Clustering of Dark Energy

Filippo Vernizzi - IPhT, CEA Saclay

(Based on work with P. Creminelli, G. D'Amico, F. Piazza, E. Sefusatti, L. Senatore)

Ringberg Meeting
25 June 2012

Can *standard* quintessence cluster?

- Canonical scalar field: $\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$
- Perturbations: $\phi = \bar{\phi}(t) + \pi(t,\vec{x})$ (Minkowski + neglect mass term)

$$\mathcal{L} = \frac{1}{2}\dot{\pi}^2 - \frac{1}{2}(\vec{\nabla}\pi)^2 \quad \Rightarrow \quad \ddot{\pi} - \nabla^2\pi = 0 \quad \Rightarrow \quad c_s^2 = 1$$

speed of sound squared

- Quintessence perturbations propagate as sound waves at speed of light.

- Thus, it **remains homogeneous** on scales smaller than the Hubble radius

Sound horizon: $$L_H = a\int_0^t \frac{c_s}{a}dt \simeq 2c_s H_0^{-1}$$

- Dark matter collapses (no pressure gradients, i.e. $c_s^2 = 0$) on all scales.

Beyond canonical field

- Scalar field action (k-essence): $S = \int d^4x\sqrt{-g}P(\phi, X), \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$
  [Amendariz-Picon et al. '99]

$$\Rightarrow \qquad T_{\mu\nu} = 2P_{,X}\,\partial_\mu\phi\partial_\nu\phi + Pg_{\mu\nu}$$ as a perfect fluid

pressure $\qquad p_Q = P$

energy density $\qquad \rho_Q = 2XP_{,X} - P$

$$\Rightarrow 2XP_{,X} = \rho_Q + p_Q \equiv (1+w)\rho_Q$$

- Expanding around background: (Minkowski + neglect mass term)

$$\mathcal{L} = (P_{,X} + 2P_{,XX}\bar{X})\dot{\pi}^2 - P_{,X}(\vec{\nabla}\pi)^2 \qquad \Rightarrow \qquad c_s^2 = \frac{P_{,X}}{2P_{,XX}\bar{X} + P_{,X}}$$

- Zero speed of sound limit: $P_{,X} \ll P_{,XX}\bar{X} \quad \Rightarrow \quad c_s^2 \to 0$

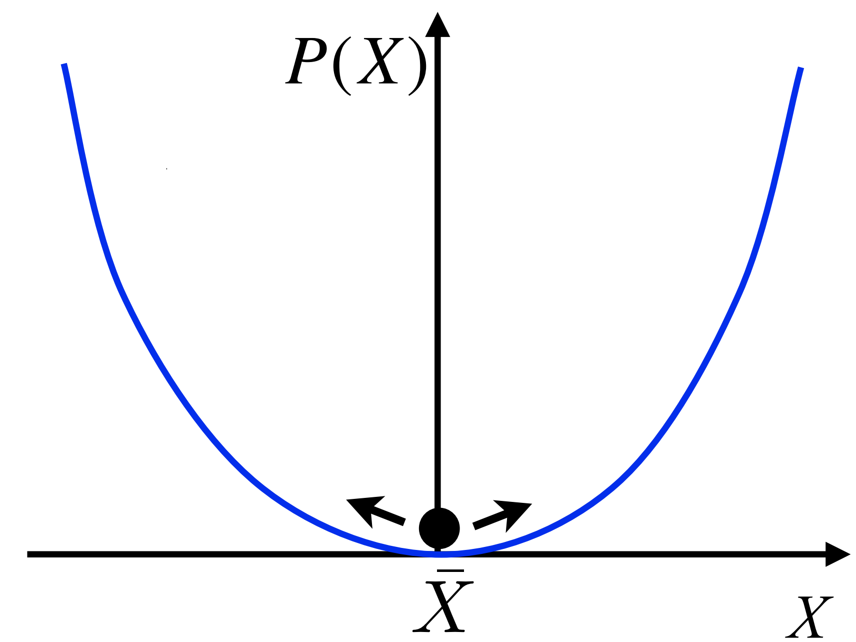
Theoretical motivations

- Shift symmetry invariance: $\phi \to \phi + \lambda \quad \Rightarrow \quad \mathcal{L} = P(X)$

EOM in expanding Universe: $\partial_t (a^3 \dot{\phi} P_{,X}) = 0$

- Solution with $\dot{\bar{\phi}} = \text{const} \quad \Rightarrow \quad \bar{X} = \text{const}^2$

and $P_{,X} \to 0 \quad \Longrightarrow \quad w \to -1 \text{ and } c_s^2 \to 0$



Ghost condensate theory: [Arkani-Hamed et al., '03, '05]

$$P(X) = \bar{P} + \frac{1}{2} P_{,XX} (X - \bar{X})^2 + \text{higher der.}$$

- Tiny breaking of shift symmentry: $\bar{P}_{,X} \ll \bar{P}_{,XX} \bar{X}$ [See also Mukohyama '06]

$$P(\phi, X) = -V(\phi) + \bar{P}_{,X}(\phi, X)(X - \bar{X}) + \frac{1}{2} \bar{P}_{,XX}(\phi, X)(X - \bar{X})^2 + \dots$$

Pressure gradients suppressed wrt density gradients:

$$\delta P|_{\delta\phi=0} \sim \bar{P}_{,X} \cdot \delta X$$

$$\delta \rho|_{\delta\phi=0} \sim \bar{P}_{,XX} \bar{X} \cdot \delta X$$

- Stable model even for *w*<-*1* (higher derivatives operators) [Arkani-Hamed et al., '05; Creminelli et al., '06]

Vanishing sound speed...

- Euler equation: $$\dot{\vec{v}} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho+p}\left[\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right] - \vec{\nabla}\Phi$$ [Creminelli et al. '10; see also Lim et al. '10]

For $c_s^2 = 0$ pressure gradients (orthogonal to the fluid 4-velocity) vanish!

