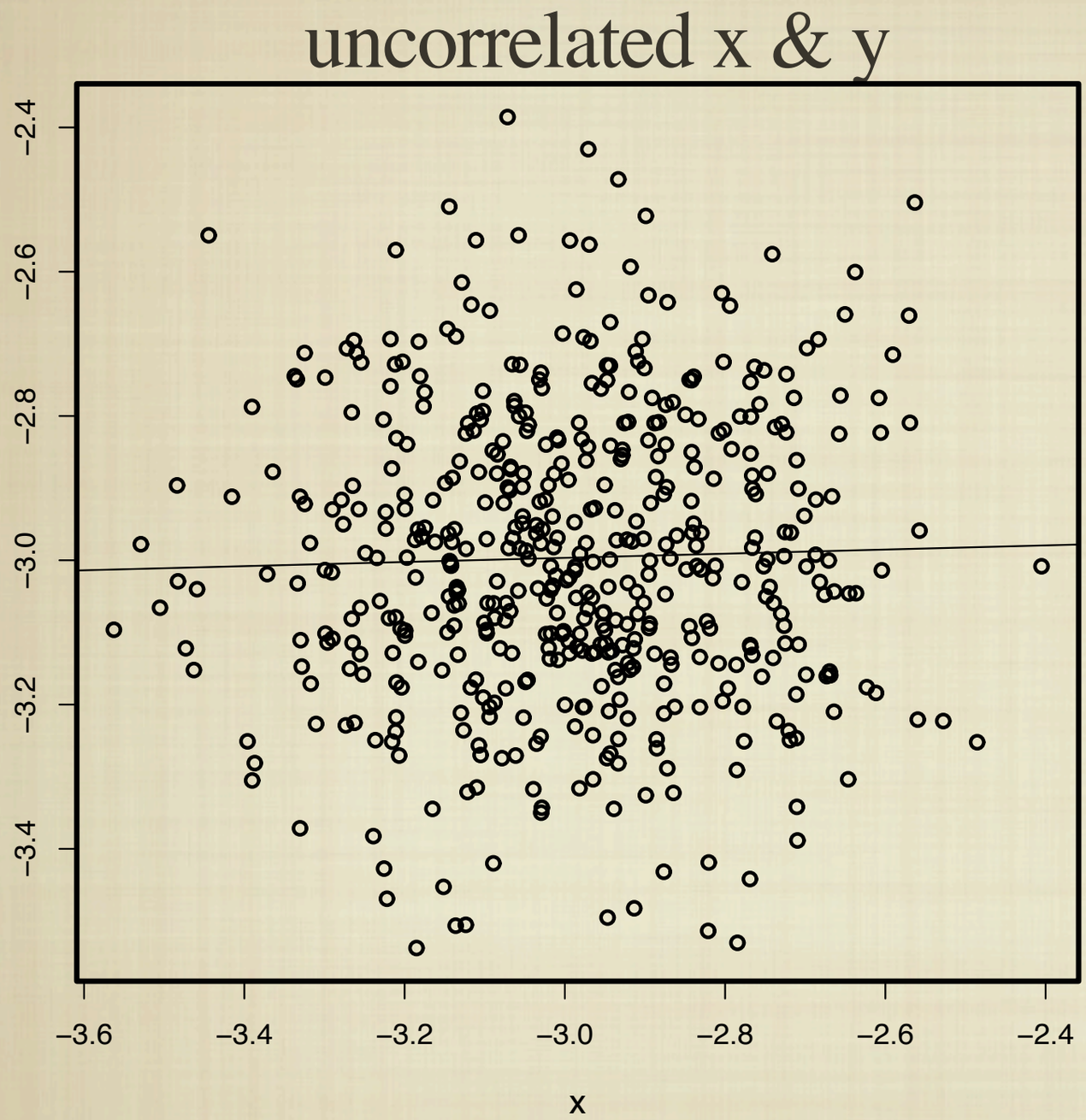
- Important to understand the underlying assumptions and limitations of a fitting method

- Linear fit: $y = \underline{A} + \underline{N}x$. OLS(y|x), OLS(x|y) and OLS (bisector) are NOT the same (Isobe + '90), due to treatment of variances (Shetty, Kelly, Bigiel '13)

- Quality control: verify accuracy of fits with synthetic data

- "Seeing is believing": By eye validity can also be effective

Testing OLS Methods



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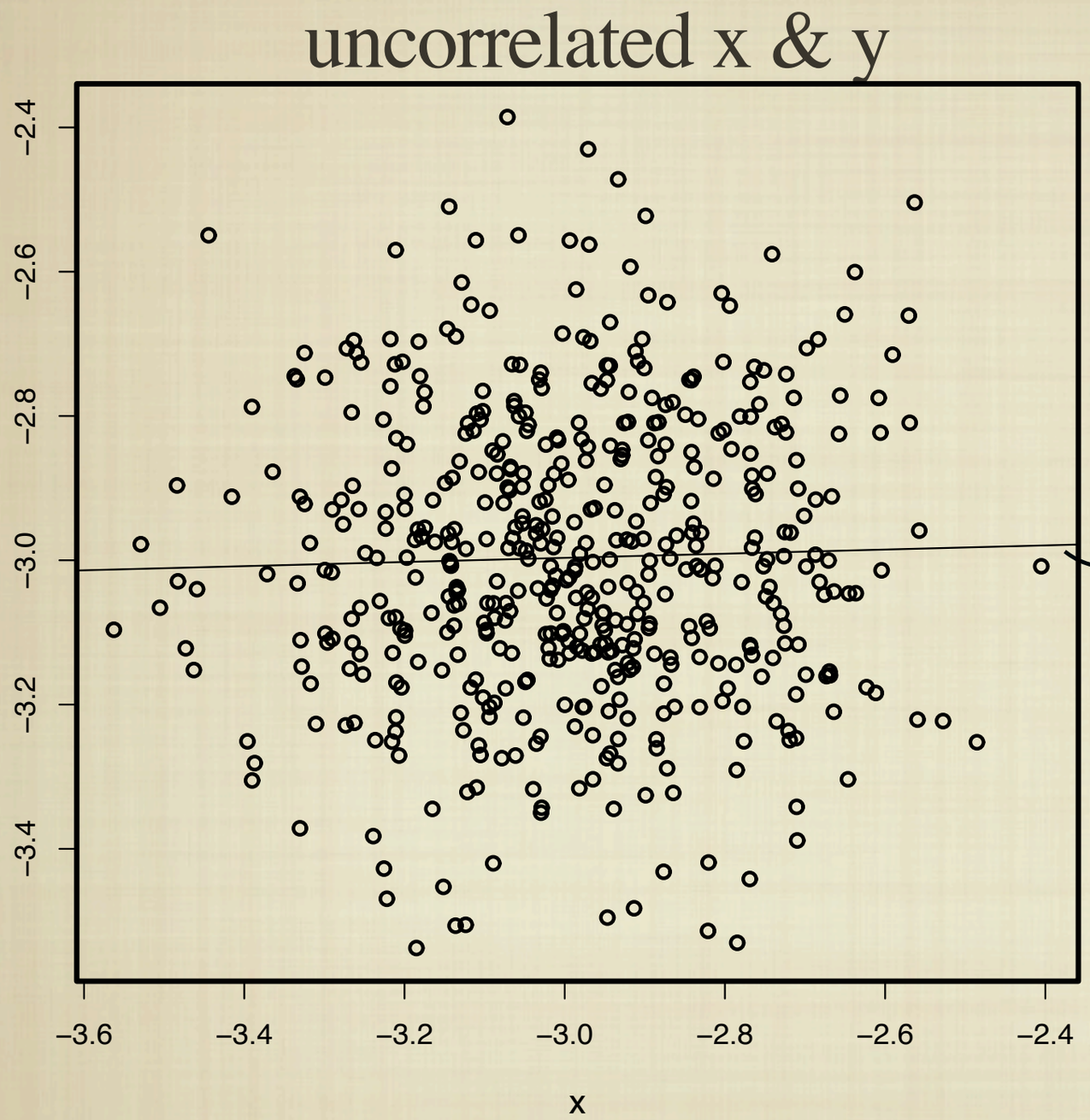
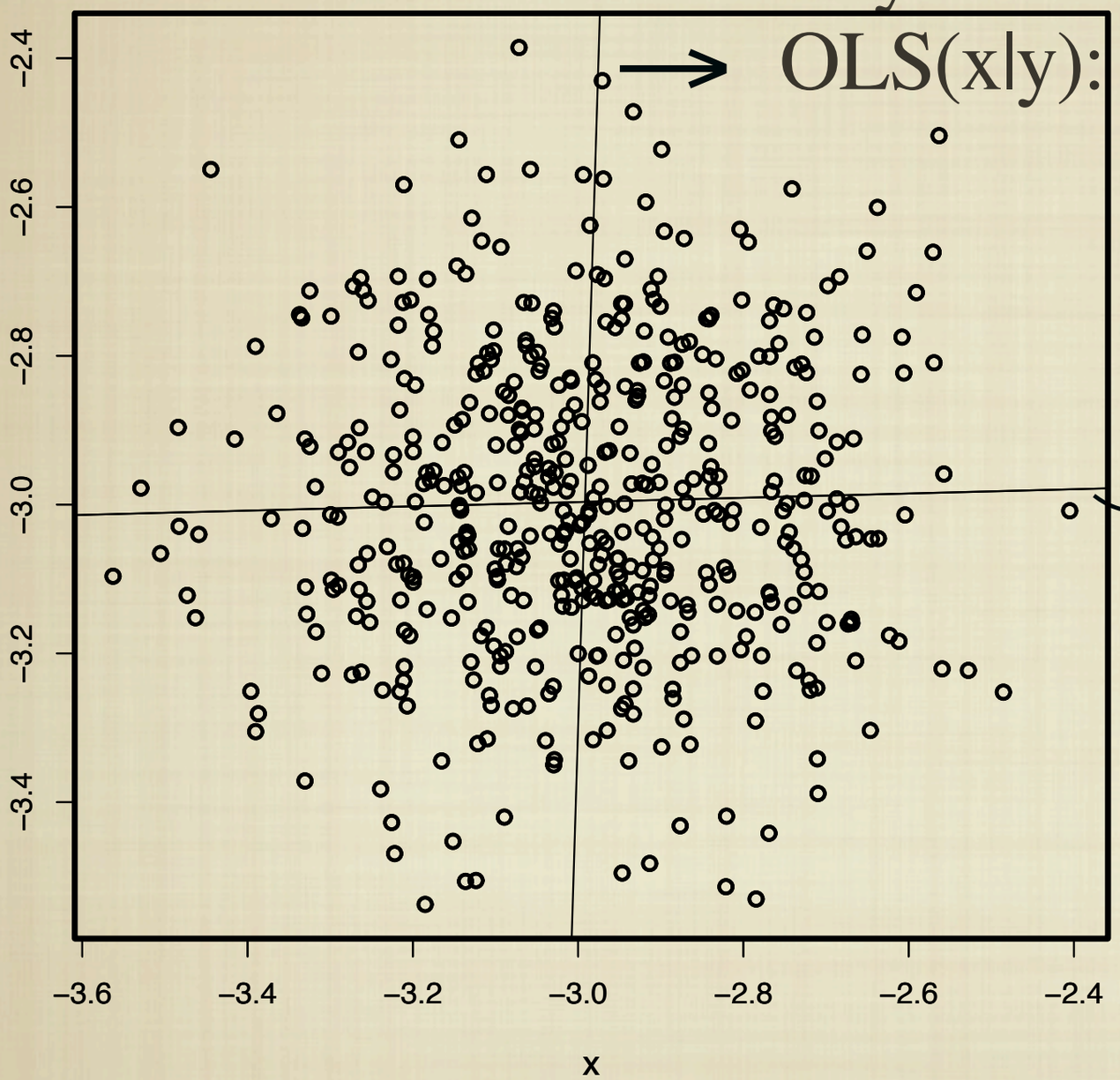


