Modified Gravity and the Radiation Dominated Epoch

Carsten van de Bruck
University of Sheffield

Lancaster-Manchester-Sheffield Consortium for Fundamental Physics

CFP

Scalar-Tensor Theories:

- Simple extensions of GR
- Describe corners of other extensions of GR (e.g. massive gravity)
- Useful for phenomenological approaches to modified gravity

Form considered here is:

$$\mathcal{S} = \int \sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\right] + S_{\rm matter}\left(\tilde{g}^{(i)}_{\mu\nu}, \chi_i\right)$$

with

$$\tilde{g}^{(i)}_{\mu\nu} = C^{(i)}(\phi)g_{\mu\nu} + D^{(i)}(\phi)\phi_{,\mu}\phi_{,\nu}$$

C is called conformal factor, D is the disformal factor. D is not well constrainted by local experiments (Brax (2012), Noller (2012), Brax et al (2012)).

Field equations:

Koivisto et al (2012); CvdB & G. Sculthorpe (2012)

$$G_{\mu\nu} = \kappa \left(T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{\mathrm{matter}} \right)$$

$$\Box\phi - \frac{dV}{d\phi} + Q = 0$$

$$\nabla^{\mu} T_{\mu\nu} = Q \nabla_{\nu} \phi$$

with

$$Q = \frac{C_{,\phi}}{2C} g^{\mu\nu} T_{\mu\nu} + \frac{D_{,\phi}}{2C} \phi_{,\mu} \phi_{,\nu} T^{\mu\nu} - \nabla_{\nu} \left[\frac{D}{C} \phi_{,\mu} T^{\mu\nu} \right]$$

FRW , conformal time:

$$\ddot{\phi} + 2\mathcal{H}\dot{\phi} + a^2\frac{dV}{d\phi} = a^2\bar{Q}$$

$$a^2\bar{Q} = -\frac{\rho}{2(C + D(\rho - \dot{\phi}/a^2))}\left[a^2\frac{dC}{d\phi}(1-3w) - 2D\left(3\mathcal{H}\dot{\phi}(1+w) + a^2\frac{dV}{d\phi} + \frac{C_{,\phi}}{C}\dot{\phi}^2\right) + D_{,\phi}\dot{\phi}^2\right]$$

Effective coupling (at background level) is

$$\beta_{\rm eff} = -\frac{Q}{\rho}$$

Perturbation equations (one species only):

$$\dot{\delta} = -(1+w)\left(\theta - 3\dot{\Phi}\right) - 3\mathcal{H}(c_s^2 - w)\delta - \frac{\bar{Q}}{\rho}\dot{\phi}\delta + \frac{\delta Q}{\rho}\dot{\phi} - \frac{\bar{Q}}{\rho}(\delta\phi)^{\cdot}$$

$$\dot{\theta} = -\mathcal{H}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{k^2}{1+w}c_s^2\delta - k^2\sigma + k^2\Psi - k^2\frac{\bar{Q}}{\rho(1+w)}\delta\phi + \frac{\bar{Q}}{\rho}\dot{\phi}\theta$$

$$(\delta\phi)^{\cdot\cdot} + 2\mathcal{H}(\delta\phi)^{\cdot} + \left(k^2 + a^2\frac{d^2V}{d\phi^2}\right)\delta\phi = (3\dot{\Phi} + \dot{\Psi})\dot{\phi} - 2\Psi\left(\frac{dV}{d\phi} + \bar{Q}\right) + a^2\delta Q$$

Perturbed coupling

$$\delta Q = -\frac{\rho}{a^2C + D(a^2\rho - \dot{\phi}^2)}\left[\mathcal{B}_1\delta + \mathcal{B}_2\dot{\Phi} + \mathcal{B}_3\Psi + \mathcal{B}_4(\delta\phi)^{\cdot} + \mathcal{B}_5\delta\phi\right]$$

$$\mathcal{B}_1 = \frac{a^2C'}{2}\left(1 - 3\frac{\delta P}{\delta\rho}\right) - 3D\mathcal{H}\dot{\phi}\left(1 + \frac{\delta P}{\delta\rho}\right) - Da^2(V' - \bar{Q}) - D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D}\right)$$

$$\mathcal{B}_2 = 3D\dot{\phi}(1+w),$$

$$\mathcal{B}_3 = 6D\mathcal{H}\dot{\phi}(1+w) + 2D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{\bar{Q}}{\rho}\right),$$

$$\mathcal{B}_4 = -3D\mathcal{H}\dot{\phi}(1+w) - 2D\dot{\phi}\left(\frac{C'}{C} - \frac{D'}{2D} + \frac{\bar{Q}}{\rho}\right),$$

$$\mathcal{B}_5 = \frac{a^2C''(1-3w)}{2} - Dk^2(1+w) - Da^2V'' - D'a^2V' - 3D'\mathcal{H}\dot{\phi}(1+w)$$
$$- D\dot{\phi}^2\left(\frac{C''}{C} - \left(\frac{C'}{C}\right)^2 + \frac{C'D'}{CD} - \frac{D''}{2D}\right) + (a^2C' + D'a^2\rho - D'\dot{\phi}^2)\frac{\bar{Q}}{\rho}.$$