($u^\mu \nabla_\mu u^\nu = 0$ if $c_s^2 = 0$)

➡ **Geodesic motion**: quintessence remains comoving with dark matter (also nonlinearly)

- Continuity equation: $$\dot{\rho}_Q + \vec{\nabla}[(\rho_Q + p_Q)\vec{v}] = 0$$

No pressure gradients but pressure is important!
No conserved particle number or current

$$\rho_m \propto \frac{1}{a^3}; \quad \rho_Q \propto \frac{1}{a^{3(1+w)}}$$

➡ **Quintessence mass**: at late time quintessence mass grows inside overdensities

Linear size of the effect [Kunz & Sapone '09]

- Linearized continuity equations:

$$\dot{\delta}_m + \frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

$$\dot{\delta}_Q - 3w\frac{\dot{a}}{a}\delta_Q + (1+w)\frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

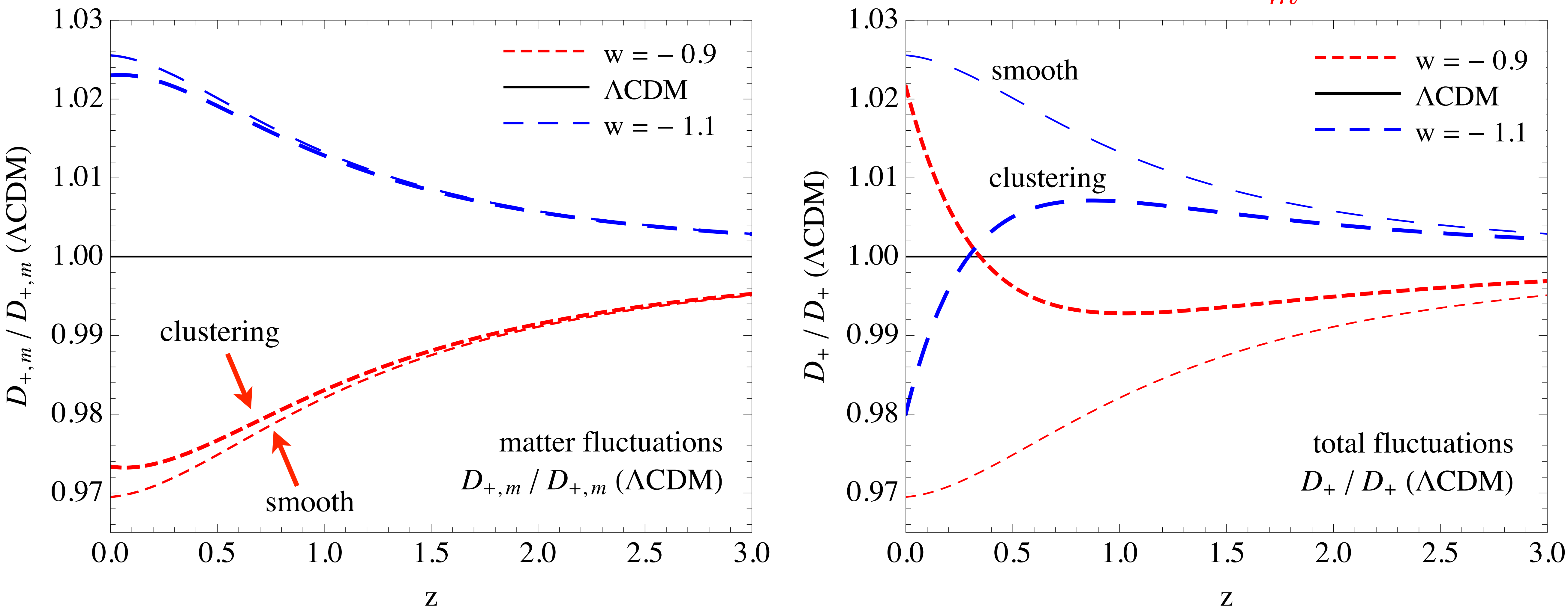
$\Rightarrow$

During dark matter dominance:

$$\delta_Q = \frac{1+w}{1-3w}\delta_{\mathrm{DM}}$$

- Linearized Euler + Poisson equations:

$$\dot{\vec{v}} + \frac{\dot{a}}{a}\vec{v} = -\vec{\nabla}\Phi \qquad \nabla^2\Phi = 4\pi G a^2 \bar{\rho}_m \left(\delta_m + \frac{\Omega_Q}{\Omega_m}\delta_Q\right)$$

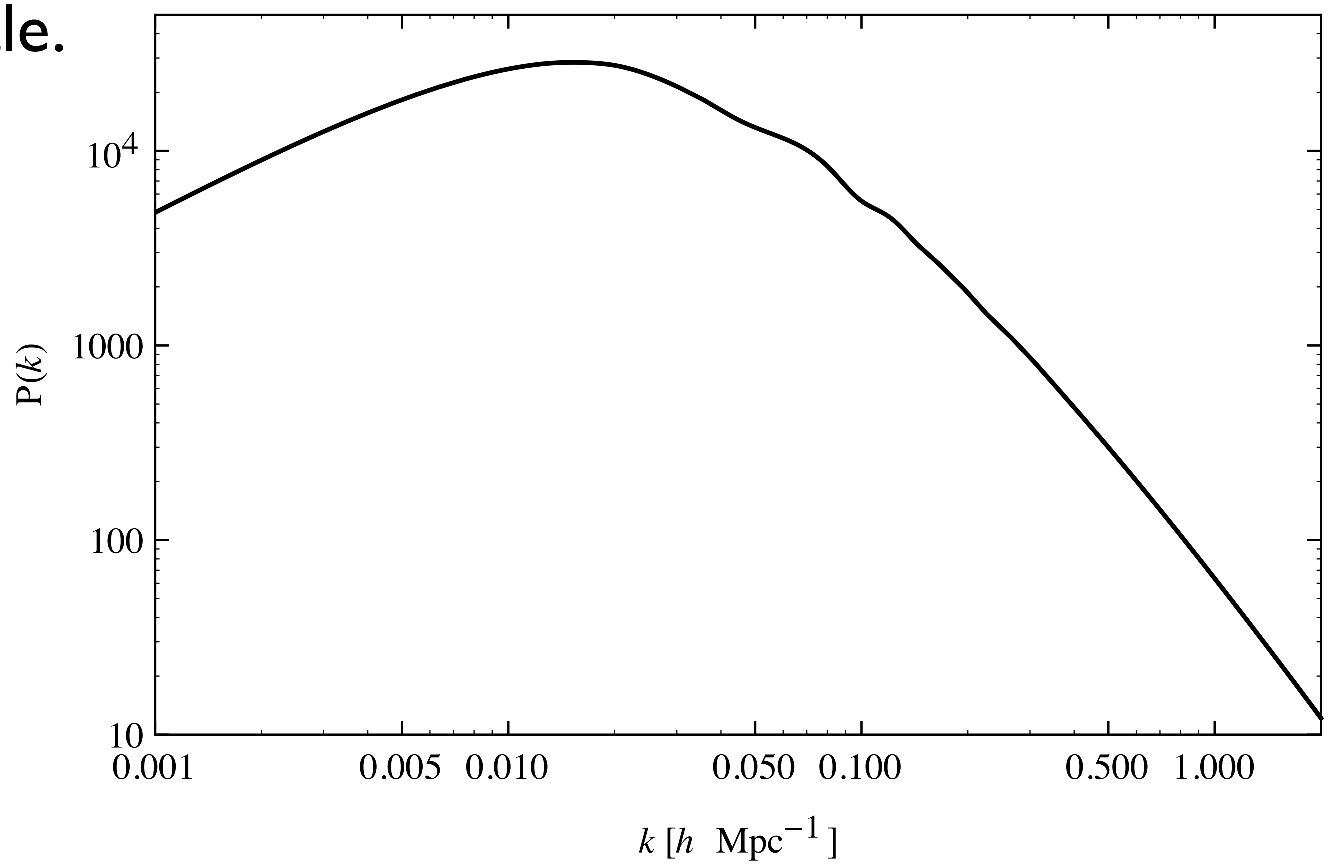


Linear vs nonlinear

- For $c_s^2 = 0$ there is no characteristic scale.