OLS(y|x): slope = 0.03 +/- 0.05

Testing OLS Methods

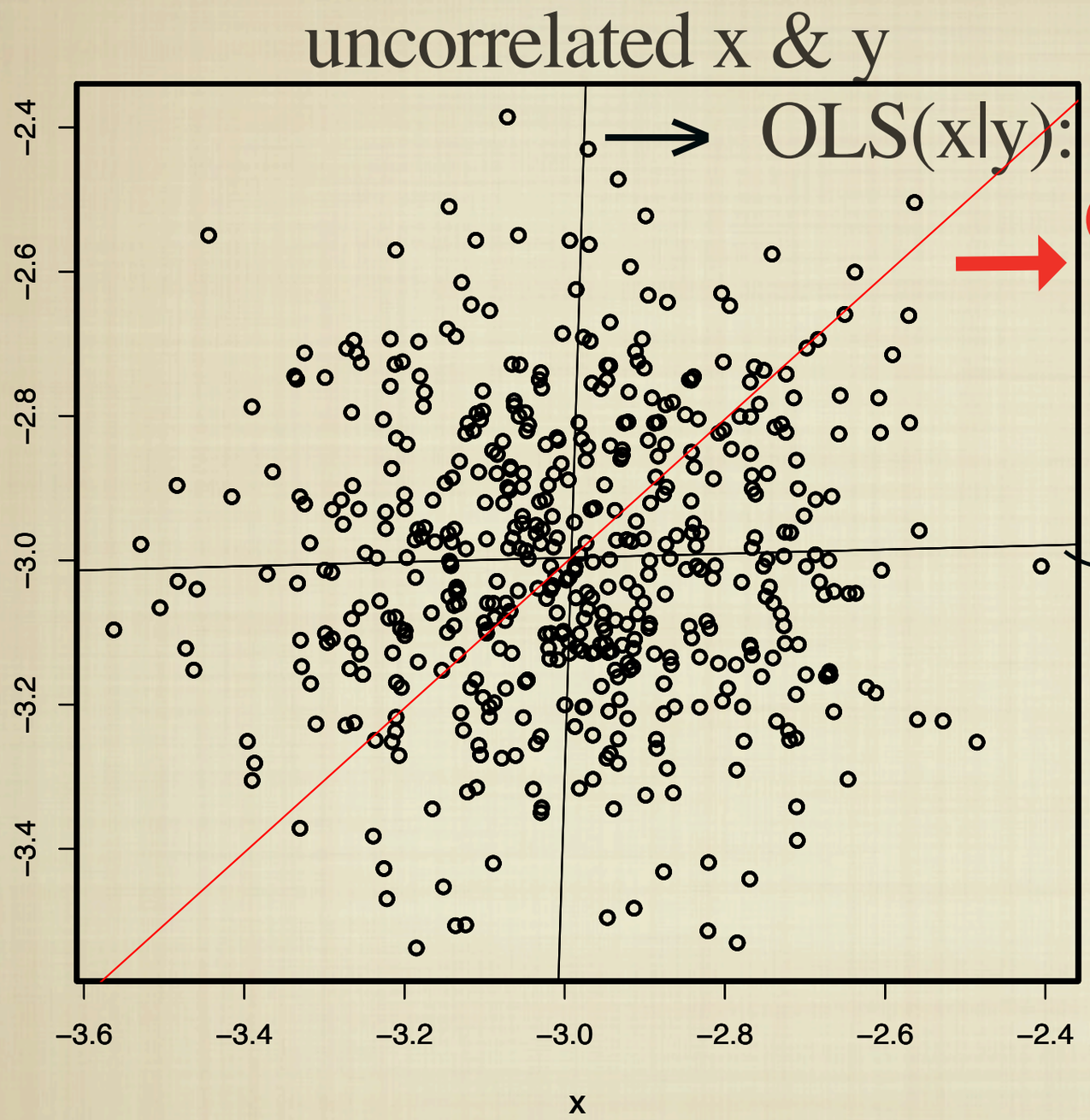
uncorrelated x & y



→ OLS(x|y): slope = 36.4 +/- 0.04

OLS(y|x): slope = 0.03 +/- 0.05

Testing OLS Methods



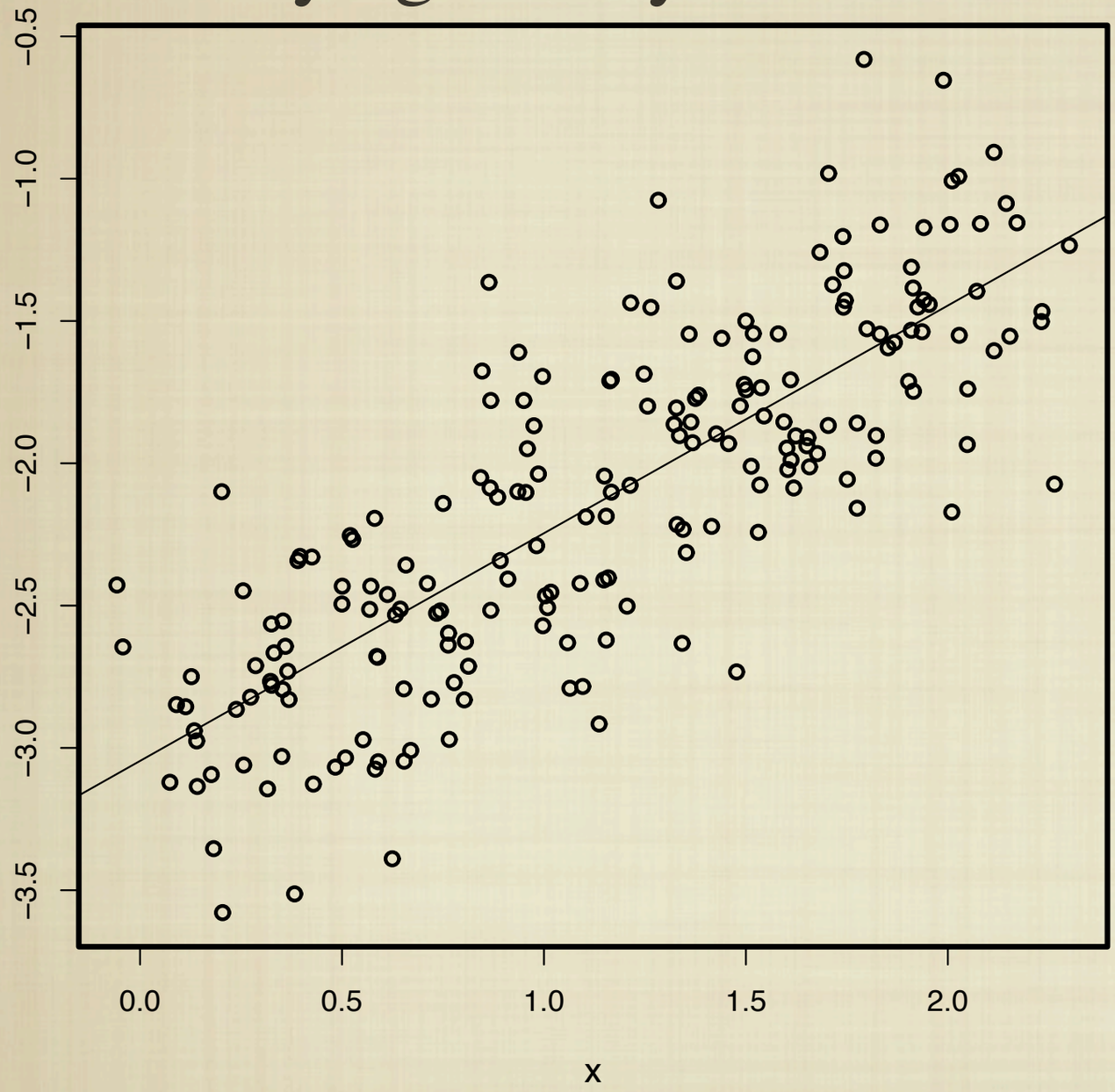
OLS(x|y): slope = 36.4 +/- 0.04

OLS(bisector): slope = 1.0 +/- 0.04

OLS(y|x): slope = 0.03 +/- 0.05

Testing OLS Methods

Underlying data: $y = -3 + 0.75x$



Shetty, Kelly, Bigiel ‘13

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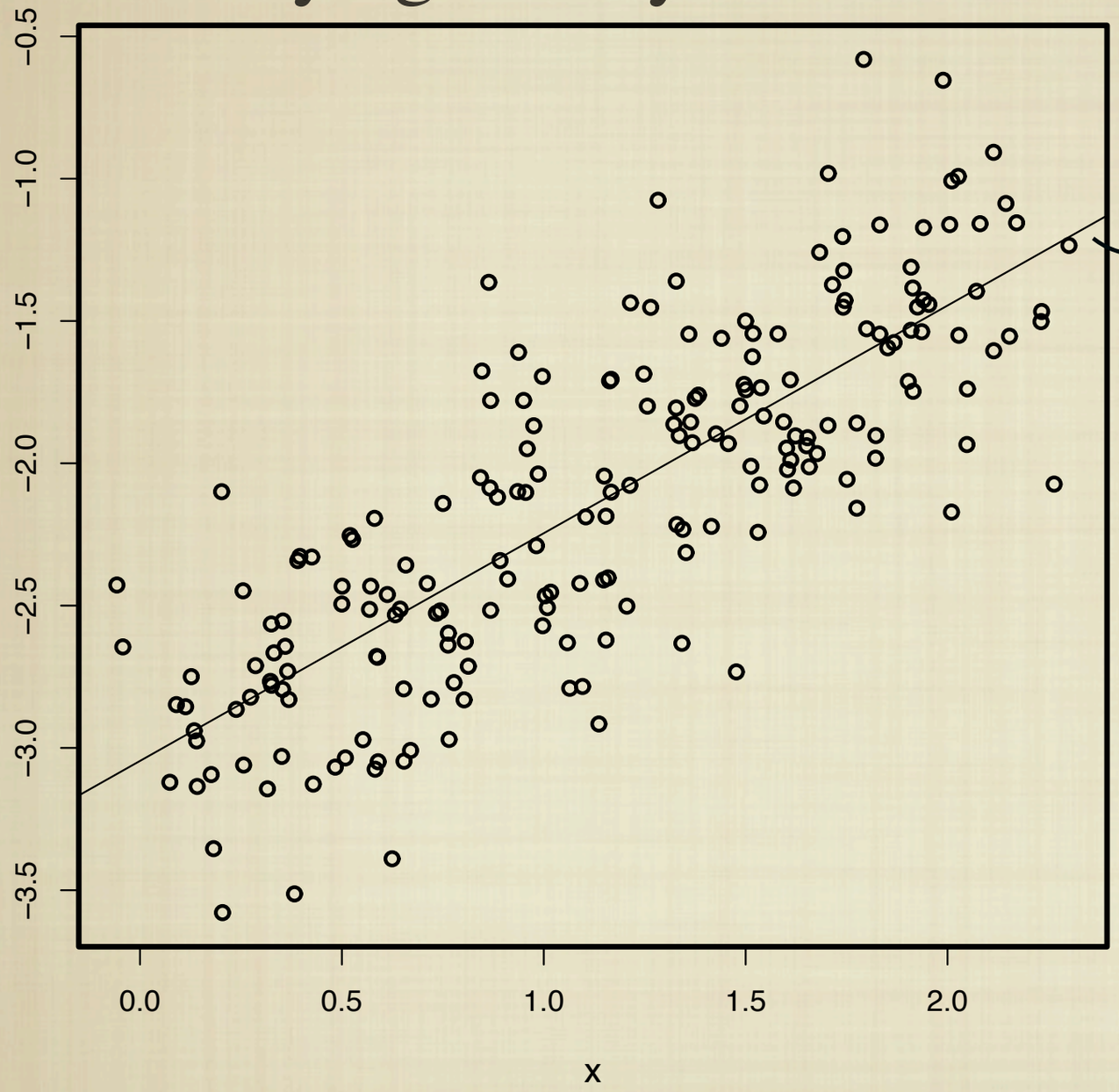

OLS(y|x): slope = 0.8 +/- 0.04

Shetty, Kelly, Bigiel '13

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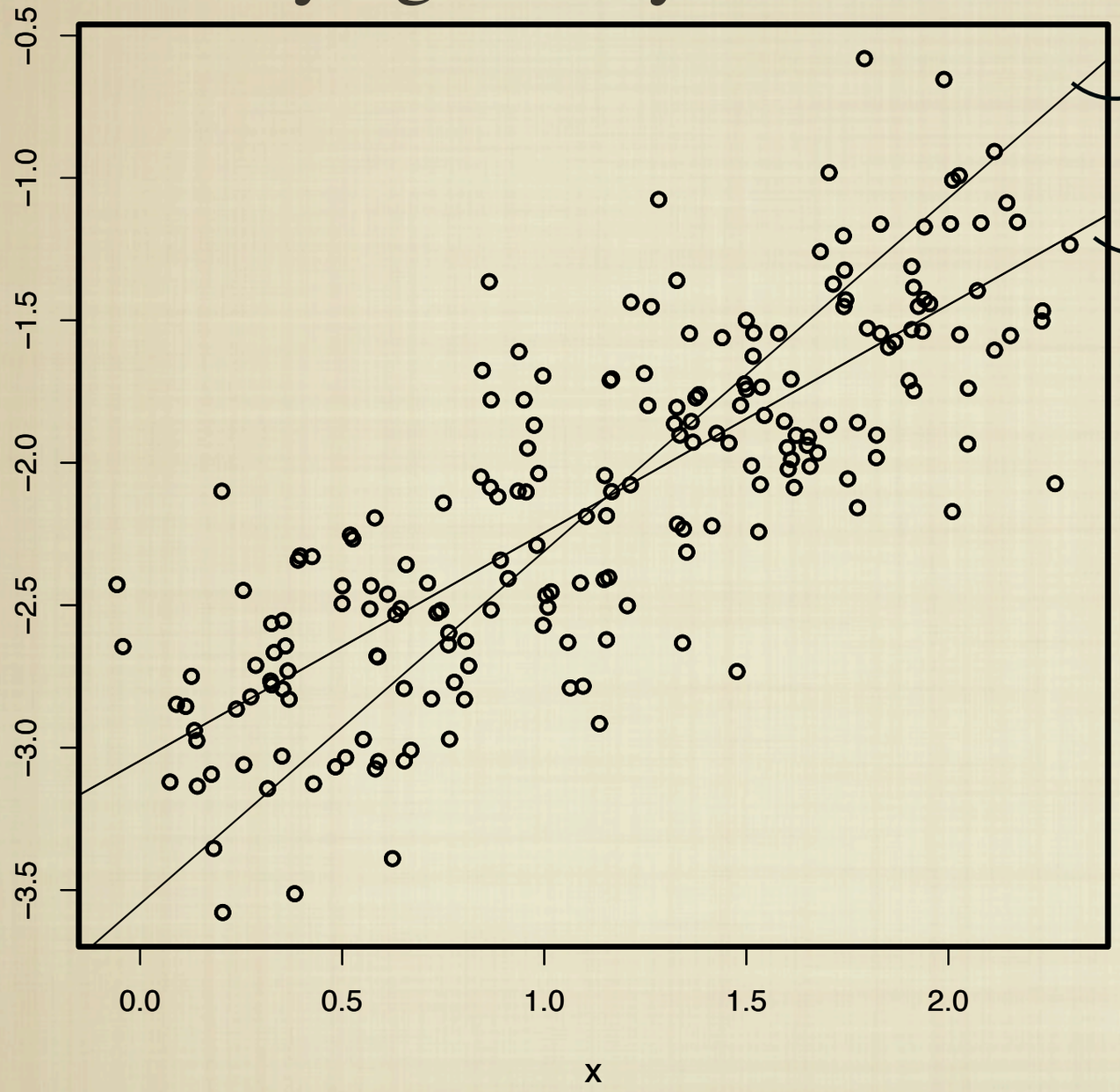

OLS(x|y): slope = 1.2 +/- 0.04

OLS(y|x): slope = 0.8 +/- 0.04

Shetty, Kelly, Bigiel ‘13

Testing OLS Methods

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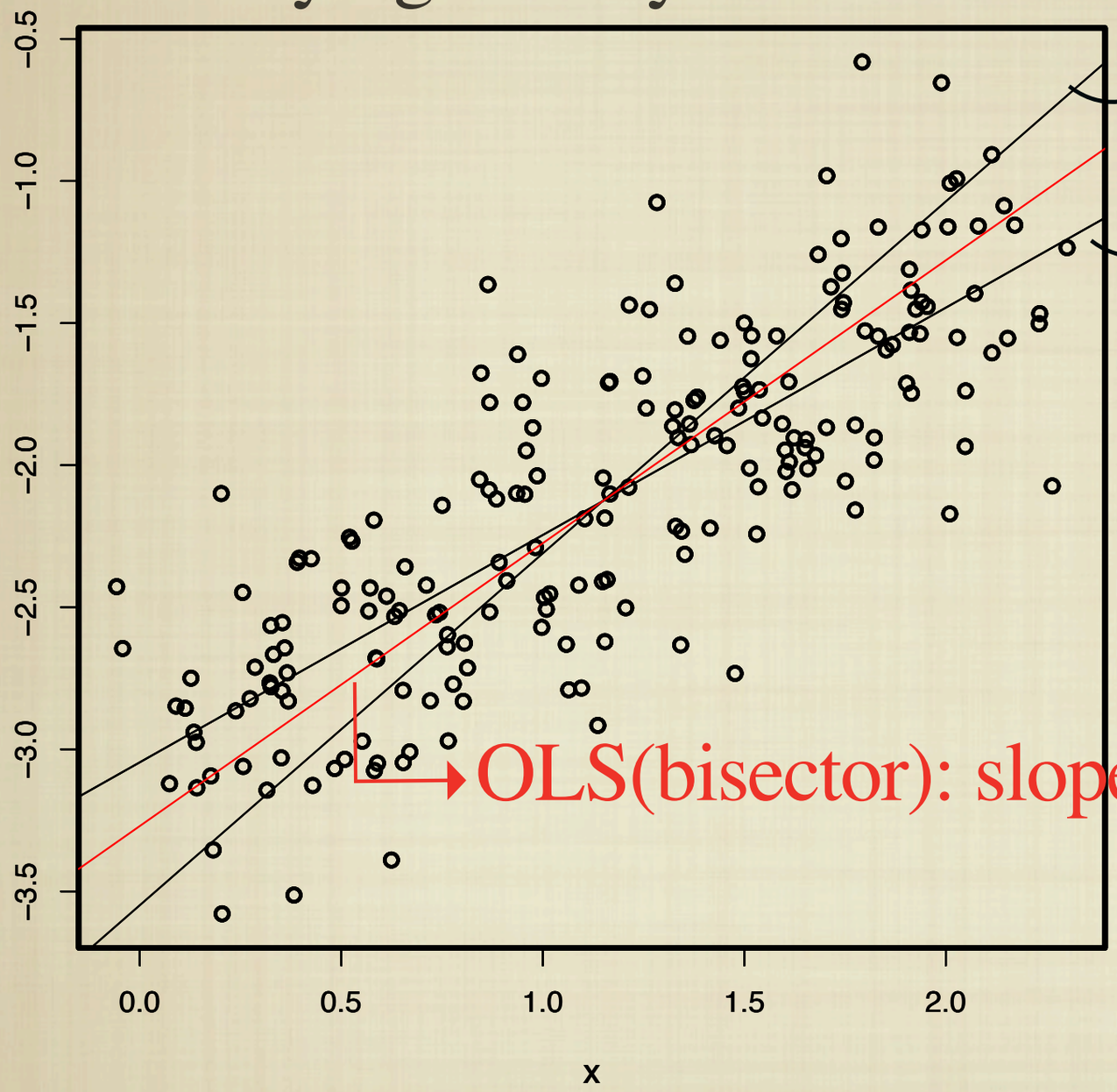

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OLS(y|x): slope = 0.8 +/- 0.04

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Shetty, Kelly, Bigiel '13

Testing OLS Methods

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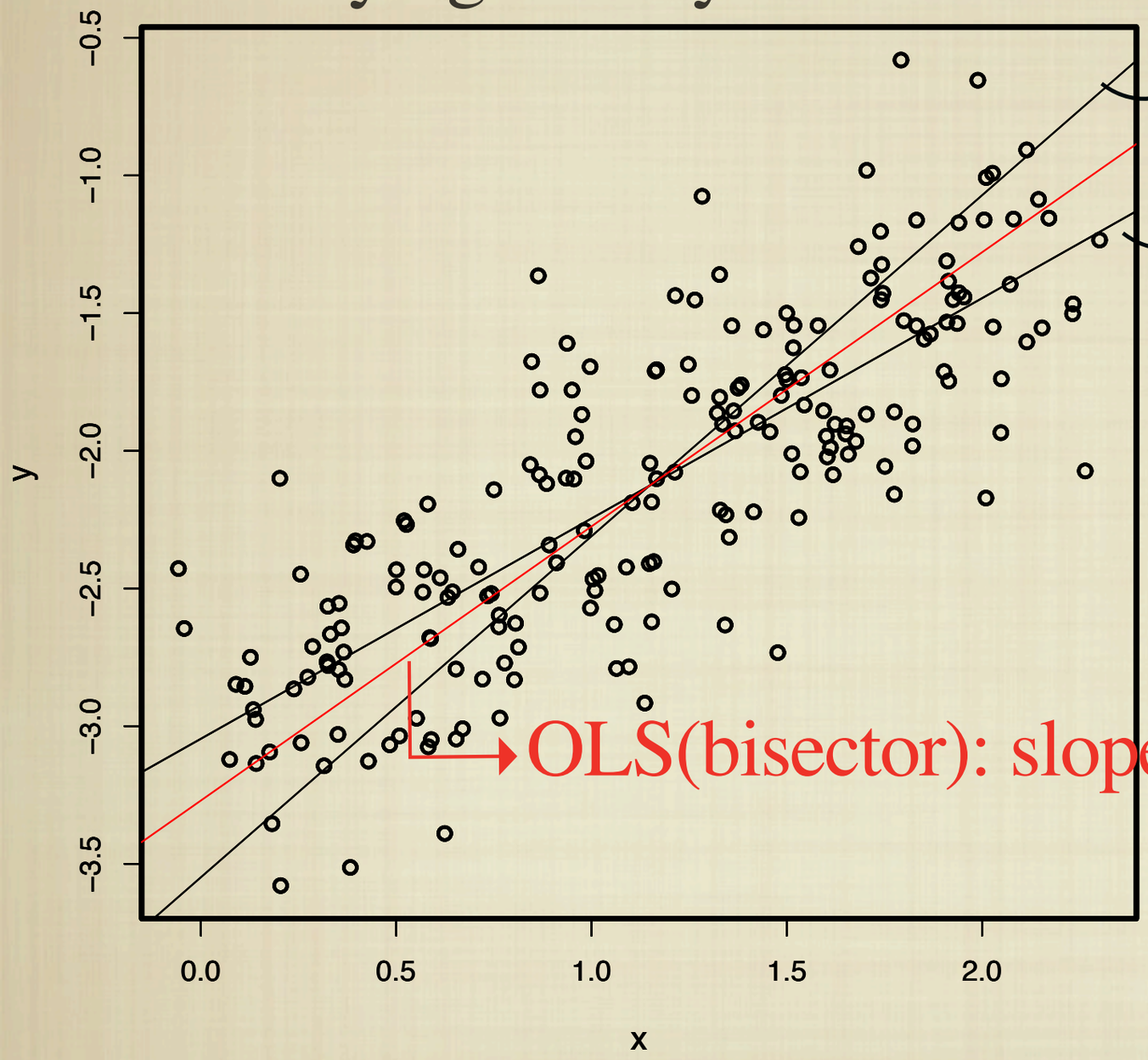

Shetty, Kelly, Bigiel '13

⇒ Bisector does not provide accurate parameter estimates of y = A + Nx

⇒ Bisector usually overestimates slope for N~1

Monte Carlo Methods

- Monte Carlo simulations can rigorously account for uncertainties

  - fit a number of realizations of the data, given the uncertainties

  - final parameter estimates are PDFs

Hierarchical Bayesian Fits

- Hierarchical Bayesian linear regression: $y = \underline{A} + \underline{N}x + \underline{\epsilon_{scat}}$
  - Estimate parameters for ***both*** individuals and ensemble. MCMC provides robust uncertainty estimates (convention to use 2σ as plausible range)
- Verify parameter estimates using synthetic data

Testing the Hierarchical Bayesian Fit

- Use synthetic dataset
- Intrinsic relationship: $\log(\Sigma_{SFR}) = A + N \log(\Sigma_{mol}) + \epsilon_{scat}$
- Include noise in both Σ_{SFR} and Σ_{mol}
- 7 Individuals within group

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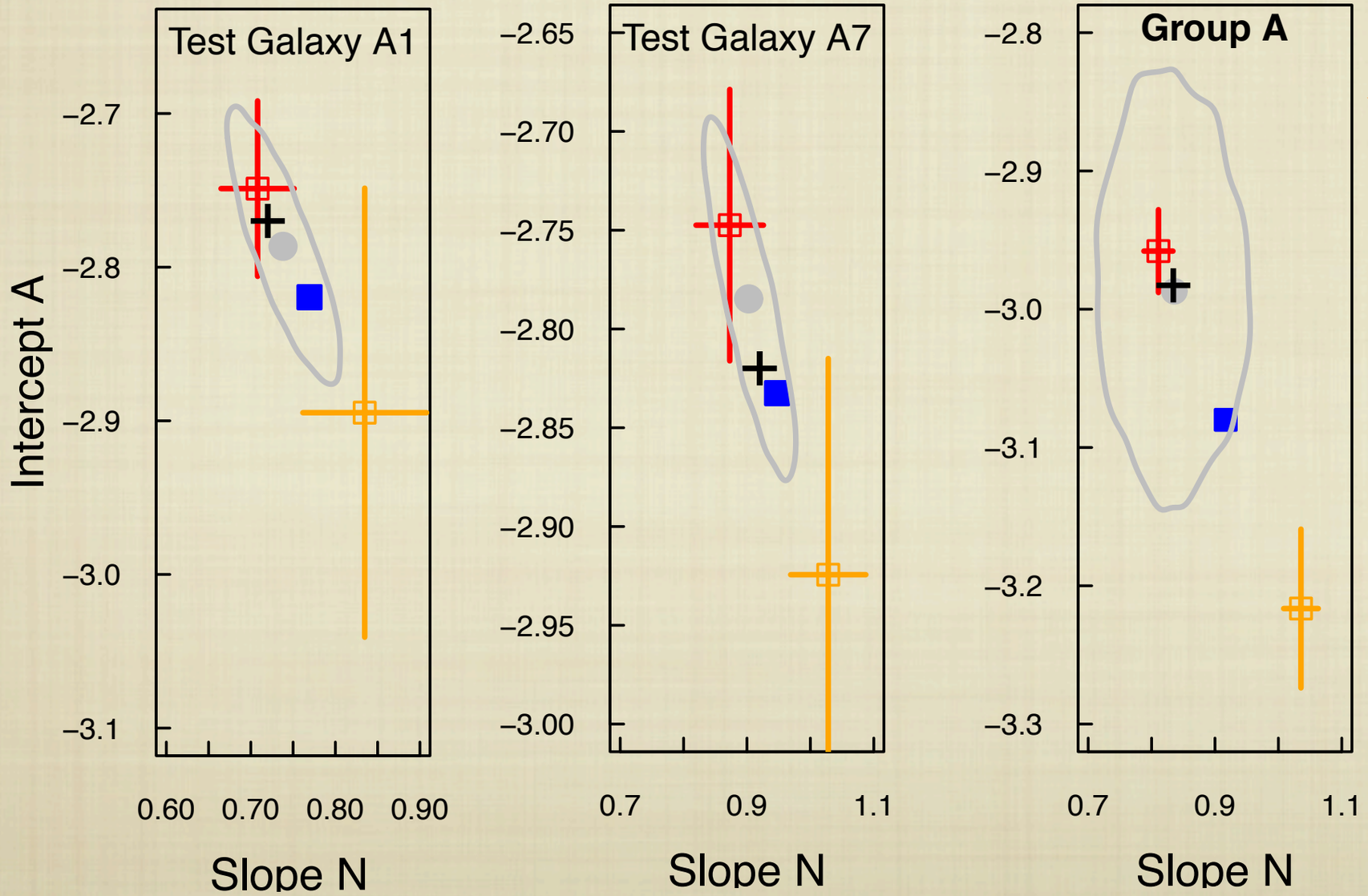


Red: OLS(y|x)