Matter power spectrum:

$$\langle \delta_{\vec{k}}\, \delta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P(k)$$

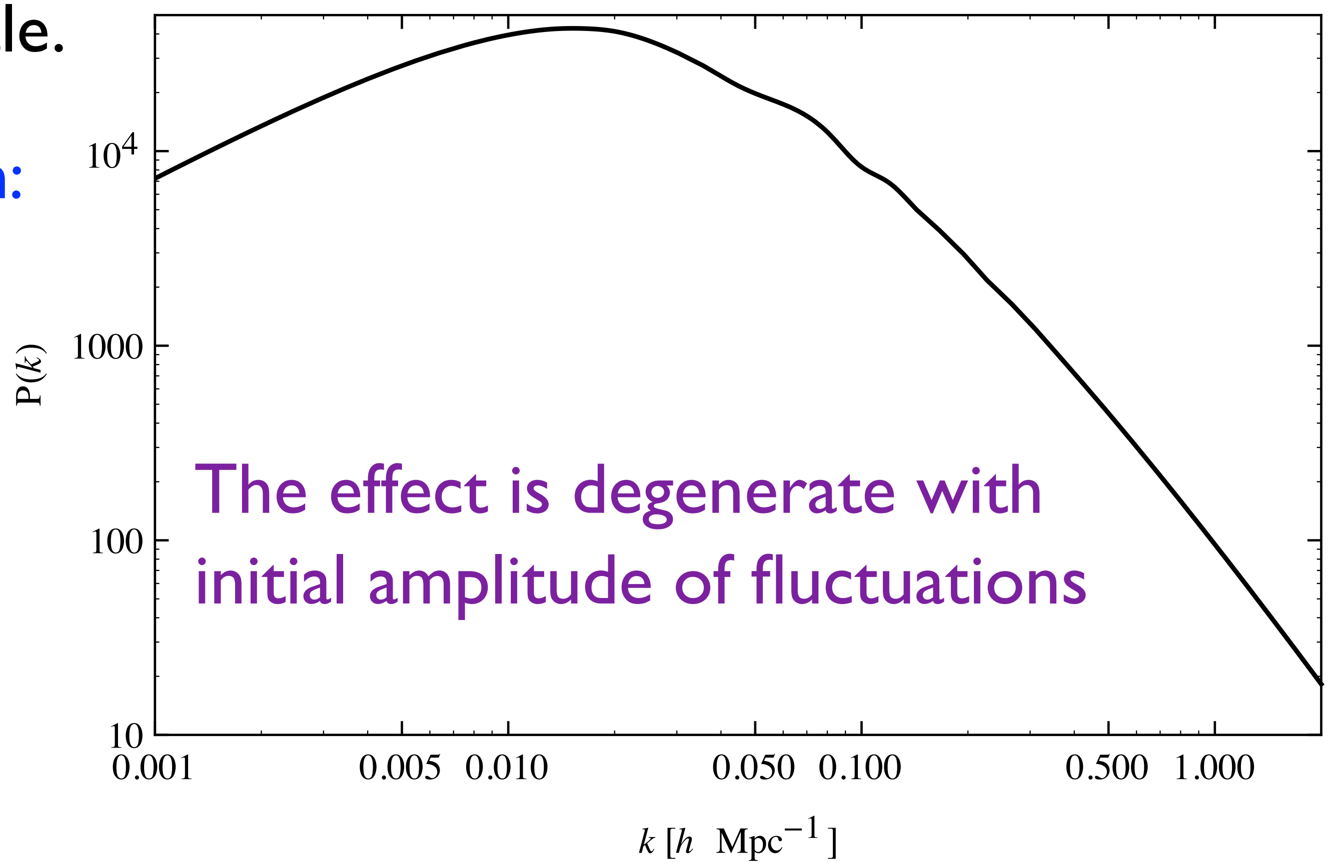


Linear vs nonlinear

- For $c_s^2 = 0$ there is no characteristic scale.

Matter + Quintessence power spectrum:

$$\langle \delta_{\vec{k}}\, \delta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P(k)$$

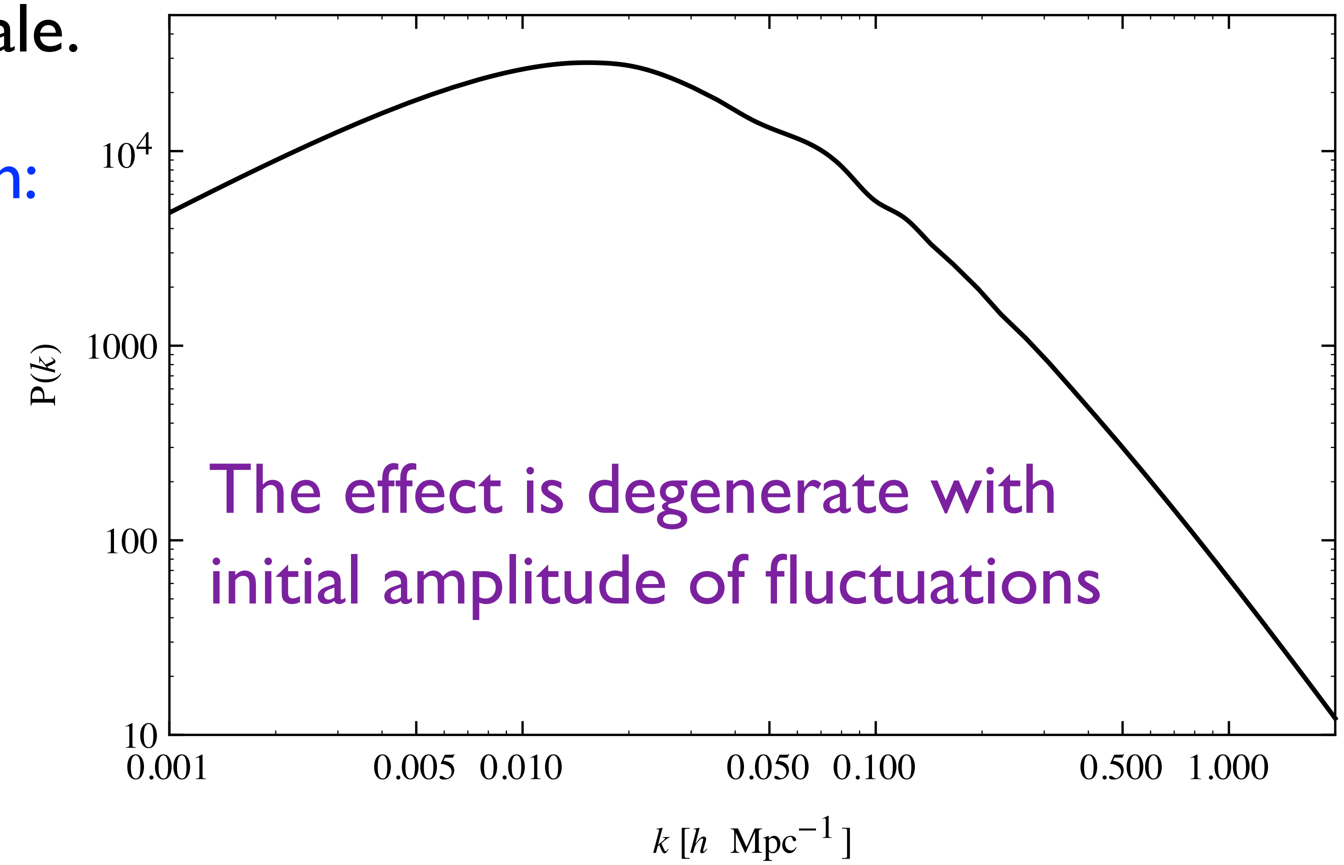


Linear vs nonlinear

- For $c_s^2 = 0$ there is no characteristic scale.

Matter + Quintessence power spectrum:

$$\langle \delta_{\vec{k}}\, \delta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P(k)$$

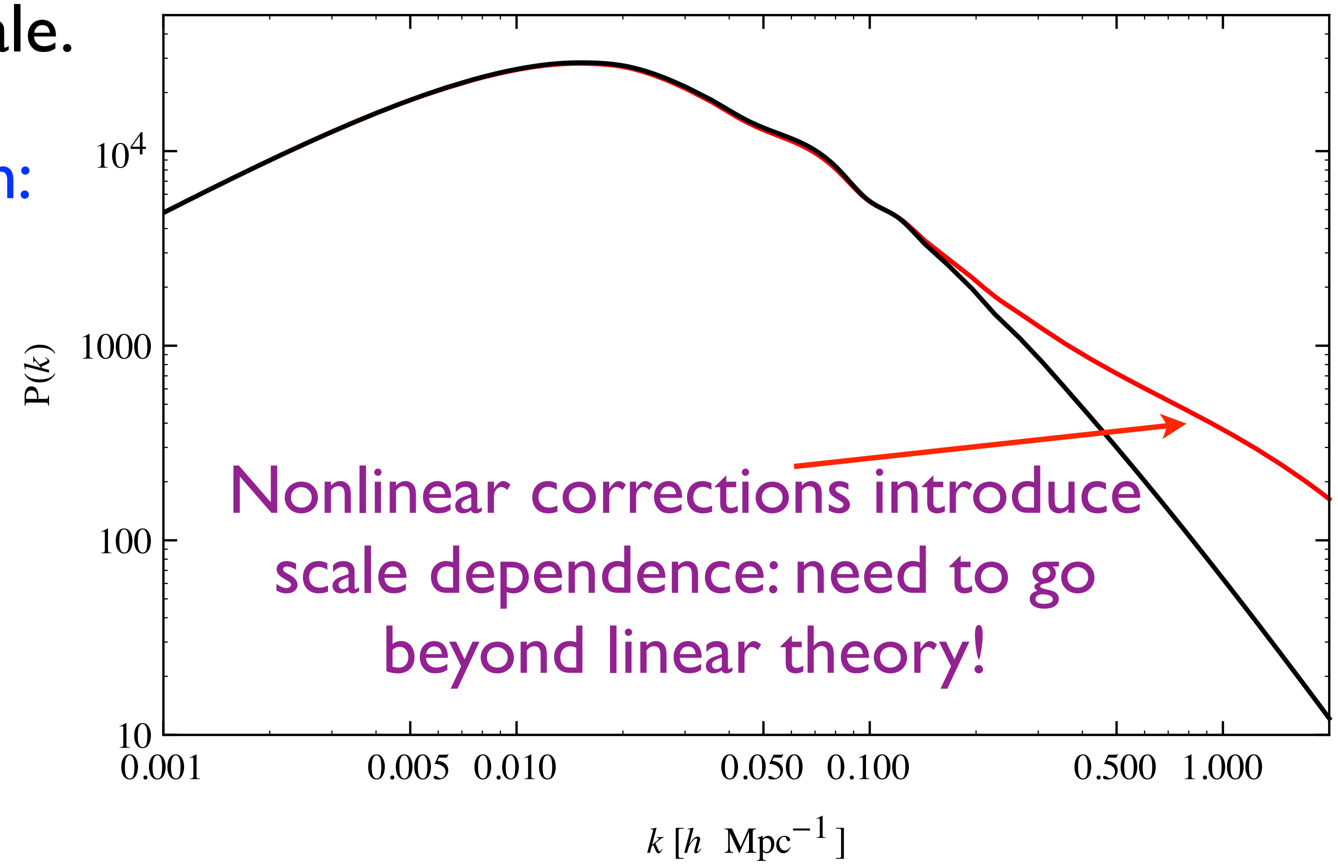


Linear vs nonlinear

- For $c_s^2 = 0$ there is no characteristic scale.

Matter + Quintessence power spectrum:

$$\langle \delta_{\vec{k}}\, \delta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') P(k)$$



In absence of numerical simulations one can resort to (semi)-analytical methods: Eulerian Perturbation Theory and perturbative nonlinear extentions

Eulerian perturbation theory

[Peebles '80; Fry '84; Bernardeau et al, '02]

Fluid approximation is very good on scales far from orbit crossing and caustics:

Continuity equation for DM: $$\frac{\partial \delta_{\vec{k}}}{\partial \tau} + \theta_{\vec{k}} = -\int \delta_D(\vec{k} - \vec{k}_1 - \vec{k}_2)\left(1 + \frac{\vec{k}_1 \cdot \vec{k}_2}{k_1^2}\right)\theta_{\vec{k}_1}\delta_{\vec{k}_2} \,,$$

Euler equation: $$\frac{\partial \theta_{\vec{k}}}{\partial \tau} + \mathcal{H}\theta_{\vec{k}} + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta = -\int \delta_D(\vec{k} - \vec{k}_1 - \vec{k}_2)\left(k^2 \frac{\vec{k}_1 \cdot \vec{k}_2}{2k_1^2 k_2^2}\right)\theta_{\vec{k}_1}\theta_{\vec{k}_2}$$

$\theta \equiv \nabla \cdot \vec{v}$ velocity divergence

- Equations can be solved order by order: $\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \ldots$

Linear solution

$$\delta^{(2)}_{\vec{k}} = \int d^3q \, F_2(\vec{k} - \vec{q}, \vec{q}) \, \delta^{(1)}_{\vec{k}-\vec{q}} \delta^{(1)}_{\vec{q}}$$

Quadratic nonlinear correction

- At tree level:

2-p function: $\langle \delta_{\vec{k}} \delta_{\vec{k}'} \rangle = \langle \delta^{(1)}_{\vec{k}} \delta^{(1)}_{\vec{k}'} \rangle$