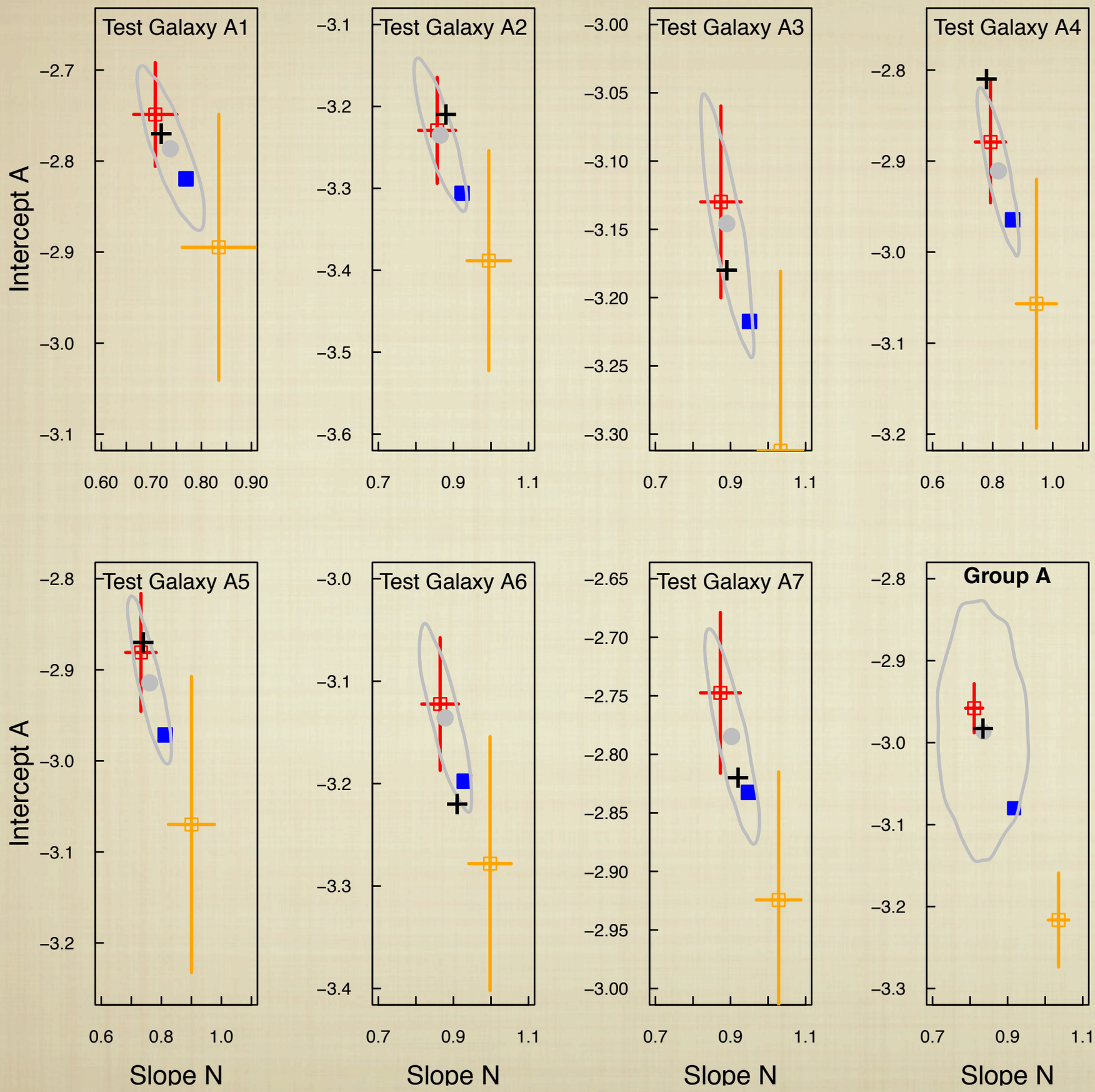
Yell: OLS(x|y)

Blue: OLS(bis)

Grey: hier. Bay.

(incl. 1σ error)

Testing the Hierarchical Bayesian Fit



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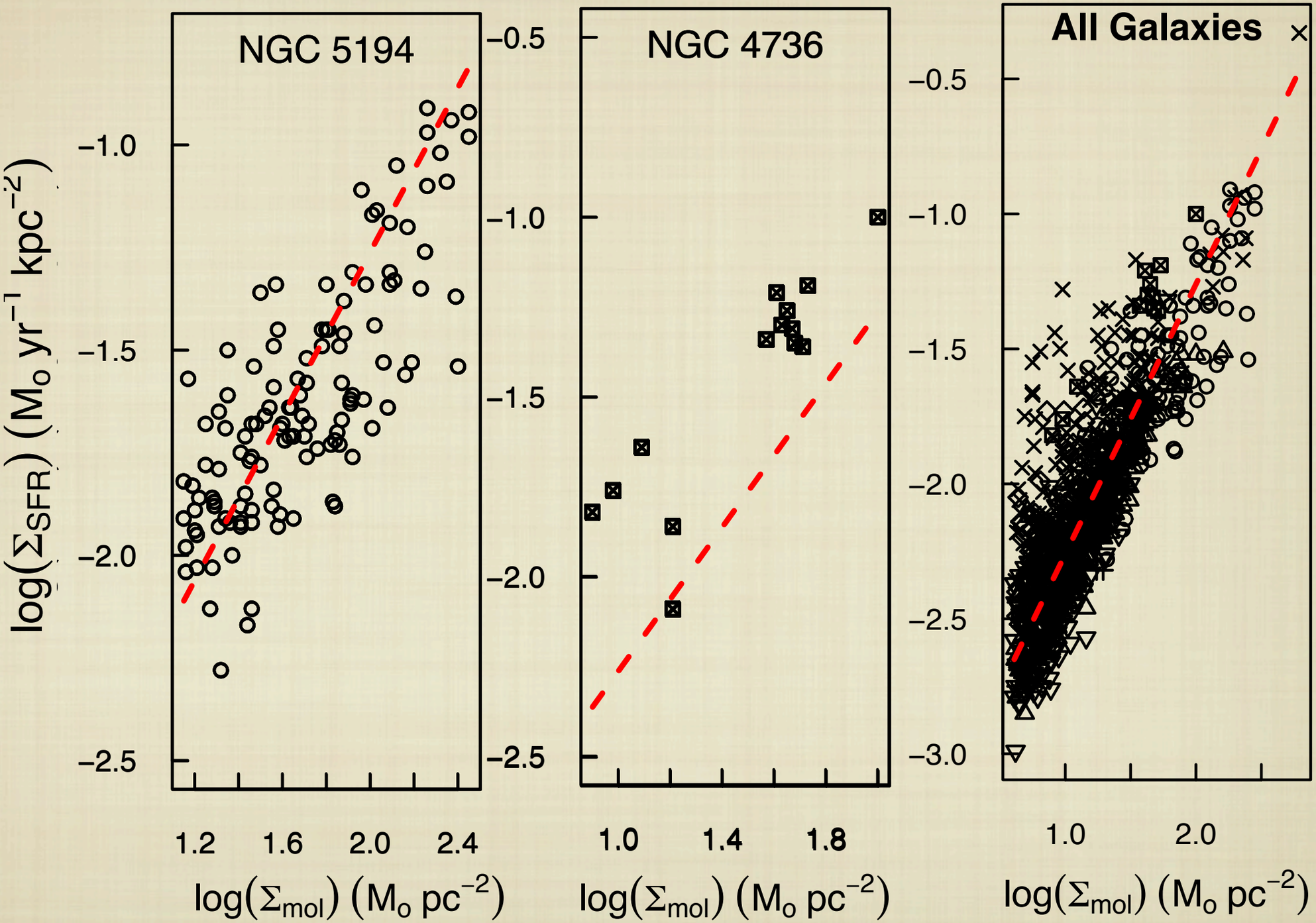
The HERACLES and STING Samples

- HERACLES (Leroy + '09): CO [(2-1), but (1-0) for M51] 24 μm SINGS (Kennicutt + '03), and UV from GALEX (Gil de Paz + '07) for 30 Galaxies

- Bigiel + '08 sub-sample of HERACLES: 750 pc regions for 7 Galaxies (Bigiel + '10)

- STING: CARMA CO (1-0) and 24 μm SINGS for 13 Galaxies (Rahman + '11, '12)

The KS Relationship of the Bigiel + ’08 Sample



Bigiel + ’08
Shetty, Kelly, Bigiel ’13

The KS Relationship of the Bigiel + ’08 Sample

Gray lines:
50 random
draws from
the Bayesian
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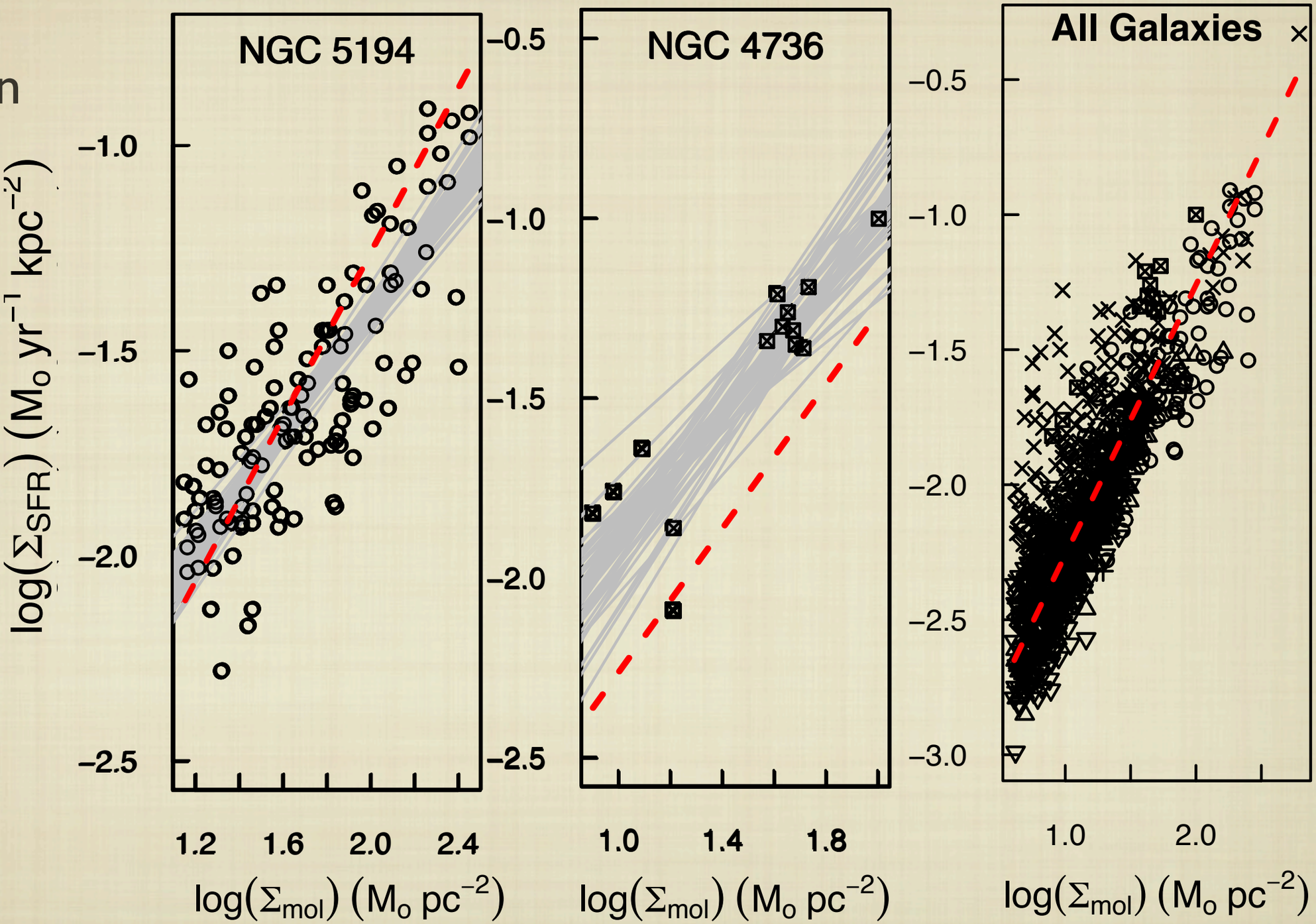

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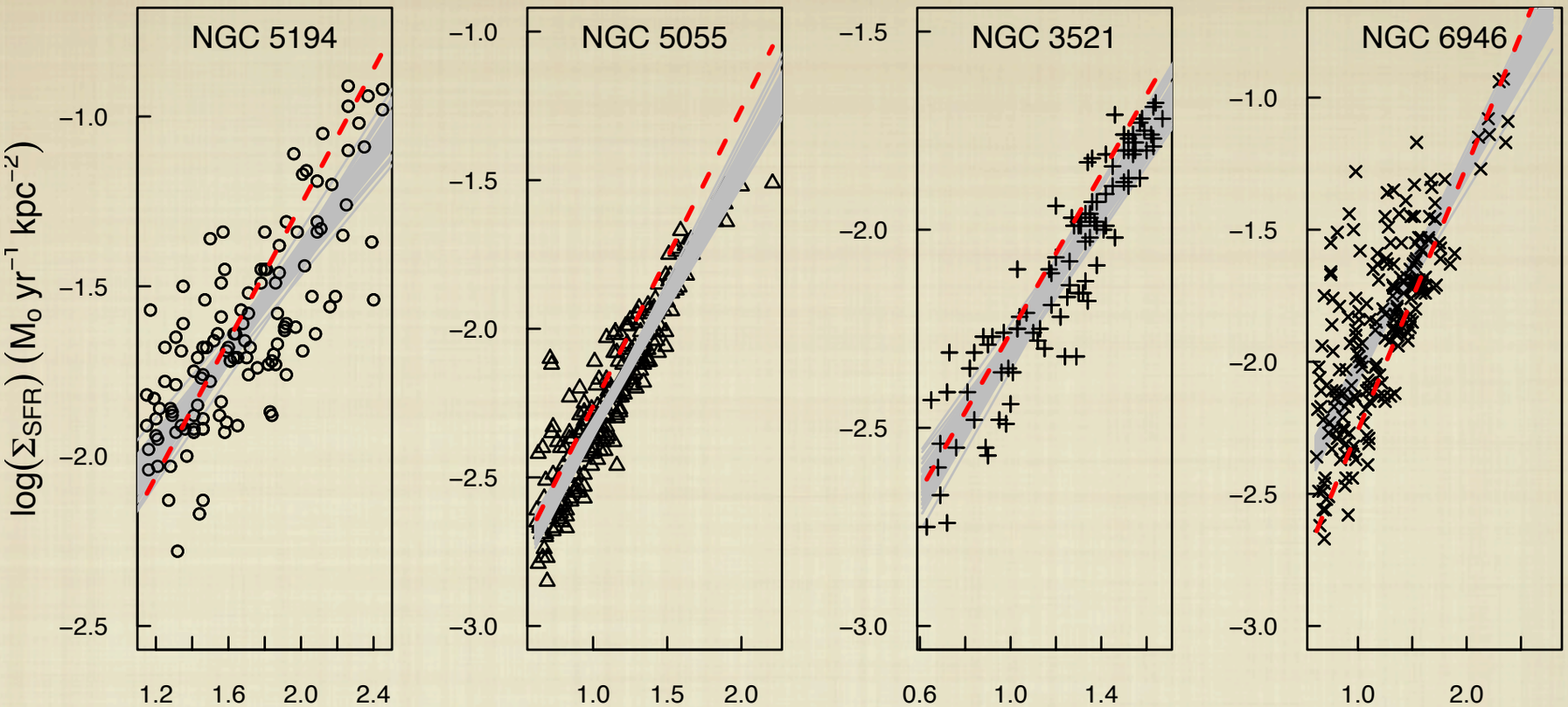

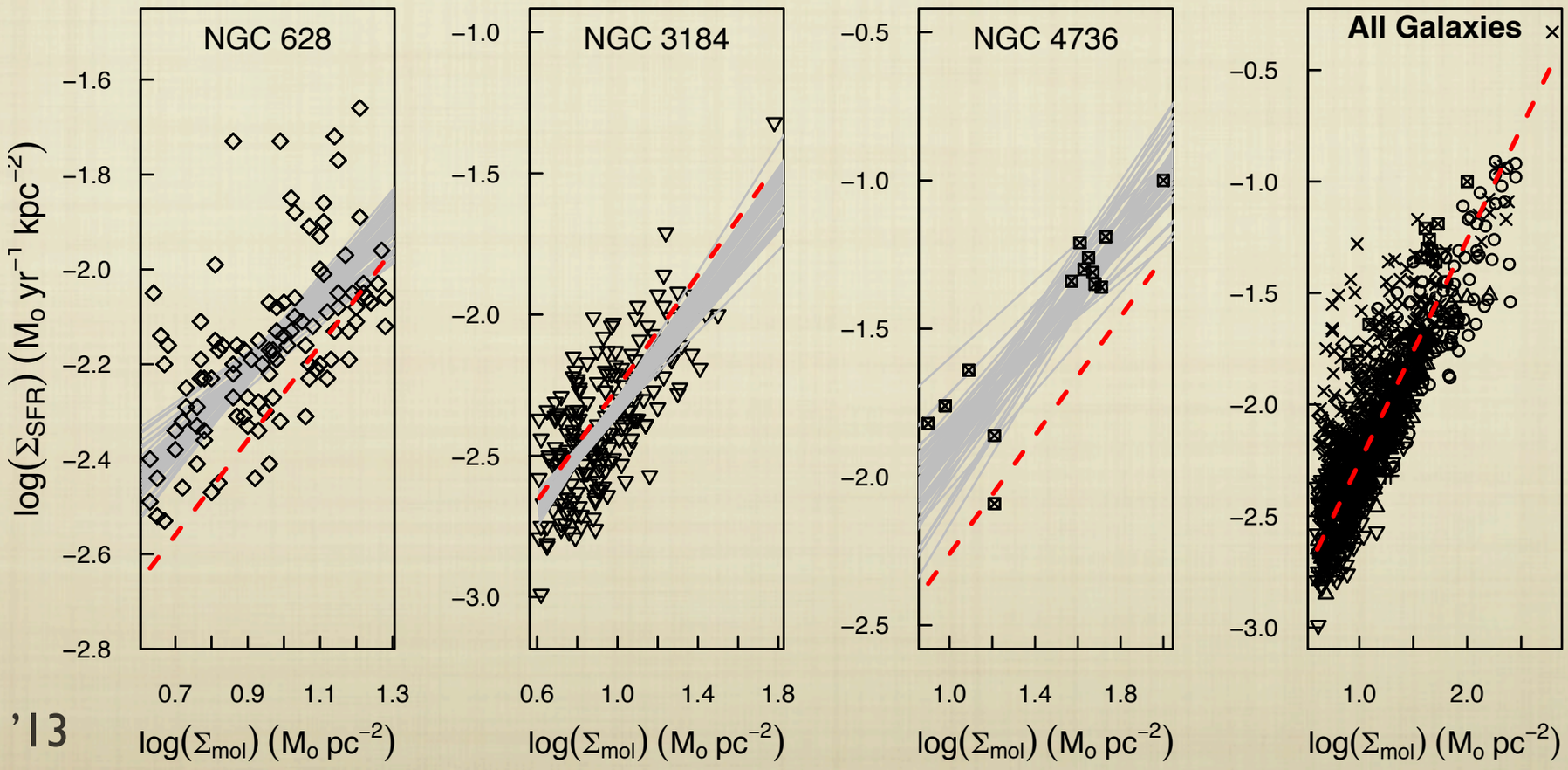

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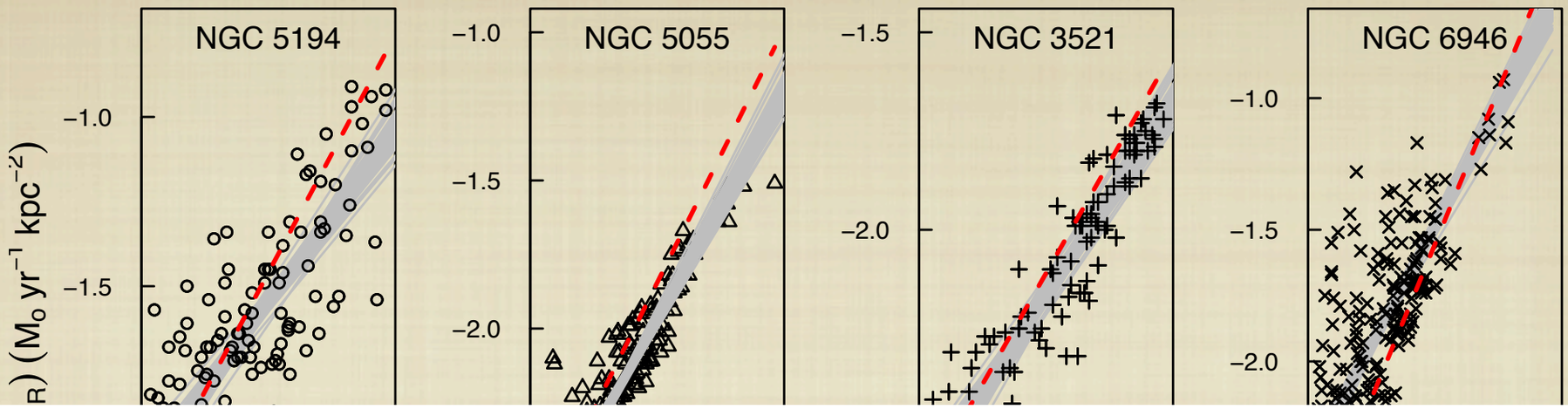