3-p function: $$\langle \delta_{\vec{k}_1} \delta_{\vec{k}_2} \delta_{\vec{k}_3} \rangle = \langle \delta^{(1)}_{\vec{k}_1} \delta^{(1)}_{\vec{k}_2} \delta^{(2)}_{\vec{k}_3} \rangle + \text{permutations}$$
$$= (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) 2 \, F_2(\vec{k}_1, \vec{k}_2) P(k_1) P(k_2) + \text{cyclic}$$

Eulerian perturbation theory

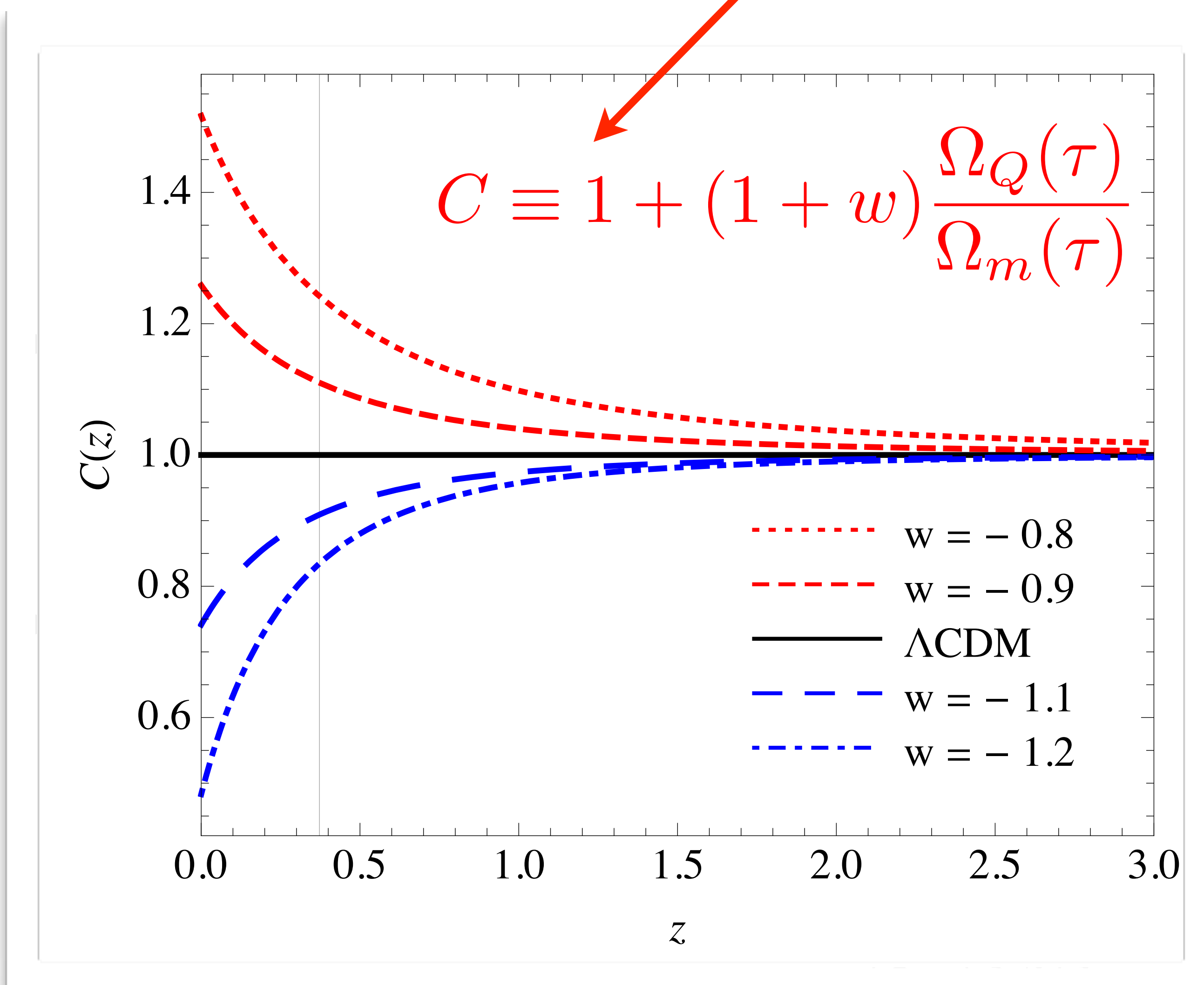
[Peebles '80; Fry '84; Bernardeau et al, '02]

Fluid approximation is very good on scales far from orbit crossing and caustics:

Continuity eq. for DM + Q: $$\frac{\partial \delta_{\vec{k}}}{\partial \tau} + C(\tau)\theta_{\vec{k}} = -\int \delta_D(\vec{k}-\vec{k}_1-\vec{k}_2)\Big(1+\frac{\vec{k}_1\cdot\vec{k}_2}{k_1^2}\Big)\theta_{\vec{k}_1}\delta_{\vec{k}_2} \,,$$

Euler equation: $$\frac{\partial \theta_{\vec{k}}}{\partial \tau} + \mathcal{H}\theta_{\vec{k}} + \frac{3}{2}\Omega_m \mathcal{H}^2 \delta = -\int \delta_D(\vec{k}-\vec{k}_1-\vec{k}_2)\Big(k^2\frac{\vec{k}_1\cdot\vec{k}_2}{2k_1^2k_2^2}\Big)\theta_{\vec{k}_1}\theta_{\vec{k}_2}$$

$\theta \equiv \nabla\cdot\vec{v}$ velocity divergence



$$\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \dots$$

tion

$$\delta^{(2)}_{\vec{k}} = \int d^3q\, F_2(\vec{k}-\vec{q},\vec{q})\,\delta^{(1)}_{\vec{k}-\vec{q}}\delta^{(1)}_{\vec{q}}$$

Quadratic nonlinear correction

$+ \text{permutations}$

$$= (2\pi)^3\delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\,2\,F_2(\vec{k}_1,\vec{k}_2)P(k_1)P(k_2) + \text{cyclic}$$

F_2 and reduced bispectrum

[Sefusatti, FV, 2011]

- In matter dominance nonlinear couplings are time-independent. Smooth quintessence and Lambda are similar to matter dominance case. Nonlinear kernels quite insensitive to smooth component.

$$F_2(\vec{k}_1, \vec{k}_2) = \frac{5}{7} + \frac{\vec{k}_1 \cdot \vec{k}_2}{2} \left(\frac{1}{k_1^2} + \frac{1}{k_2^2} \right) + \frac{2}{7} \frac{(\vec{k}_1 \cdot \vec{k}_2)^2}{k_1^2 k_2^2}$$

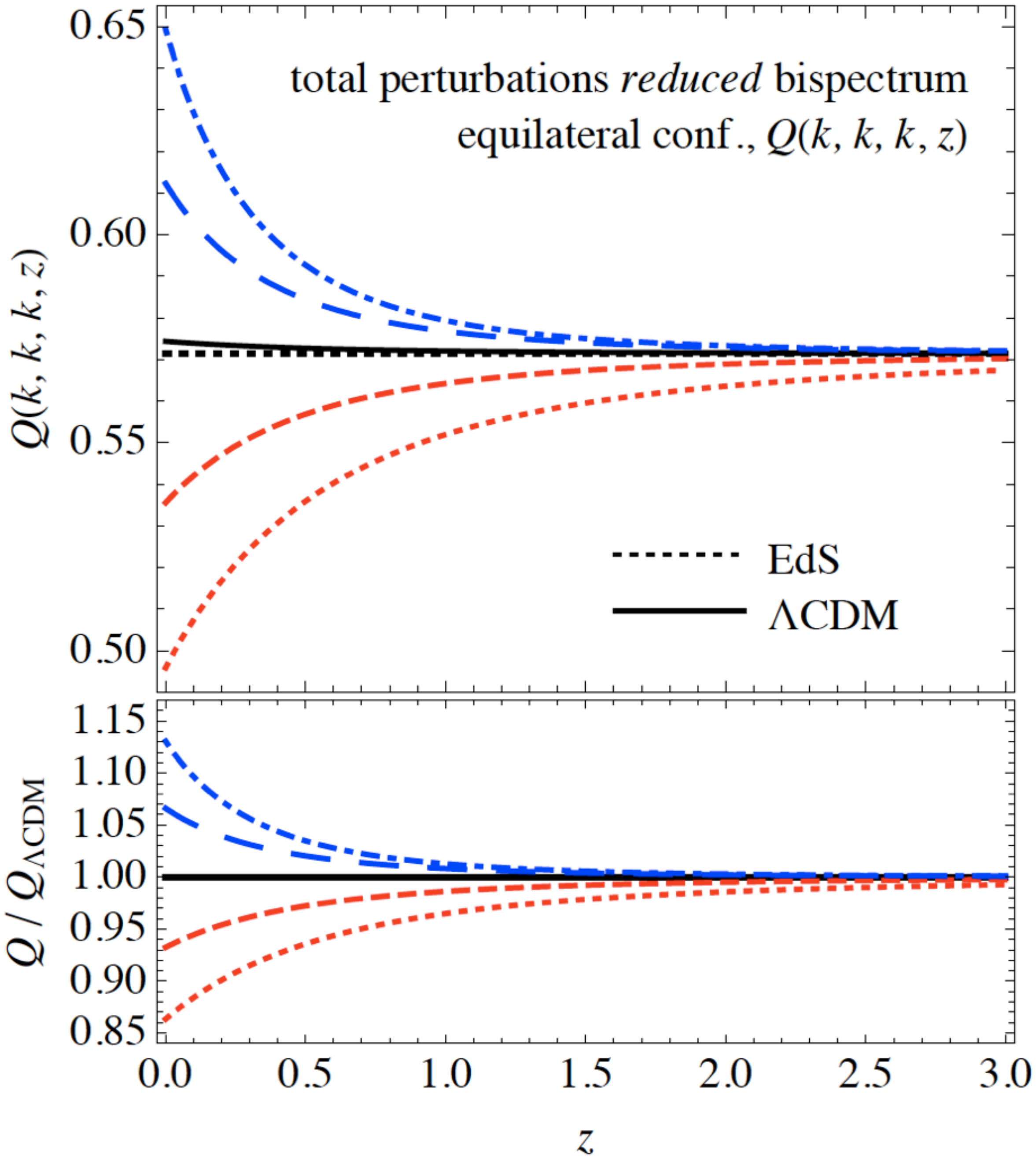
- Large time dependent corrections for clustering quintessence only:

$$F_2(\vec{k}_1, \vec{k}_2) = (1 - \mathcal{O}(\epsilon)) \frac{5}{7} + (1 - \epsilon) \frac{\vec{k}_1 \cdot \vec{k}_2}{2} \left(\frac{1}{k_1^2} + \frac{1}{k_2^2} \right) + (1 - \mathcal{O}(\epsilon)) \frac{2}{7} \frac{(\vec{k}_1 \cdot \vec{k}_2)^2}{k_1^2 k_2^2}$$

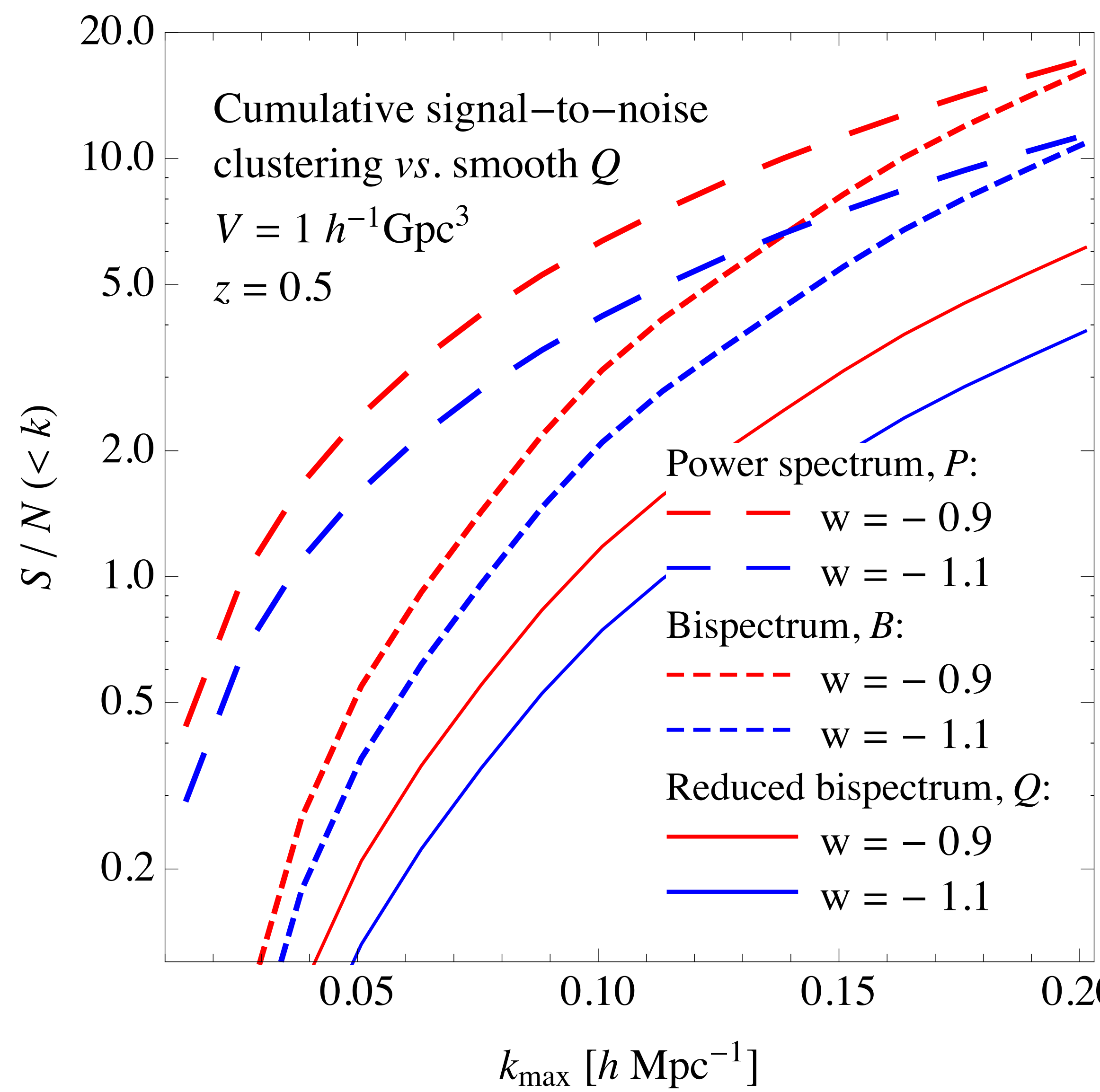
where $\epsilon(\tau) = \delta \rho_Q^{\rm lin} / \delta \rho^{\rm lin}$