Subject	Bayes A	Bayes $2\sigma_A$	Bayes N	Bayes $2\sigma_N$	Bayes $\sigma_{\rm scat}$
NGC 5194 (M51)	-2.84	$[-3.0, -2.7]$	0.72	[0.62, 0.83]	0.06
NGC 5055	-3.20	$[-3.3, -3.1]$	0.87	[0.79, 0.95]	0.04
NGC 3521	-3.20	$[-3.4, -3.0]$	0.90	[0.76, 1.03]	0.05
NGC 6946	-2.81	$[-2.9, -2.7]$	0.78	[0.70, 0.86]	0.11
NGC 628	-2.89	$[-3.1, -2.6]$	0.76	[0.51, 0.95]	0.05
NGC 3184	-3.24	$[-3.4, -3.1]$	0.92	[0.79, 1.10]	0.05
NGC 4736	-2.83	$[-3.2, -2.4]$	0.92	[0.67, 1.20]	0.08
Group Parameters	$\mathbf{-3.00}$	$\mathbf{[-3.3, -2.7]}$	**0.84**	**[0.63, 1.0]**	0.14

Bigiel + '08
Shetty, Kelly, Bigiel '13

The KS Relationship of the Bigiel + '08 Sample

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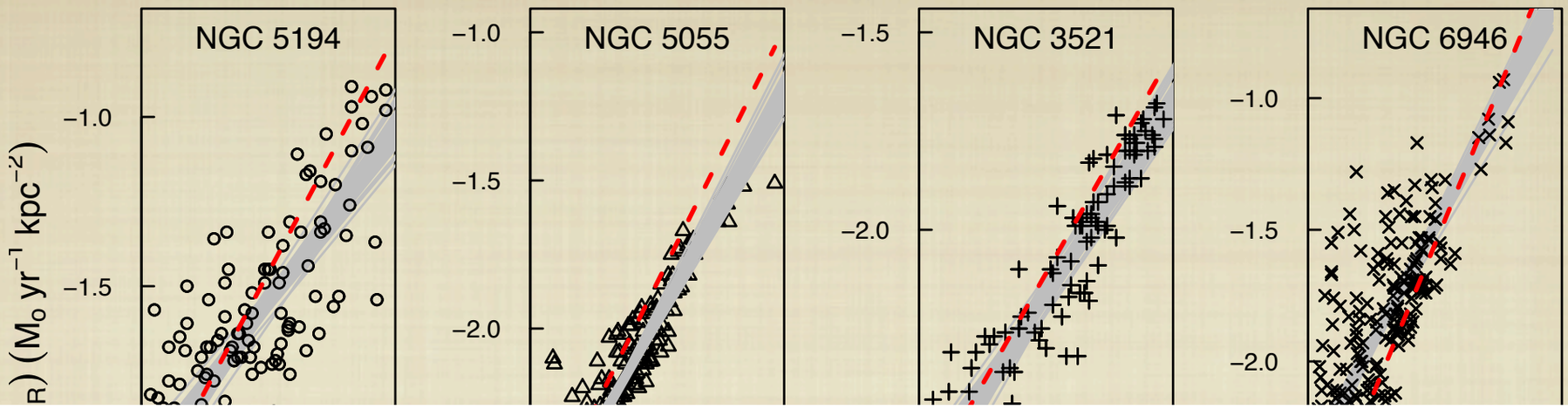

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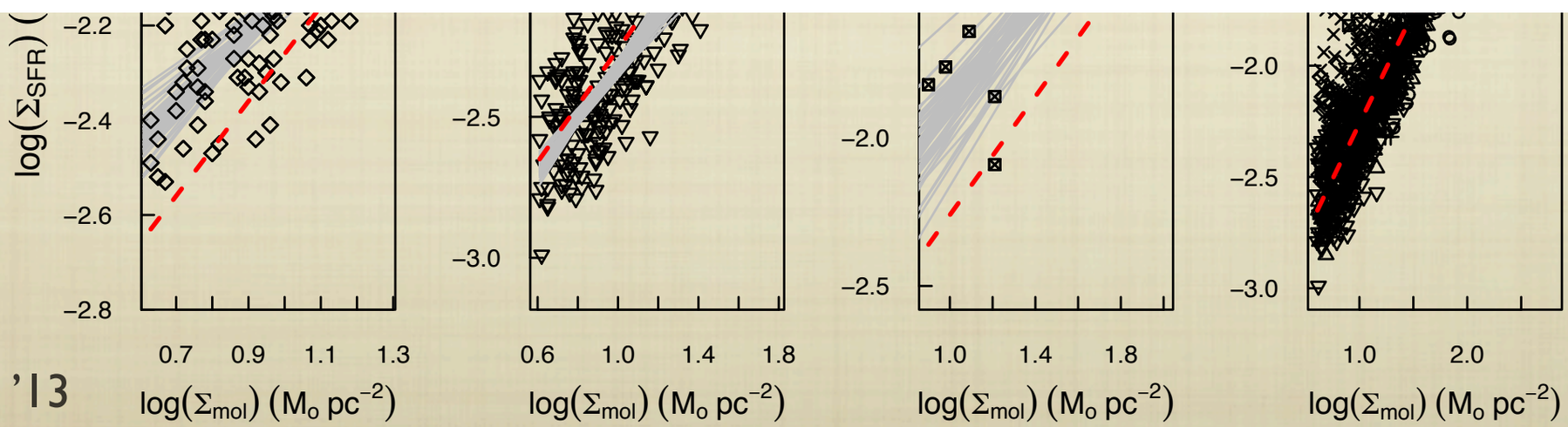
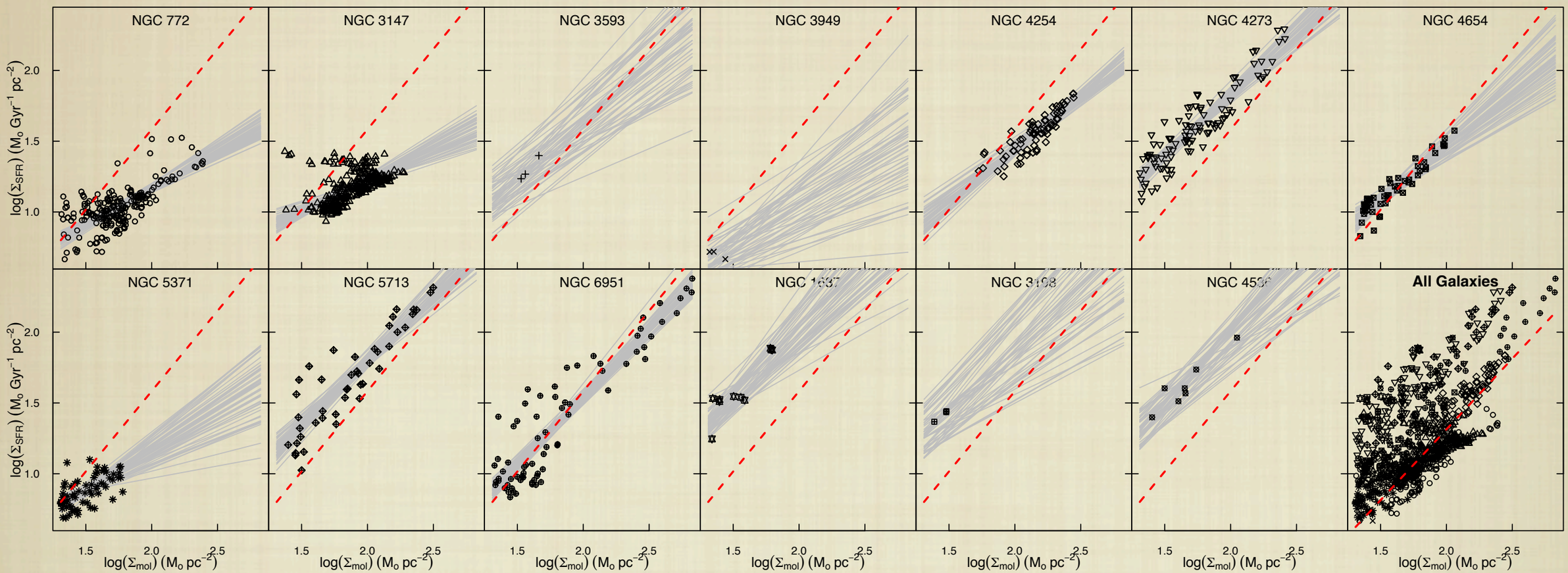


Bigiel + '08
Shetty, Kelly, Bigiel '13

The KS Relationship of the STING Sample



Rahman +'11, '12,
Shetty + '13b

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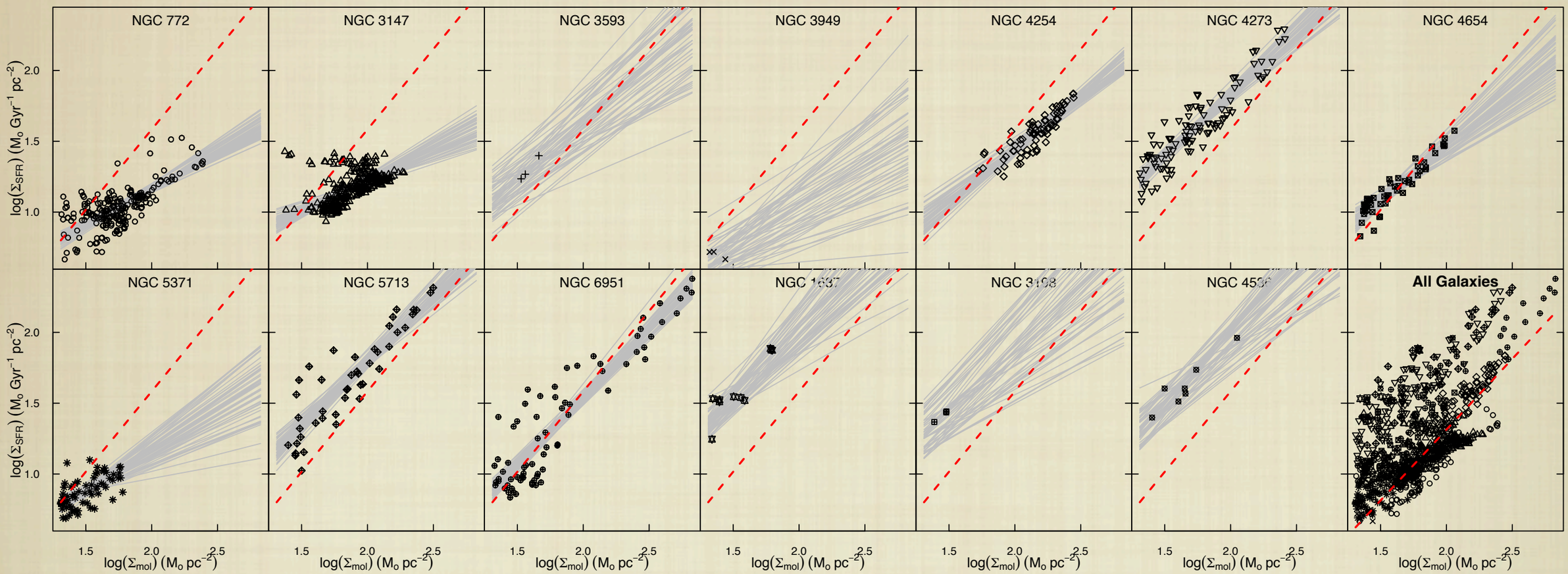

Individual slopes range from 0.42 - 0.95
Mean Slope = 0.76; 2σ=[0.58 - 0.94]

Rahman +'11, '12,
Shetty + '13b

Importance of hierarchical fits

- Even without hierarchical fit, Bayesian fit slope of Bigiel + '08 sample [0.88 - 0.95], indicating sub-linear KS relationship (Shetty, Kelly, Bigiel '13)
- By eye fit...?

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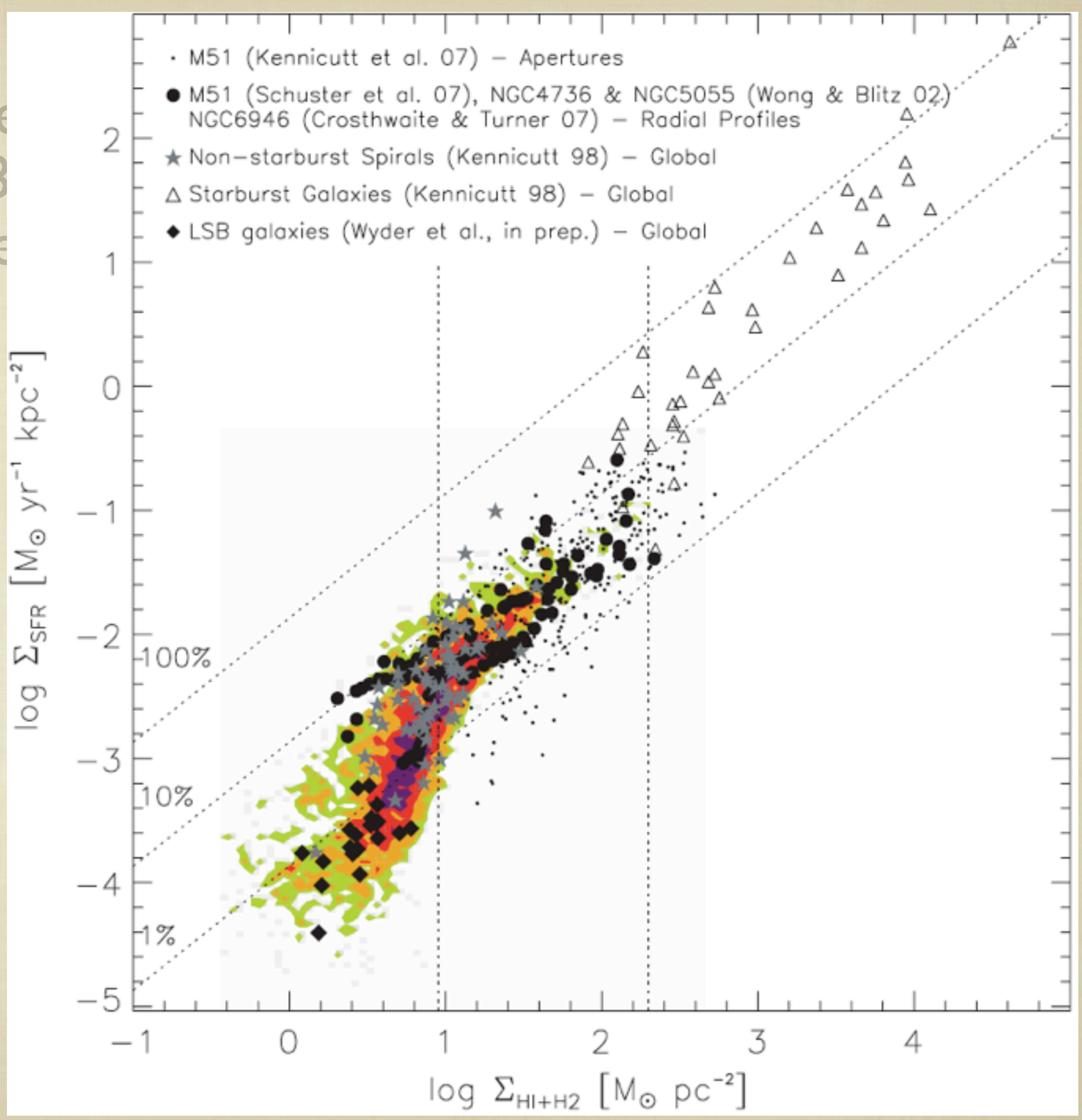

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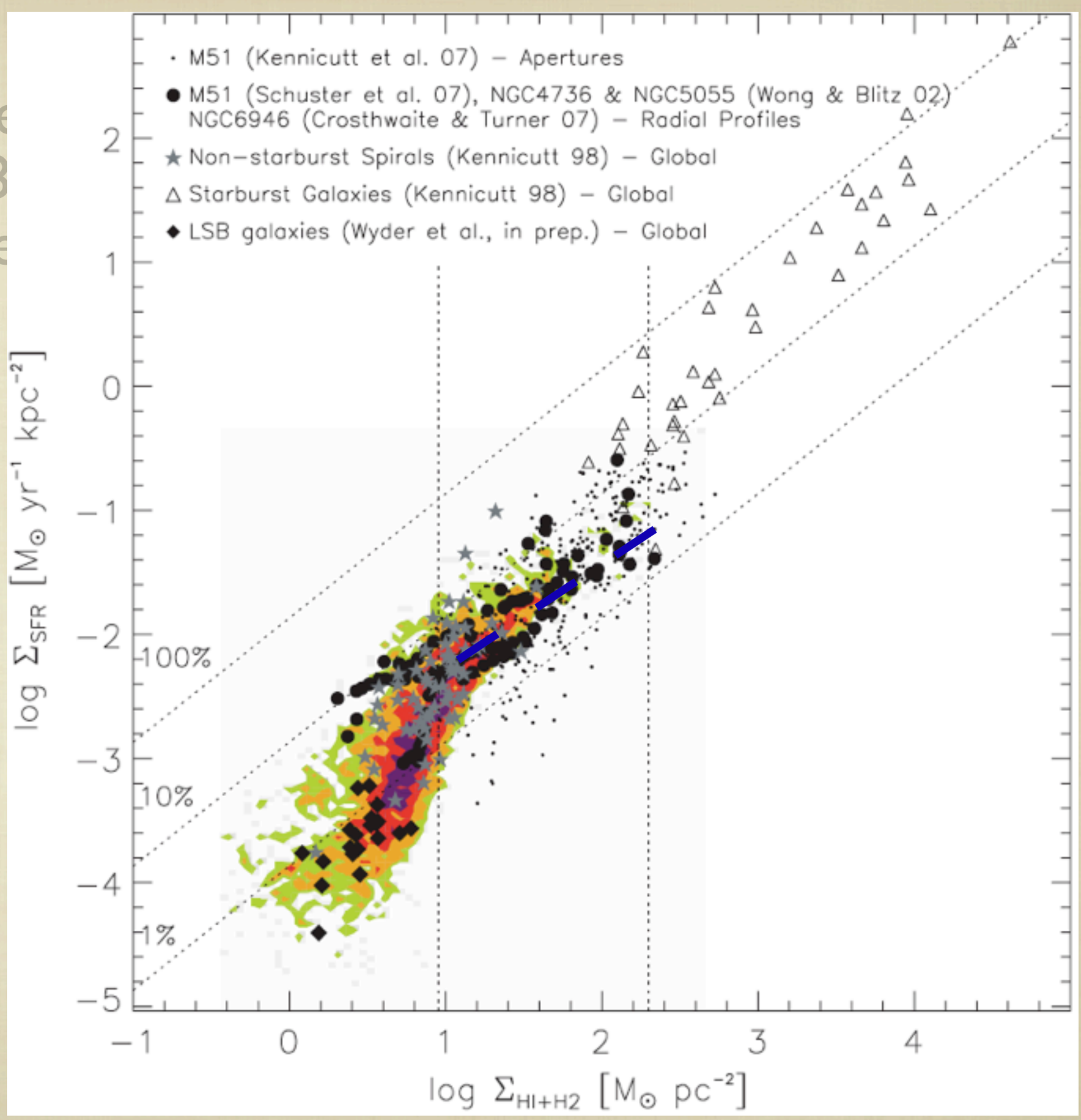

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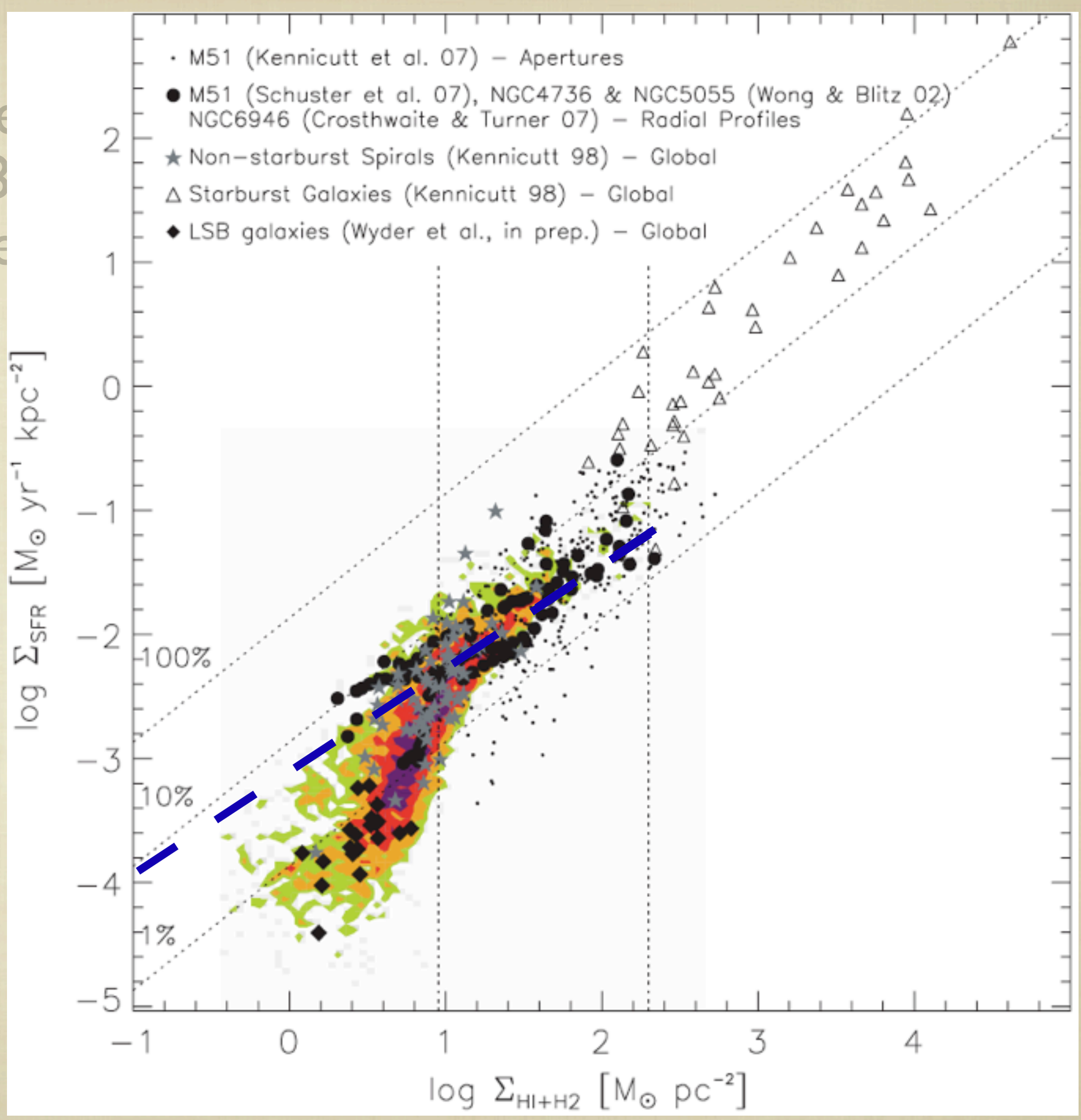
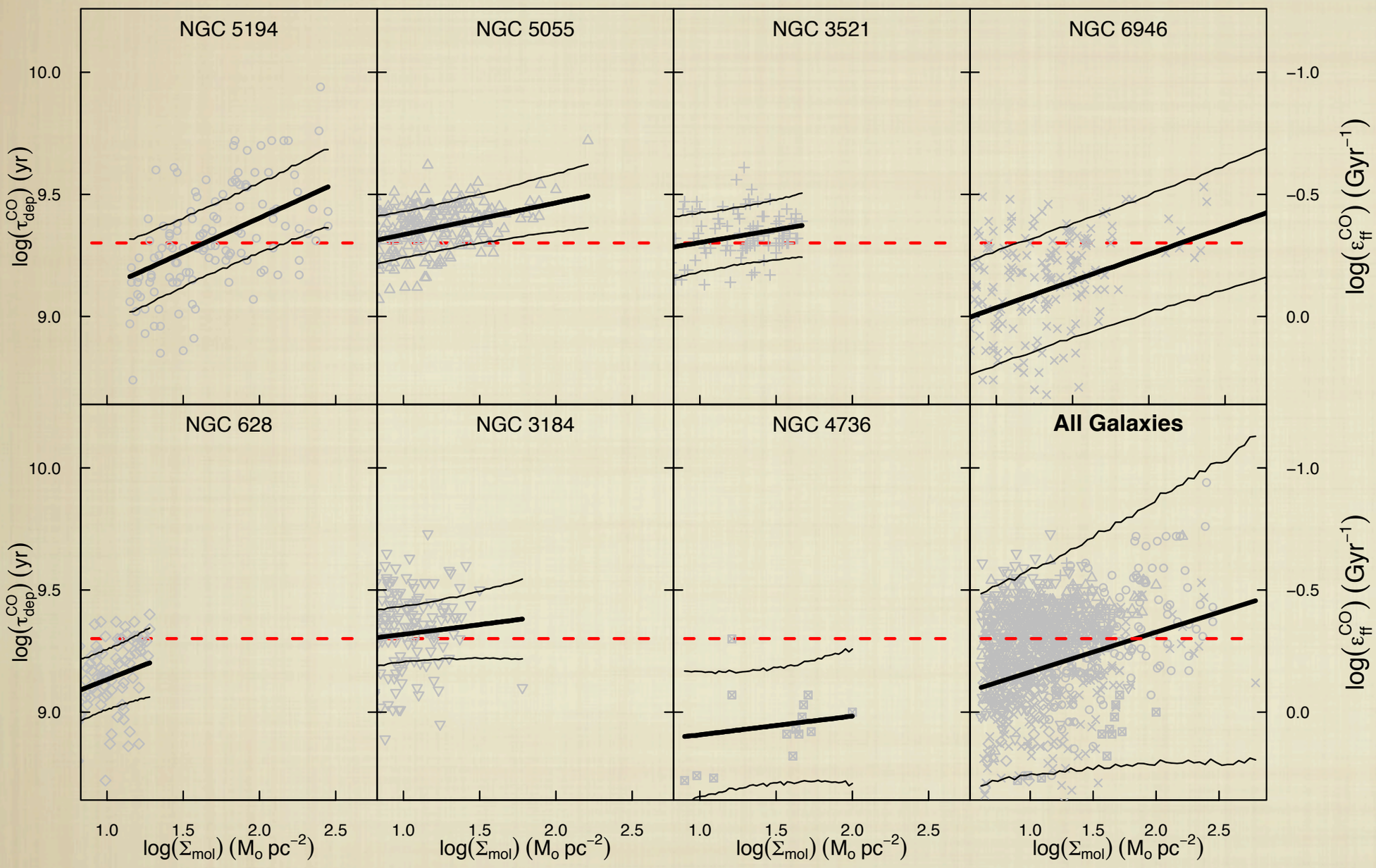


Depletion times $T_{DEP}=\Sigma_{GAS}/\Sigma_{SFR}$



Bigiel + ’08 HERACLES sub-Sample

Shetty + ‘13a,b

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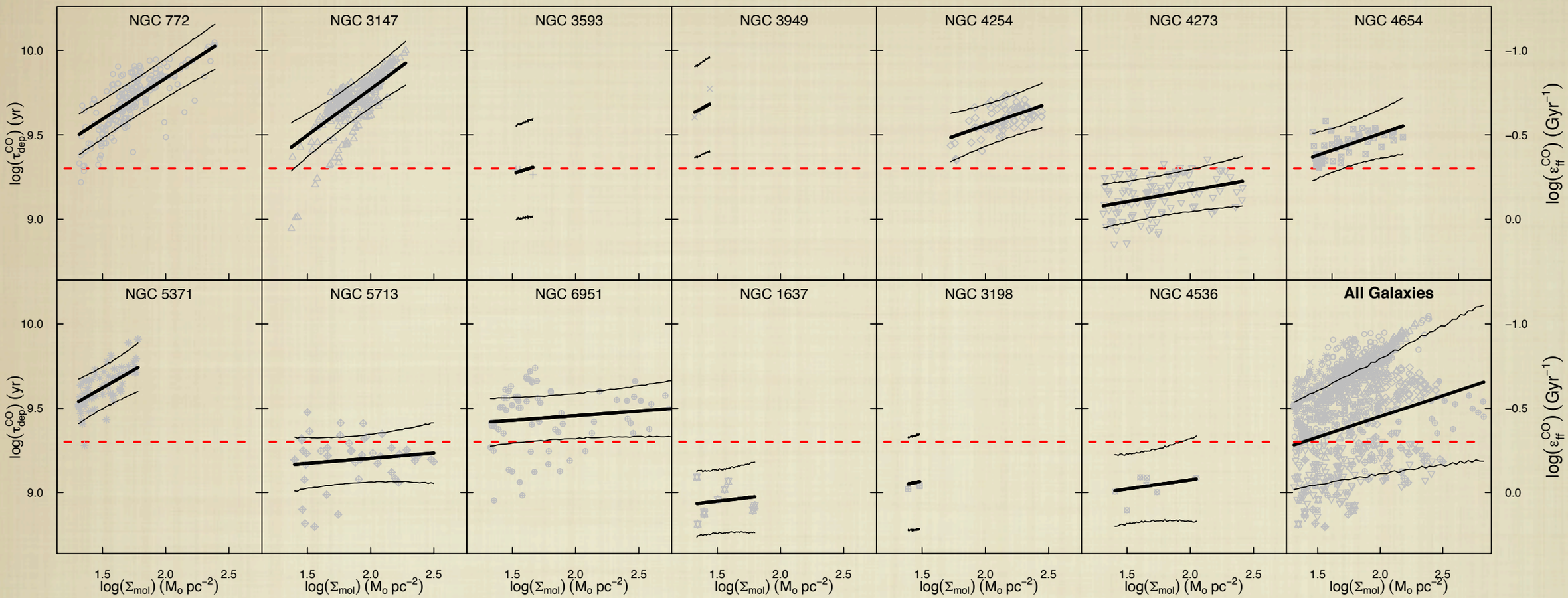

Rahman + ’12 STING Sample

Shetty + ‘13a,b

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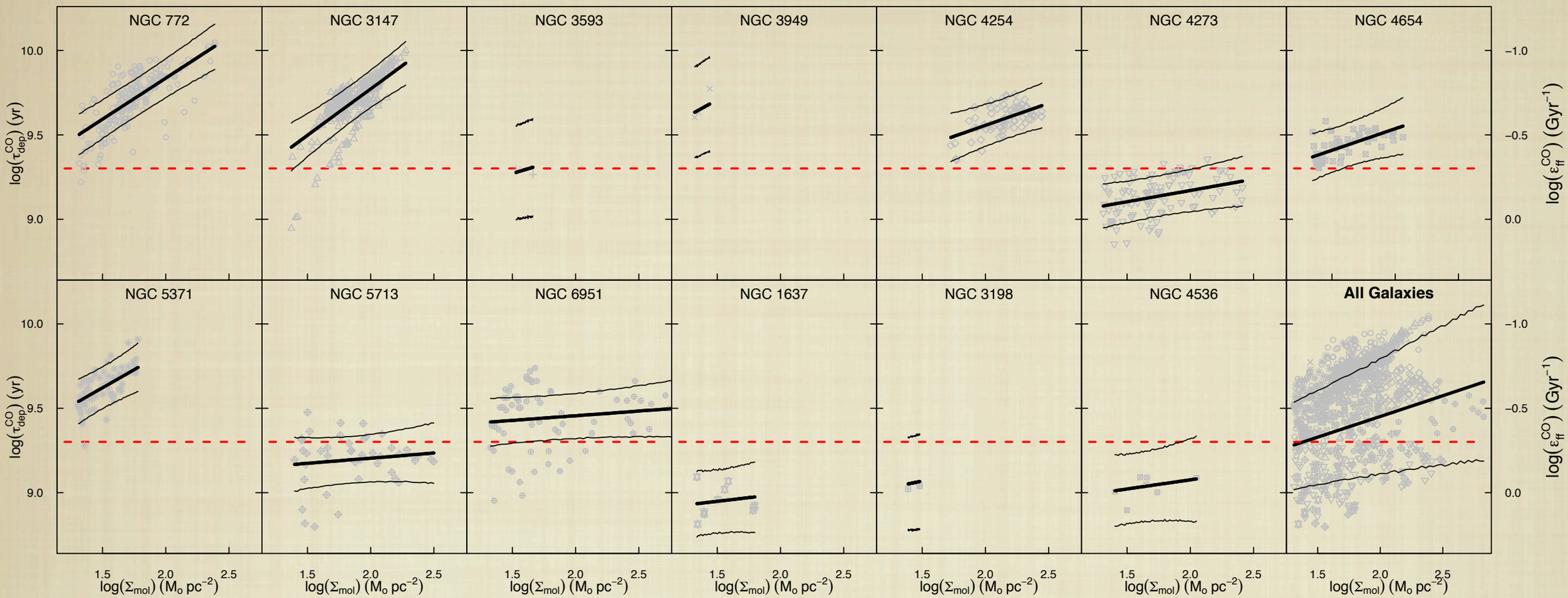

Rahman + '12 STING Sample

⇒ No Universal depletion time

(Bigiel+'08, Leroy+'09, Schruba+'11, Rahman+'12, etc...)

⇒ t_{dep} = 2 Gyr is not a representative timescale

Shetty + '13a,b

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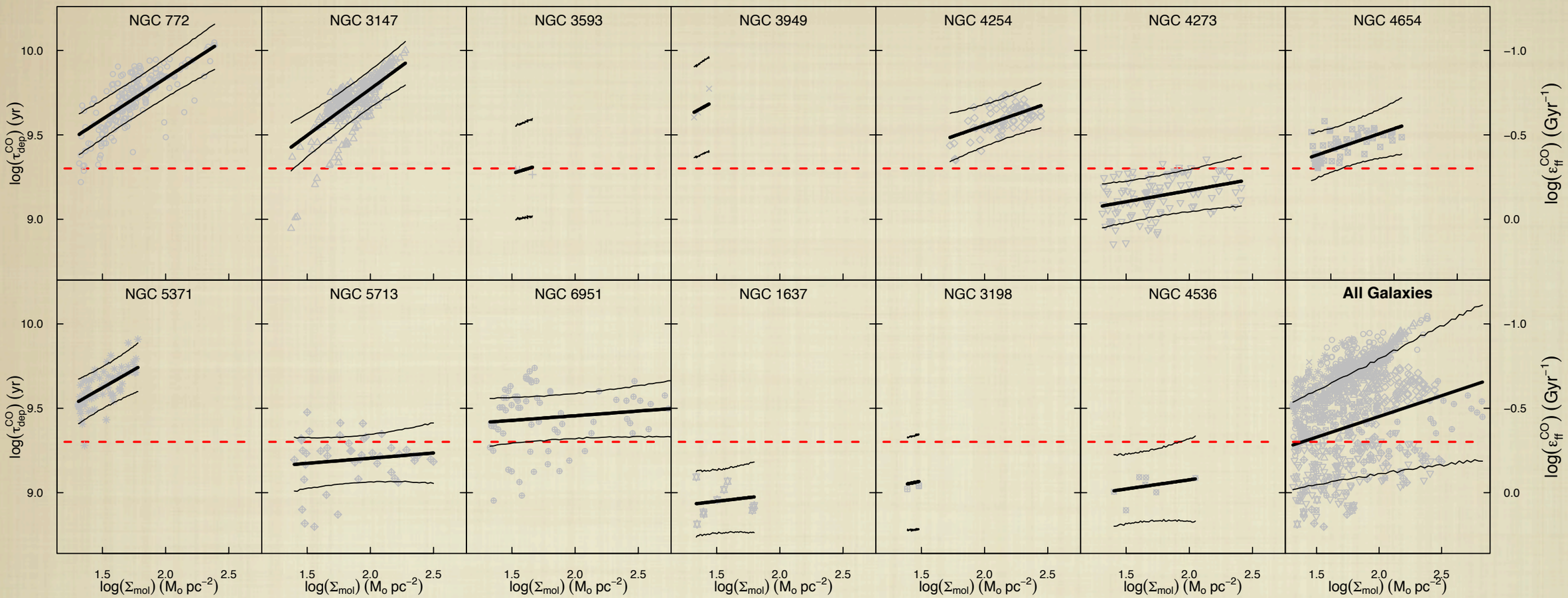

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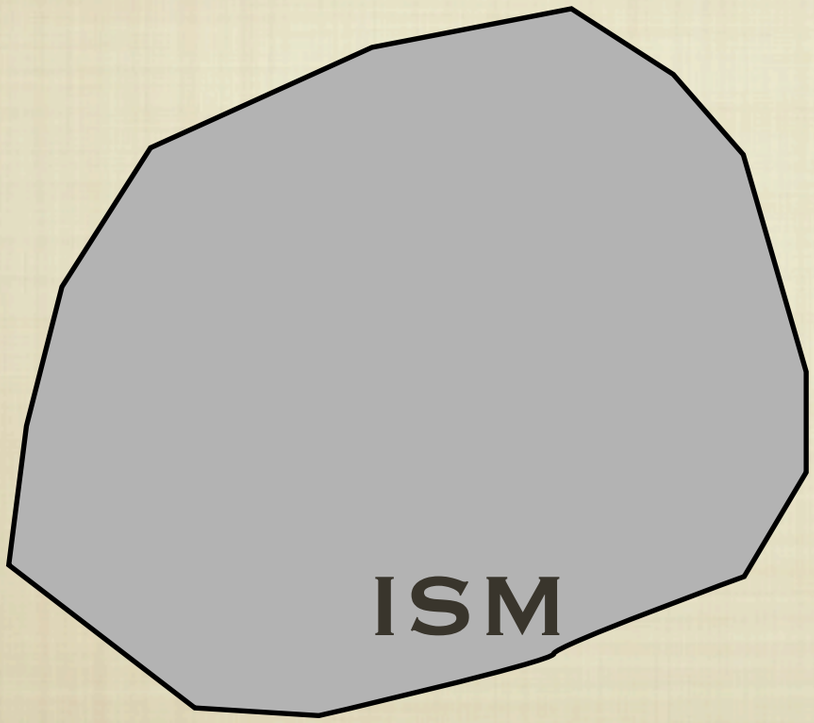
⇒ t_{dep} = 2 Gyr is not a representative timescale

⇒ t_{dep} increases with Σ_{gas}, as traced by CO

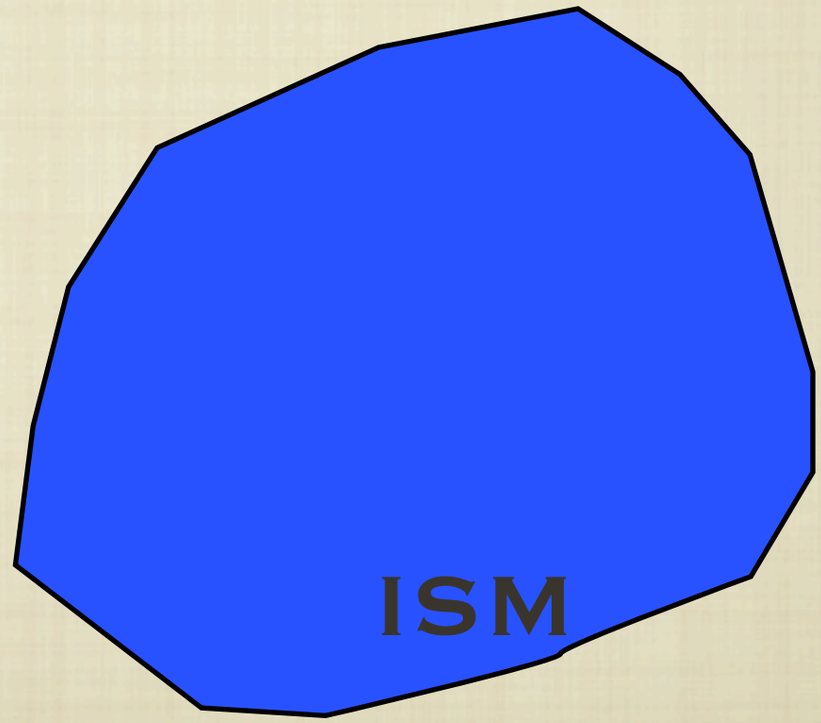
Shetty + '13a,b

Interpreting the KS Slope

- No evidence for an "Universal" KS slope: magnetic fields, stellar content, metallicity, molecular gas fraction all affect SF properties of given galaxy
- Significant evidence for sub-linear relationship, for most individual galaxies and for both ensembles: CO tracing some gas that is not associated with star formation?