- F_2 can be observed in the reduced bispectrum:

$$\mathcal{Q}(k_1, k_2, k_3) = \frac{\text{3-p fun.}}{\text{2-p fun.}} = \frac{2F_2(\vec{k}_1, \vec{k}_2) P(k_1) P(k_2) + 2 \text{ cyclic}}{P(k_1) P(k_2) + 2 \text{ cyclic}}$$



[Sefusatti, FV, 2011]



- Cumulative signal-to-noise

$$\left(\frac{S}{N}\right)^2 = \sum_{k_1,k_2,k_3 \geq \frac{2\pi}{V^{1/3}}}^{k_{\max}} \frac{[Q(c_s=0) - Q(c_s=1)]^2}{\Delta Q^2}$$

Nonlinear corrections to PS

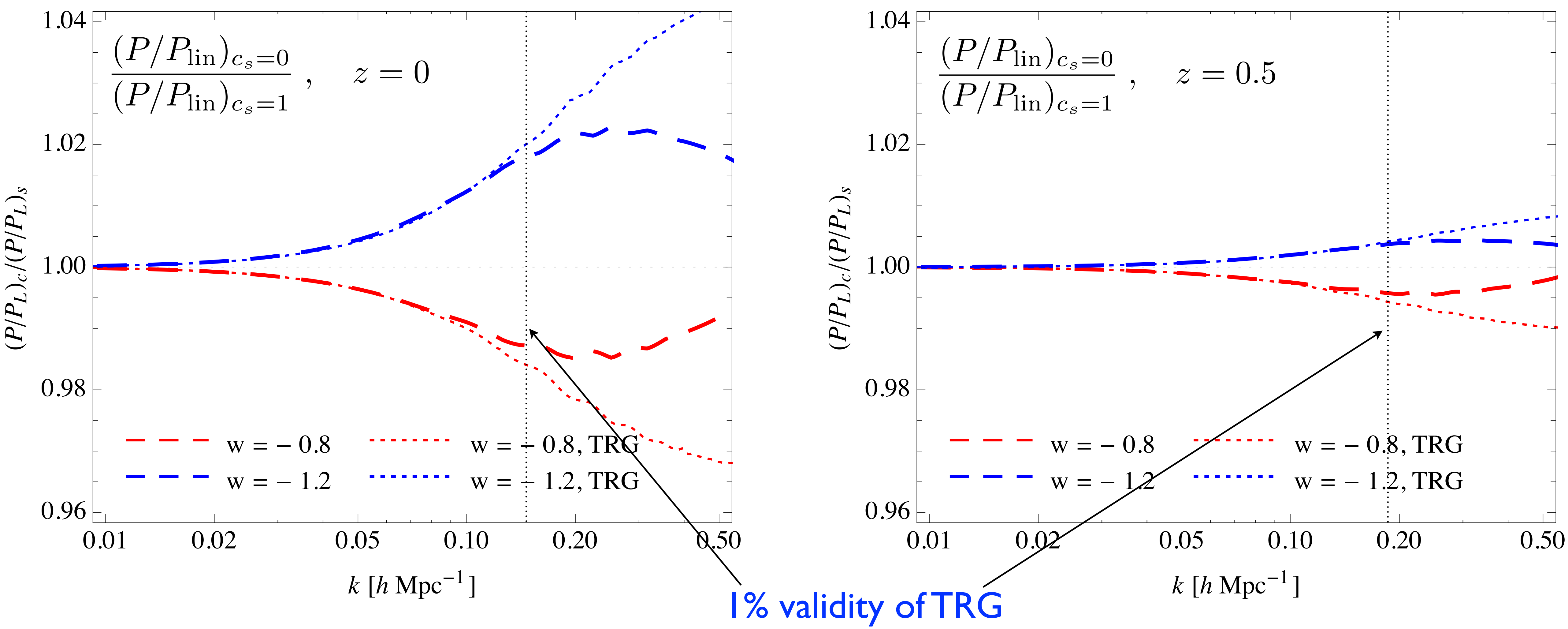
[D'Amico, Sefusatti '11;
Bonne, Sefusatti, FV in prog.]

$$\delta_{\vec{k}} = \delta^{(1)}_{\vec{k}} + \delta^{(2)}_{\vec{k}} + \ldots \qquad \langle\delta\delta\rangle = \langle\delta^{(1)}\delta^{(1)}\rangle + \langle\delta^{(2)}\delta^{(2)}\rangle + \langle\delta^{(1)}\delta^{(3)}\rangle + \ldots$$

linear power spectrum

1-loop, non-linear corrections

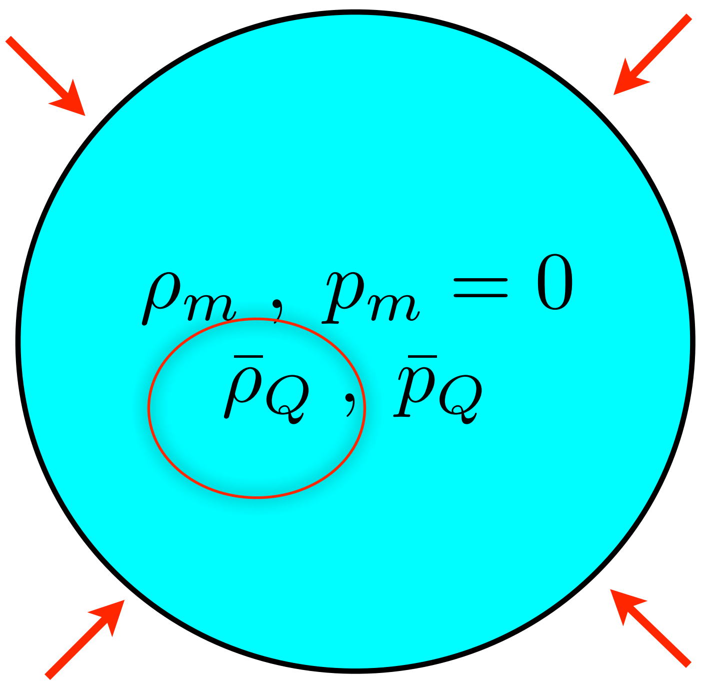
- 1-loop corrections to the power spectrum:



Spherical collapse

- Quintessence affects the spherical collapse model:

- Both density and pressure remain homogeneous and follow the outside Hubble flow:



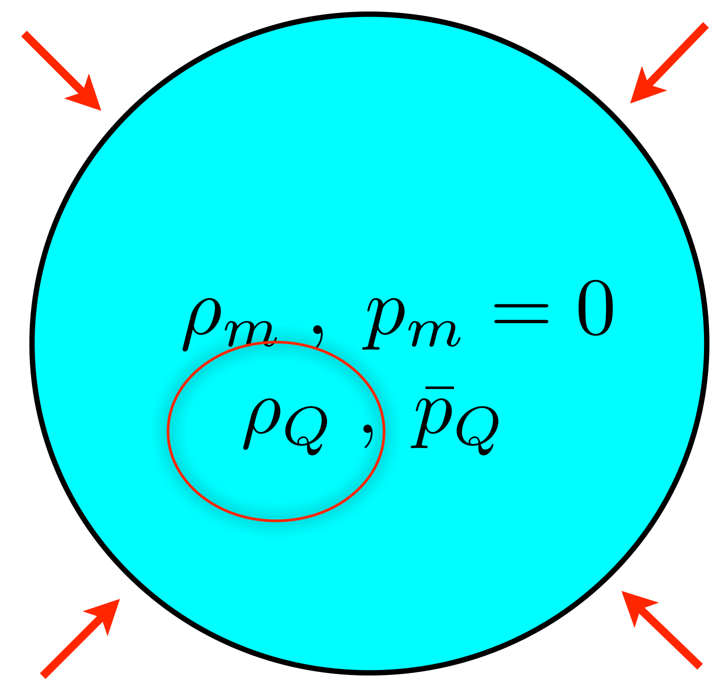
$$c_s^2 = 1$$

$$\bar{\rho}_m \; , \; \bar{p}_m = 0$$
$$\bar{\rho}_Q \; , \; \bar{p}_Q$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \bar{\rho}_Q + 3\bar{p}_Q)$$

No FRW universe inside [Wang & Steinhardt '98]

- Quintessence density follows dark matter flow but pressure remains as outside:



$$c_s^2 = 0$$

$$\bar{\rho}_m \; , \; \bar{p}_m = 0$$
$$\bar{\rho}_Q \; , \; \bar{p}_Q$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \rho_Q + 3\bar{p}_Q)$$

Exact FRW universe inside!

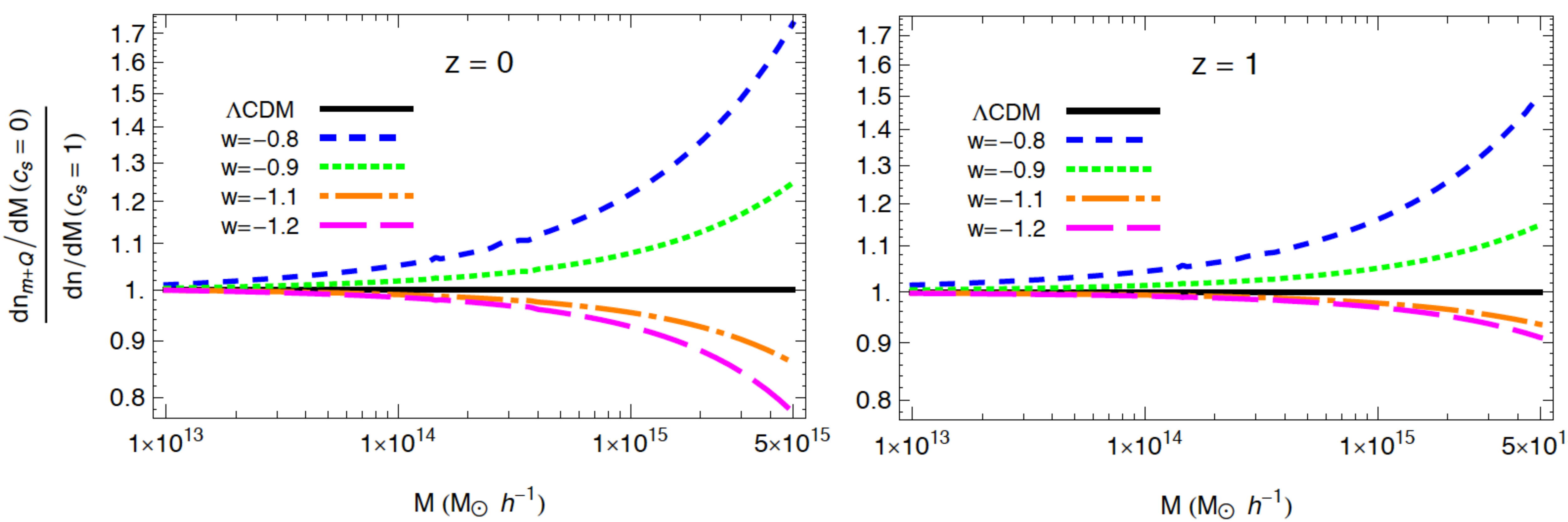
Quintessence "mass"

- Evolution equation inside the overdensity: $\dot{\rho}_Q + 3\frac{\dot{R}}{R}(\rho_Q + \bar{p}_Q) = 0$
- Large overdensities behave as DM:

$$\delta_Q \gg |1+w| \quad \Rightarrow \quad \delta\dot{\rho}_Q + 3\frac{\dot{R}}{R}\delta\rho_Q \approx 0$$

Conserved quintessence mass inside halos!

$$M_Q = \frac{4\pi R^3}{3}\delta\rho_Q \approx (1+w)\left.\frac{\Omega_Q}{\Omega_{\rm DM}}\right|_{z_{\rm coll}} \cdot M_{\rm DM}$$



Summary

✓ Quintessence can have zero sound speed! Simplest phenomenological alternative to the smooth case.

✓ Nonlinearities break degeneracy with initial amplitude: 1) effects of ~ 5% (for $|1+w|=0.1$) on the bispectrum absent in smooth case; 2) nonlinear corrections to the power spectrum reduced for $w>-1$ (enhanced for $w<-1$).

✓ Virialized objects contain new mass component: effect on mass function for more massive objects.

To do:

- You observe tracers: bias & redshift-space distortion.
- Caustics and virialization.
- Cluster abundance analysis.
- Numerical simulations.