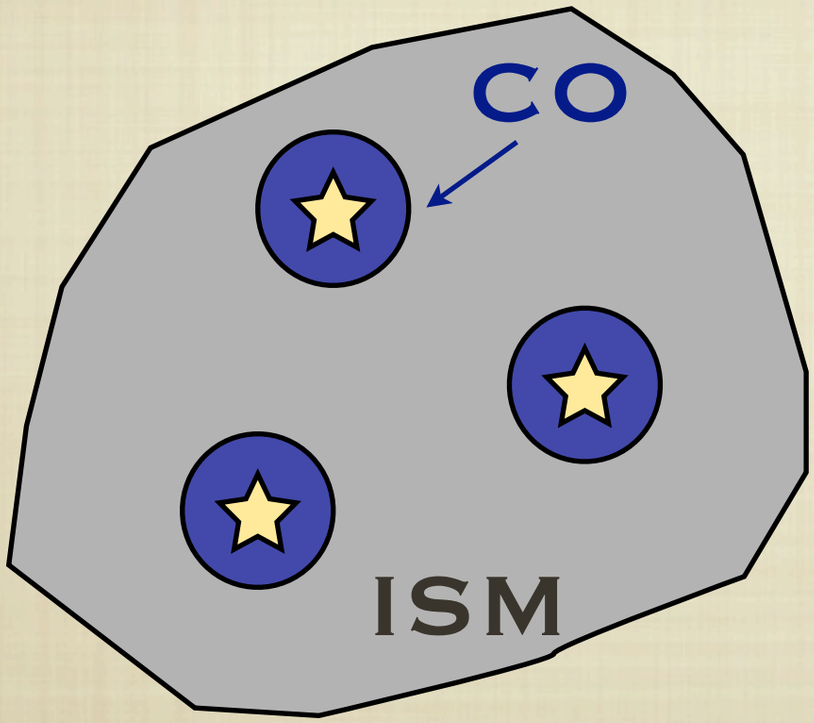
Outer Disk or Low Metallicity



Inner Disk or High Metallicity

Interpreting the KS Slope

- No evidence for an "Universal" KS slope: magnetic fields, stellar content, metallicity, molecular gas fraction all affect SF properties of given galaxy
- Significant evidence for sub-linear relationship, for most individual galaxies and for both ensembles: CO tracing some gas that is not associated with star formation?



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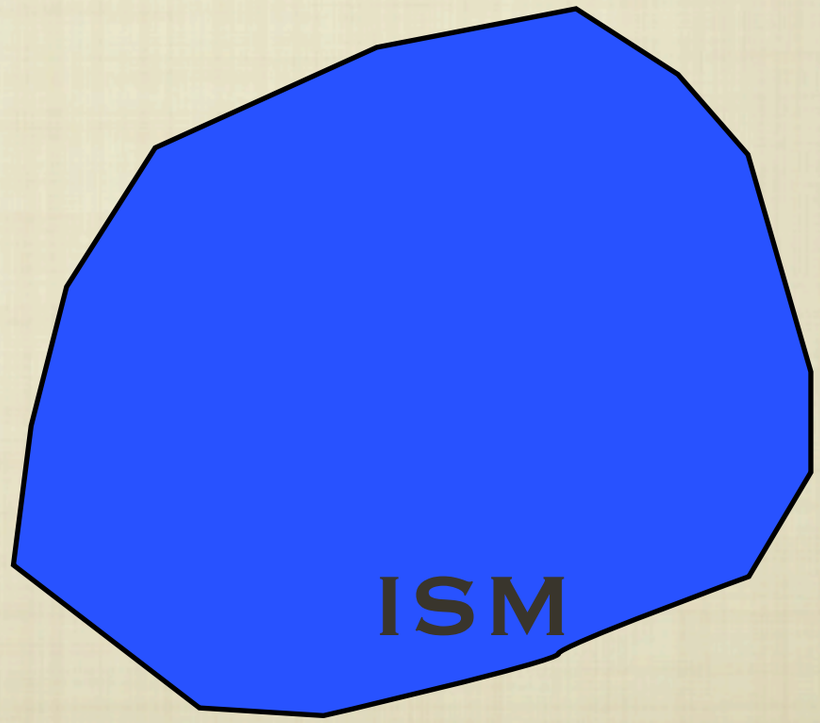

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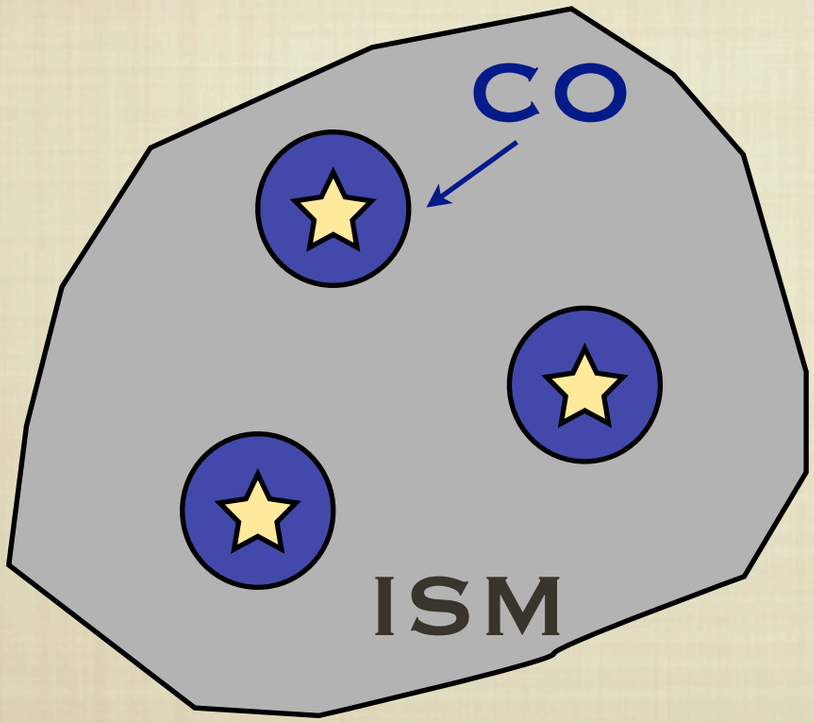

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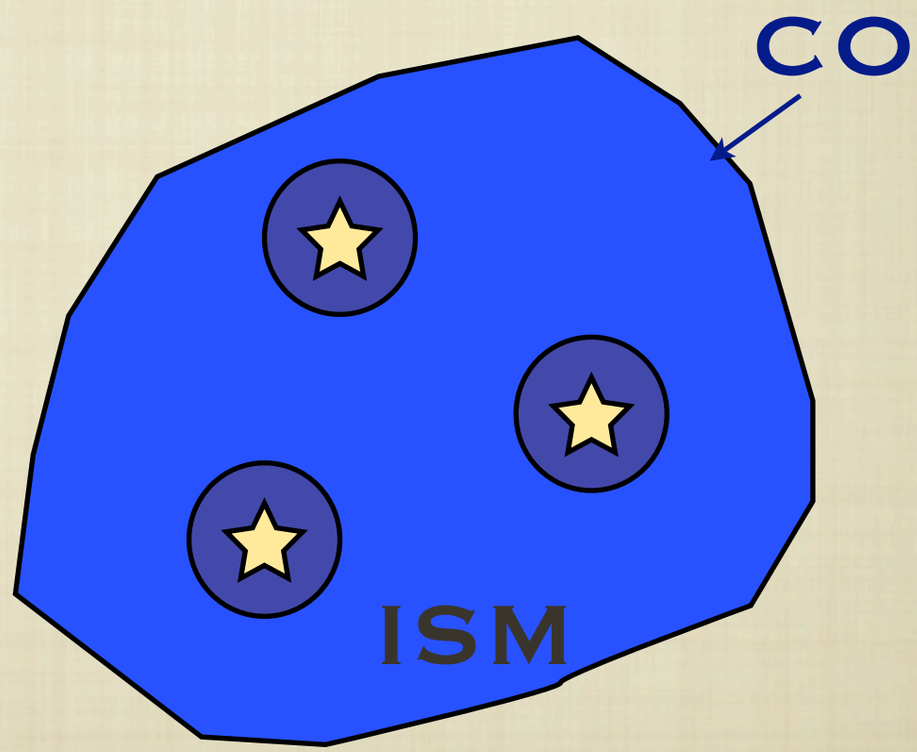
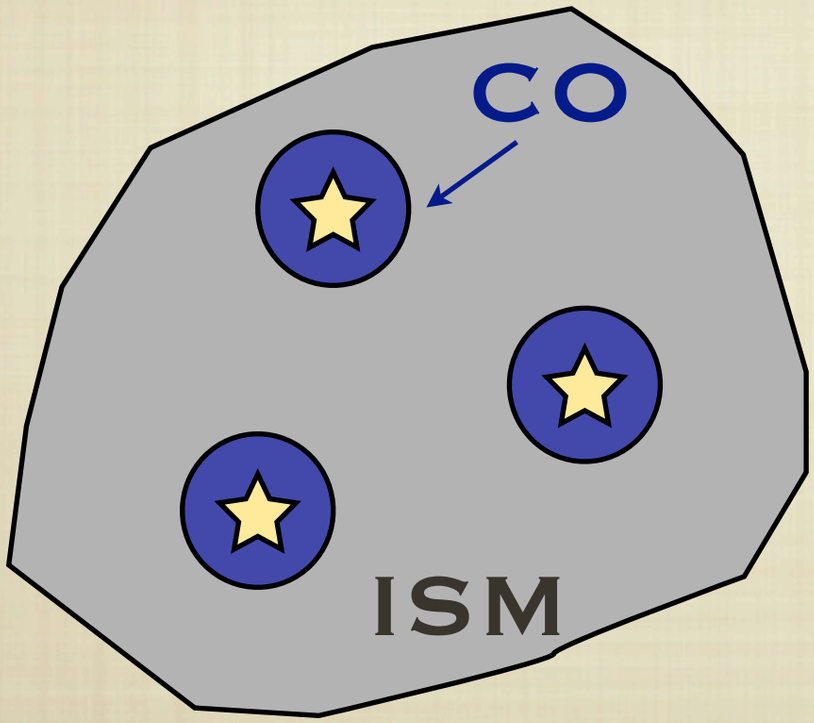


Inner Disk or High Metallicity

Interpreting the KS Slope

- M51 the most sublinear slope (0.72) in Bigiel + '08 sample (see also Blanc + '09): diffuse CO emission? (PAWS, Pety, Hughes, Meidt, Schinnerer)



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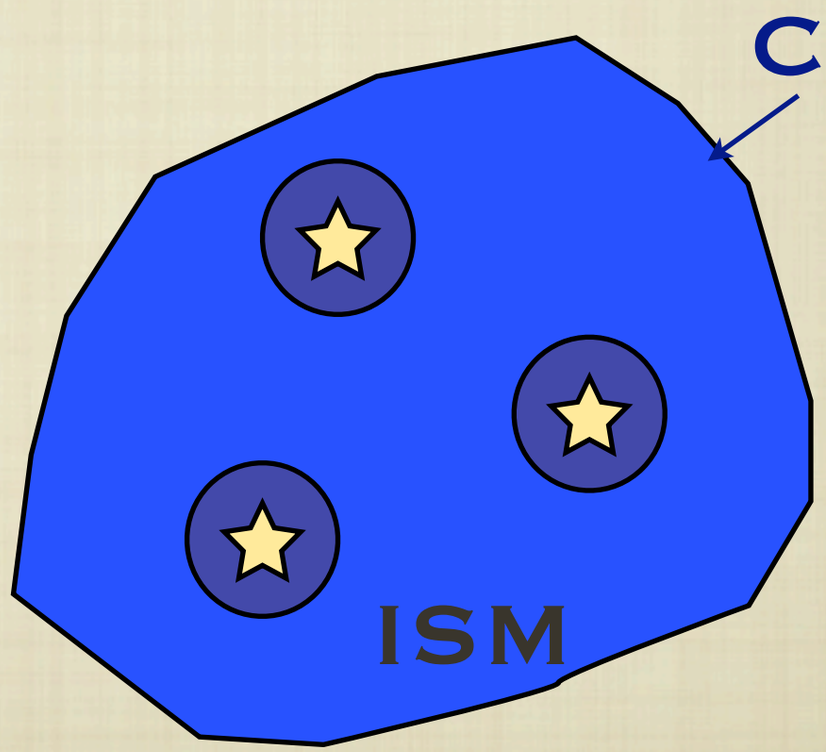
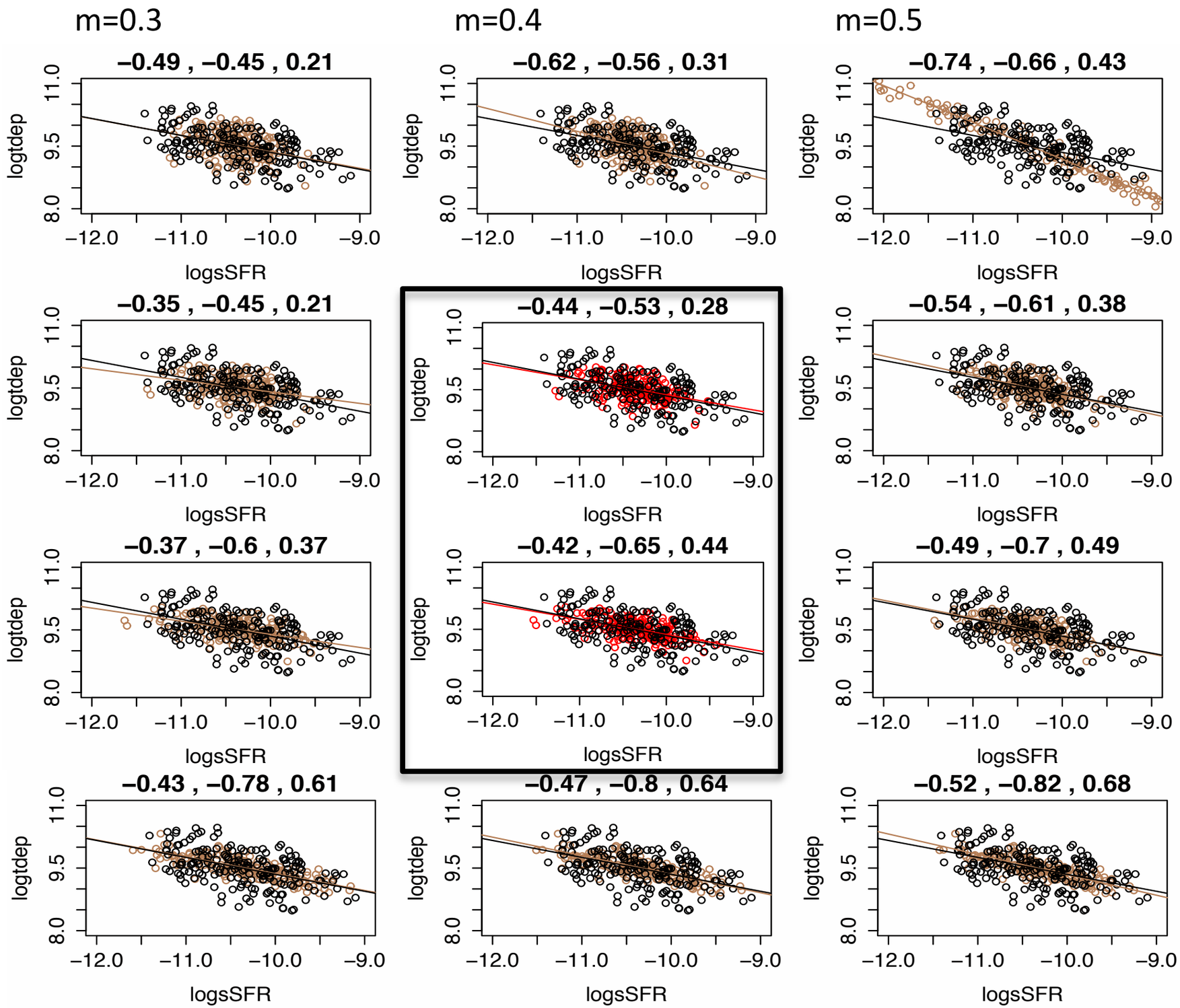


Inner Disk or High Metallicity

Caveats

- Diffuse emission? (Kennicutt + '07, Liu + '11)
- Conversion factors...?
  - e.g. X factor, old stellar population, IMF...?
  - Correlations b/w parameters (Shetty, Kelly, Bigiel '13)?
    - slope and scatter depend on scale
      - Schruba + '11, Kruijssen & Longmore In Prep
- Single dish for STING (checked NGC 4254 in Shetty + '13b)

COLD GASS (Saintonge + '12)



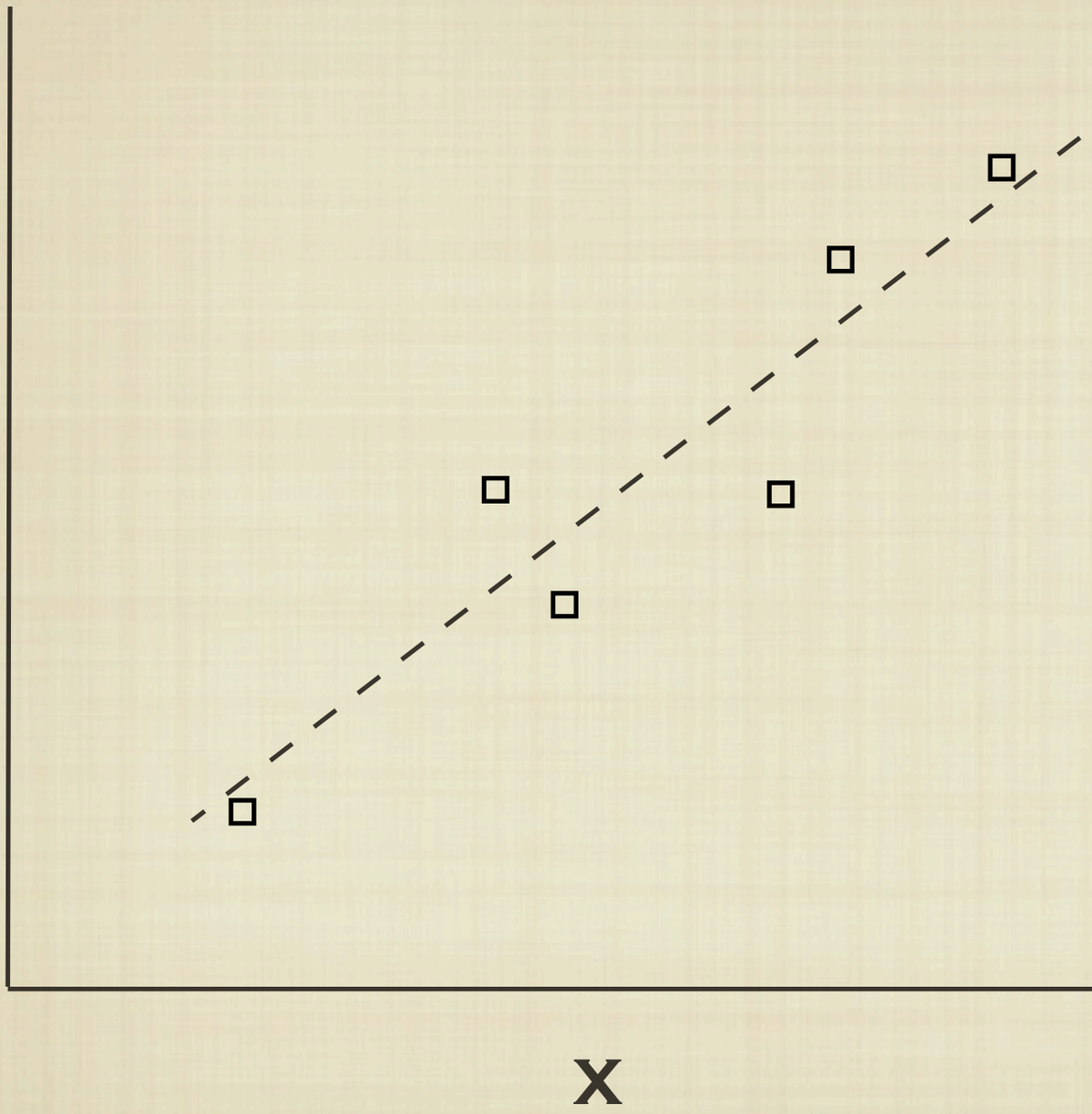
Hopkins + '13, In Prep

(see poster)

Summary

- Need to fit data carefully
  - noise leads to biases
  - bisector provides inaccurate estimates
  - hierarchical fits for hierarchical data
  - Monte Carlo (& Bayesian) provide accurate estimates
- 2 datasets strongly indicate a mean sub-linear KS relationship, and sub-linear slopes for most galaxies
- No "Universal" KS law
  - t_{dep} = 2 Gyr not representative
  - t_{dep} increases with CO traced surface density
  - indication of diffuse CO component?

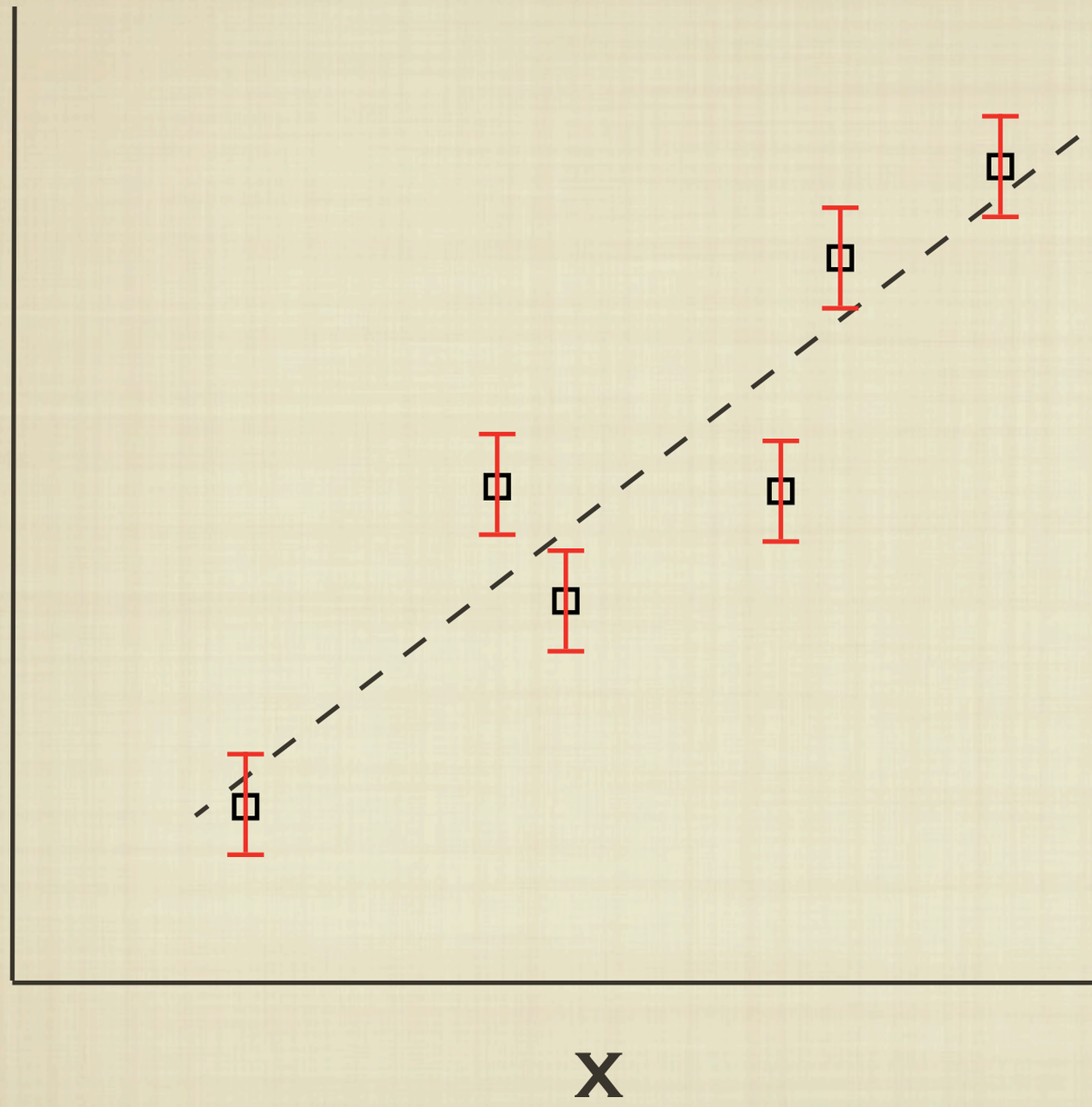
Monte Carlo Methods



$$y_i = A + N x_i$$

$$\chi^2 = \sum_{i=1}^{m} \left[\frac{\hat{y}_i - y_i(A, N)}{\sigma_y} \right]^2$$

Monte Carlo Methods



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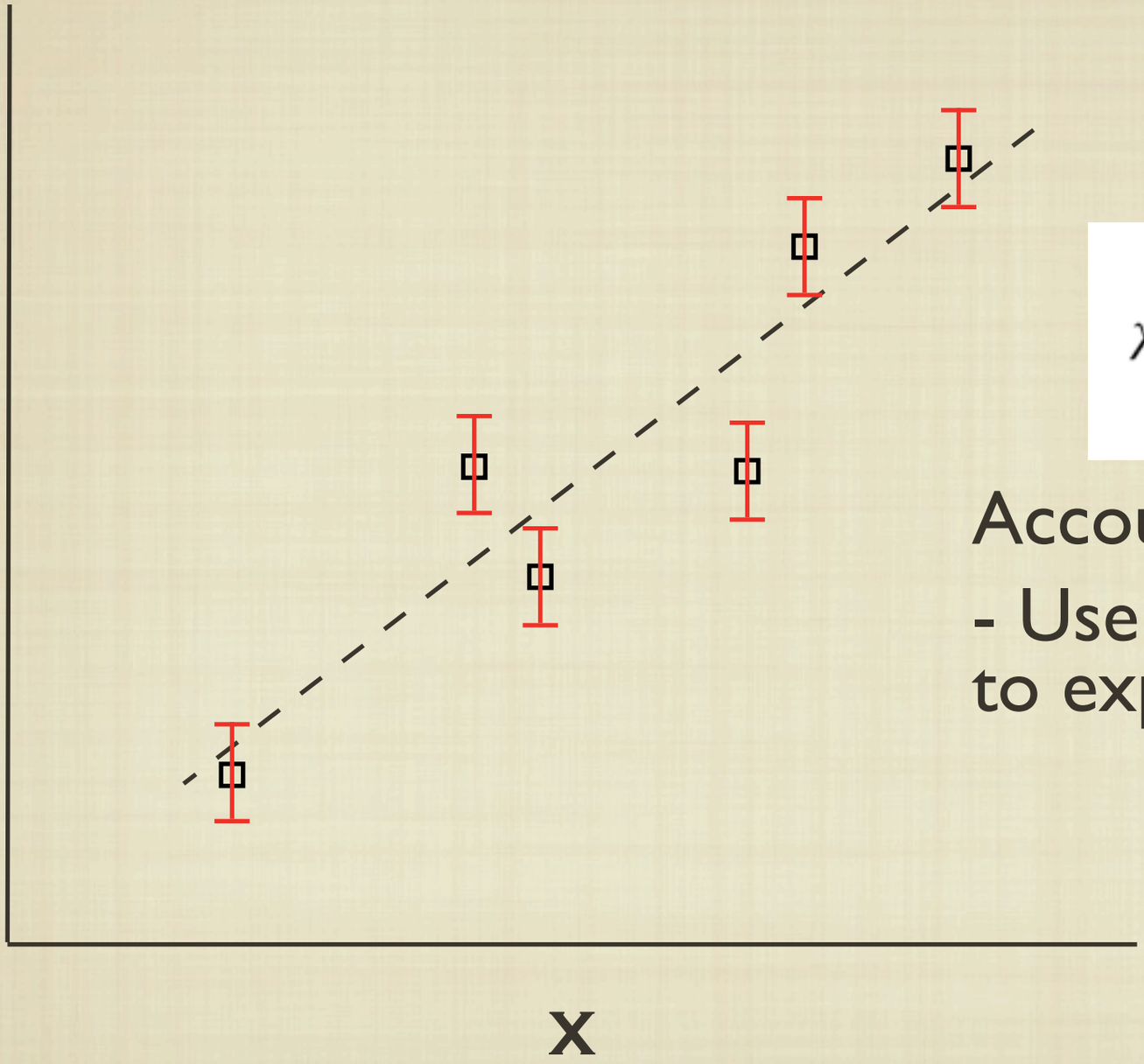

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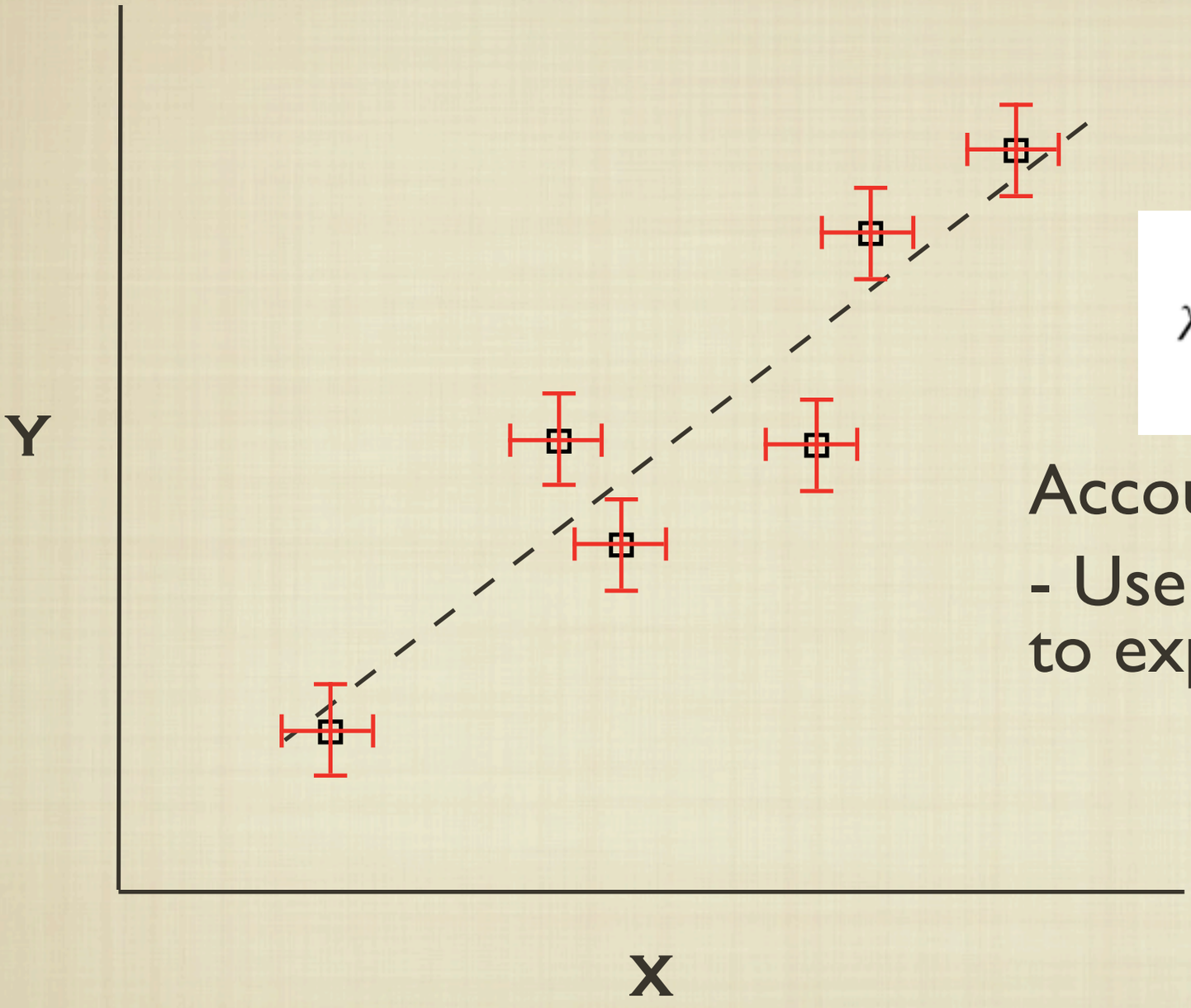
Account for Uncertainty in y?

- Use Monte Carlo simulations to explore possible y values.

Monte Carlo Methods



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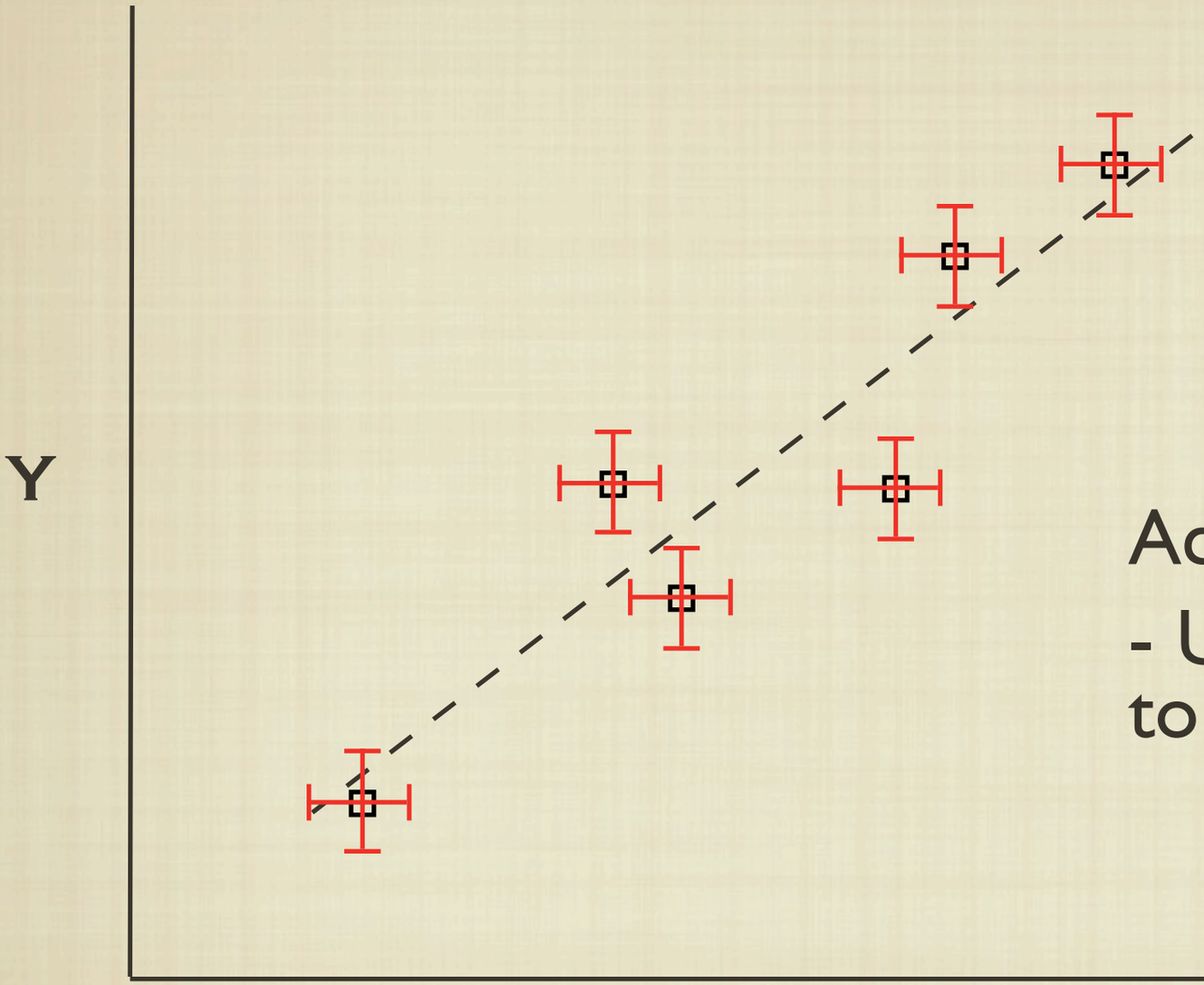

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Account for Uncertainty in y?

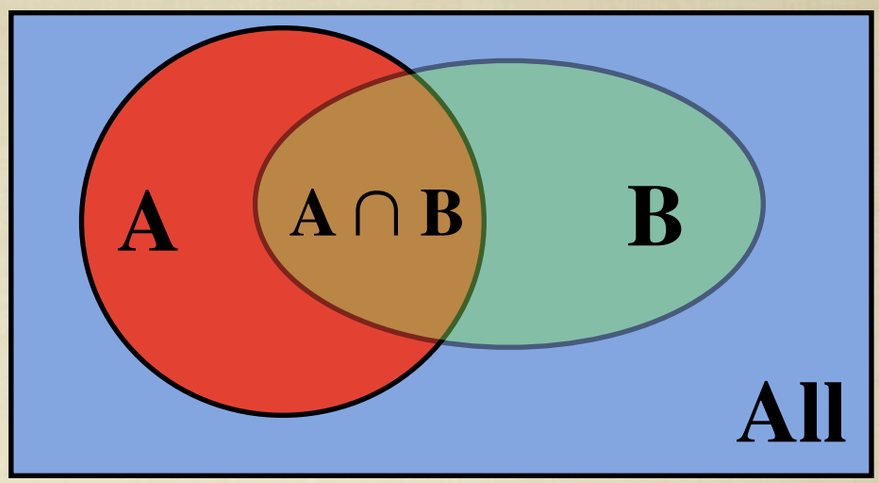
- Use Monte Carlo simulations to explore possible y values.

Account for Uncertainty in x?

- Monte Carlo simulations explore uncertainties in both x and y values.

Bayesian Inference

- **Bayes' Theorem:** $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$



- **Consider data (D), model (M): P(M|D)**
- **Model M can set the range of plausible parameters:**
  $M = (A_j, N_j, \mu_A, \mu_N ...)$

- **Modeling should also account for uncertainties in data**

$$\log \hat{\Sigma}_{\rm SFR} = \log \Sigma_{\rm SFR} + \epsilon_{\rm SFR}$$
$$\log \hat{\Sigma}_{\rm mol} = \log \Sigma_{\rm mol} + \epsilon_{\rm mol}$$

- **Relate M and D by KS equation & uncertainties**

$$\log(\Sigma_{\rm SFR}) = A + N \log(\Sigma_{\rm mol}) + \epsilon_{\rm scat}$$

Shetty, Kelly, Bigiel 2013

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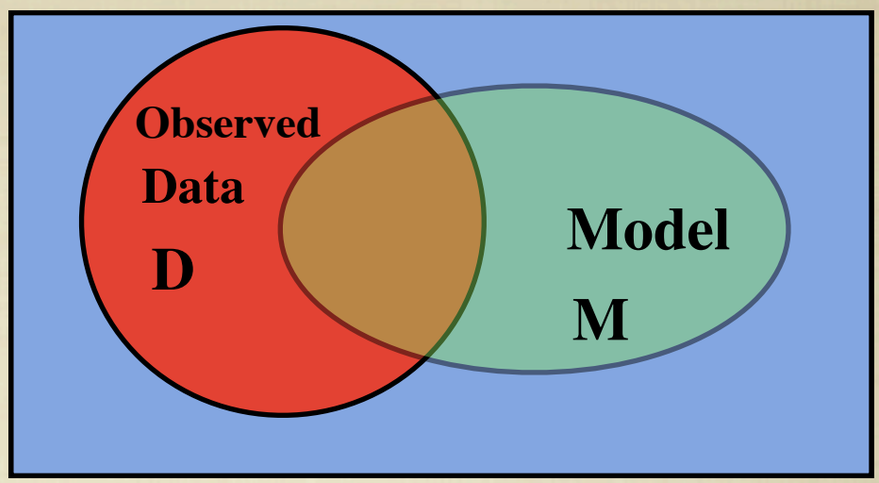

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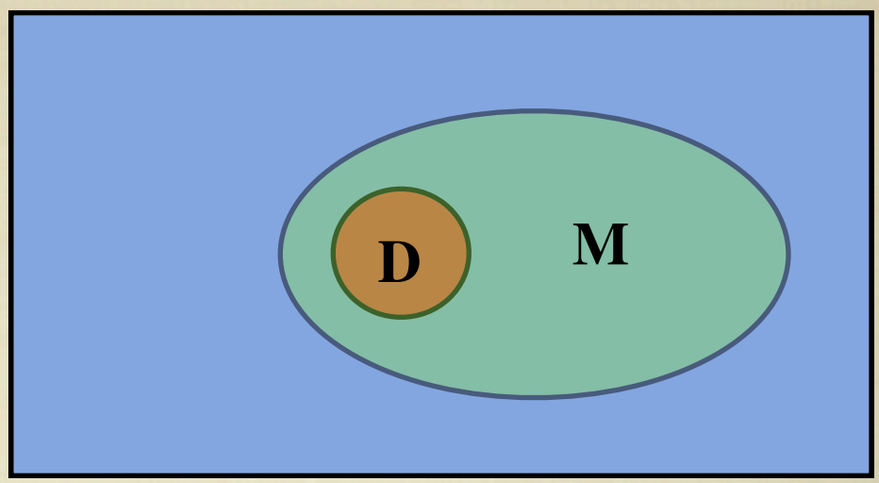

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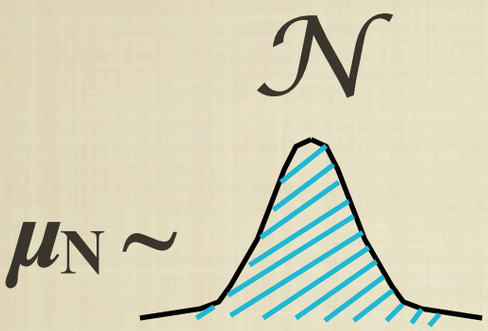

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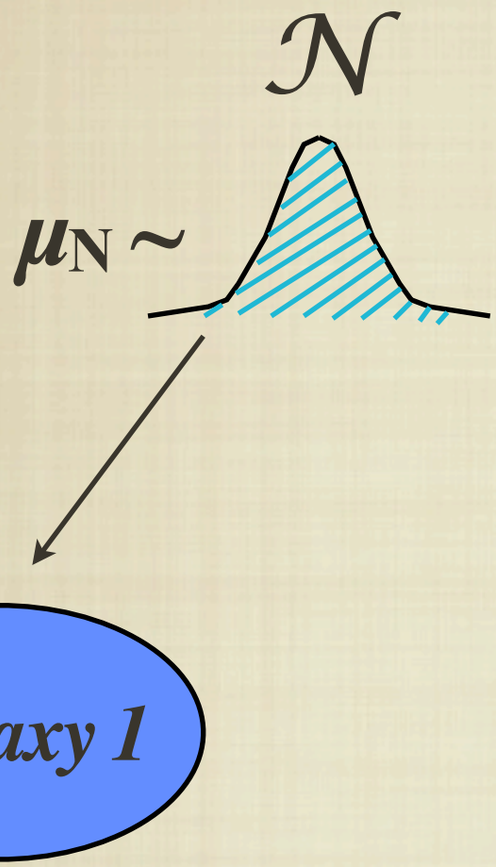

$$N_j \mid \mu_N, v_N \sim \mathcal{N}(\mu_N, v_N)$$

Similarly for A_j, etc...

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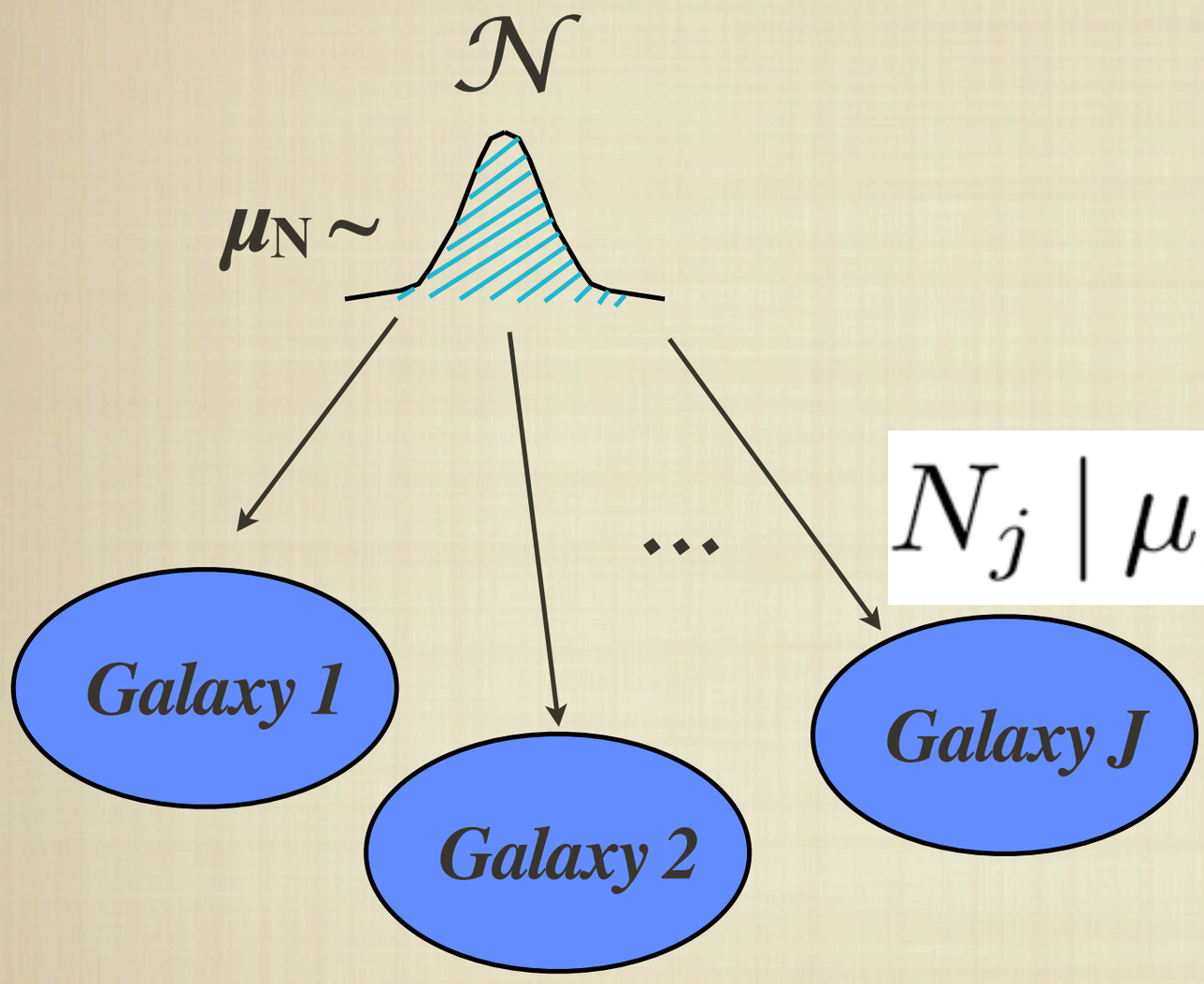

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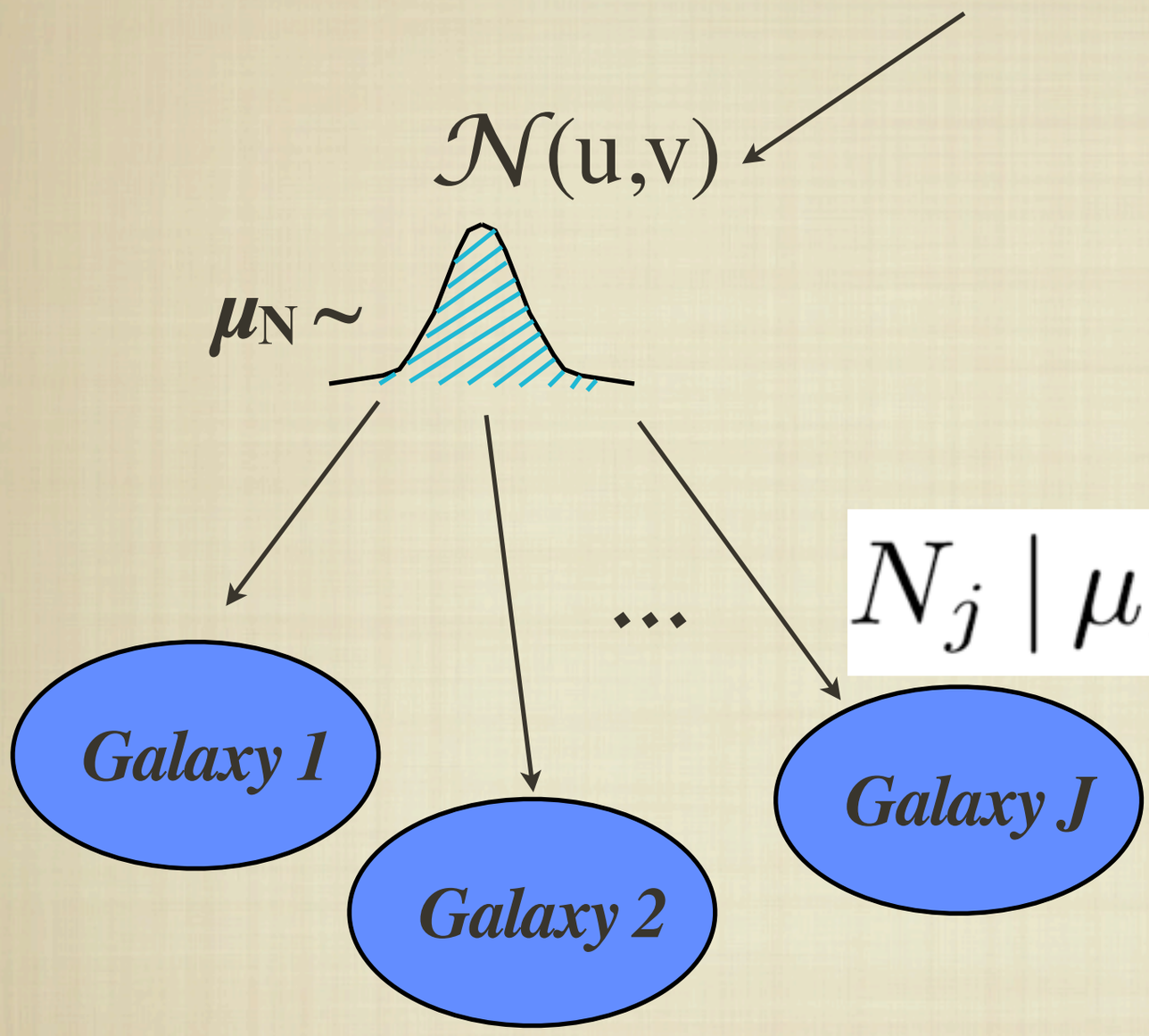
Hierarchical Modeling

Additional level for population parameters



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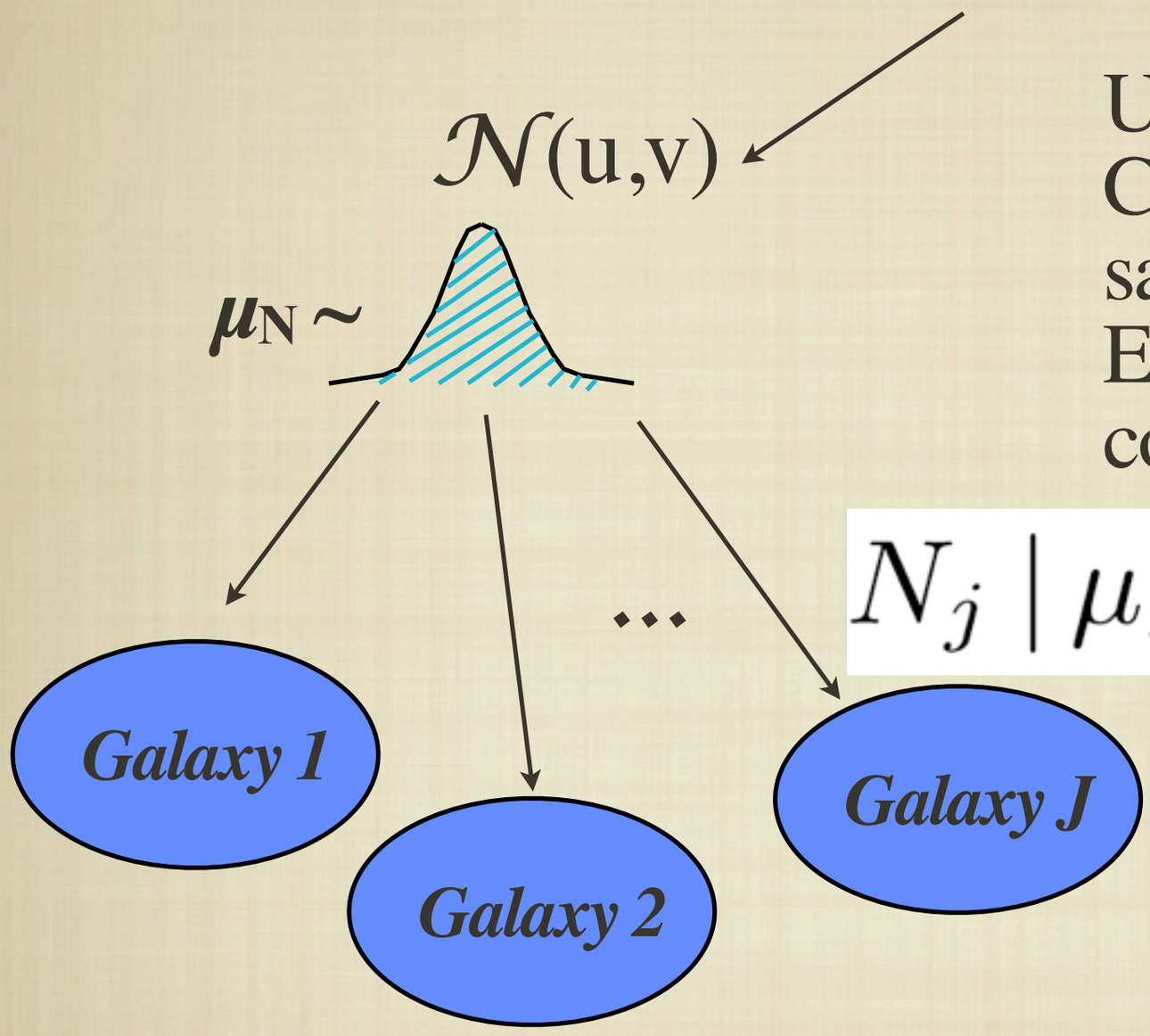

Using Markov Chain Monte Carlo (MCMC) techniques to sample all distributions. Evaluate the likelihood, and construct the posterior PDF.

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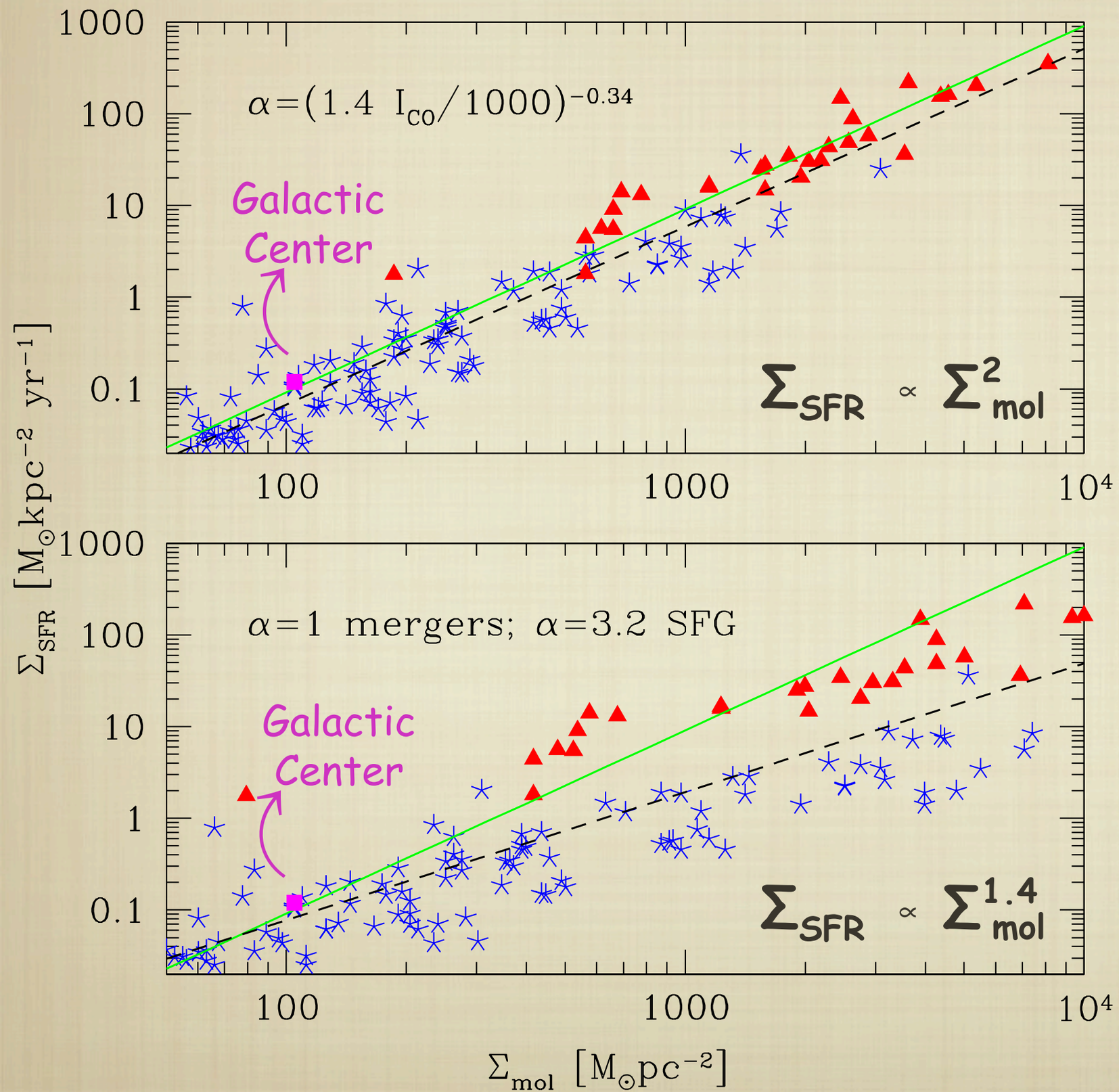
Self-Regulation in Observed Systems?

Ostriker & Shetty '11, Shetty & Ostriker '12



Star forming galaxies and merger systems from Genzel + '10

Galactic Center from Yusef-Zadeh + '09

Continuous X factor? (Shetty + 11a,b, Narayanan + 11, 12)