Consider coupling to baryons here. Take Newtonian limit, find equation for radiation density contrast. One finds

$$\delta_\gamma \propto \exp\left(-\frac{k^2}{k_D^2}\right)\exp\left(\pm ik\tilde{r}_s\right)$$

$$k_D^{-2} = \frac{1}{6}\int_0^\eta \frac{1}{an_e\sigma_T}\left(\frac{1}{1+R}\left(\frac{16}{15}+\frac{R^2}{1+R}\right)\right)d\eta'$$

$$\tilde{r}_s = \int_0^\eta \tilde{c}_s d\eta'$$

$$\tilde{c}_s^2 = c_s^2\left(1-\frac{9\Omega_b\beta_b^2 R\mathcal{H}^2}{k^2+a^2m^2}\right) \qquad c_s^2 = \frac{1}{3}\frac{1}{1+R} \qquad R = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$$

Coupling is now

$$\beta_b = -\frac{\mathcal{B}_1}{a^2 C + D(a^2\rho - \dot{\phi}^2)}$$

$$\mathcal{B}_1 = \frac{a^2 C'}{2} - 3D\mathcal{H}\dot{\phi} - Da^2(V' - \bar{Q}) - D\dot{\phi}^2\left(\frac{C'}{C} - \frac{D'}{2D}\right)$$

- Modified sound speed reduces to Brax&Davis (2011) one for pure conformal case.
- Coupling and (effective) mass could be complicated function of time in general.
- Note that mass contains B_5 term.

$$\tilde{c}_s^2 = c_s^2 \left(1 - \frac{9\Omega_b \beta_b^2 R \mathcal{H}^2}{k^2 + a^2 m^2}\right)$$

Sound speed can be complicated function of time, depending on the details of the theory. Expression above can go negative (issue neglected here - instabilities possible). Note that

$$9\Omega_b \beta_b^2 R \mathcal{H}^2 \approx 1.5 \beta_b^2 10^{-5} \mathrm{Mpc}^{-2}$$

and therefore larger couplings are needed to modify sound speed significantly.

Modified sound speed changes sound horizon:

$$\tilde{r}_s = \int_0^\eta \tilde{c}_s d\eta \qquad \tilde{c}_s^2 = c_s^2 \left(1 - \frac{9\Omega_b \beta_b^2 R \mathcal{H}^2}{k^2 + a^2 m^2}\right)$$

Changes, e.g. position of peaks, since $l \propto \frac{1}{\tilde{r}_s}$

Position of first peak well know (220.1±0.8), so assuming ΛCDM evolution for most of the time, sound horizon cannot vary too much. (Other effects of scalar on CMB to be explored too!)

CMB distortion due to dissipation of acoustic waves:

- Injection of energy into baryon-photon fluid
- Processes happen for $2\times10^6 \gtrsim z \gtrsim 5\times10^4$.
- Probe scales $k \gg 1\ \mathrm{Mpc}^{-1}$
- Chemical potential created

$$\frac{1}{e^{\frac{h\nu}{kT}}-1} \rightarrow \frac{1}{e^{\frac{h\nu}{kT}+\mu(\nu)}-1}$$

Constraints $|\mu| < 9\times10^{-5}$

Time evolution governed by (Hu et al (1992,1994))

$$\frac{d\mu}{dt} = -\frac{\mu}{t_{\rm DC}(z)} + 1.4\frac{dQ/dt}{\rho_\gamma}$$

$$t_{\rm DC} = 2.06 \times 10^3 3 \left(1 - \frac{Y_p}{2}\right)^{-1} (\Omega_b h^2)^{-1} z^{-9/2} s$$

$$\mu = 1.4 \int_{z_f}^{z_i} dz \frac{dQ/dz}{\rho_\gamma} e^{-(z/z_{\rm DC})^{5/2}}$$

$$z_{\rm DC} = 1.97 \times 10^6 \left(1 - \frac{1}{2}\frac{Y_p}{0.24}\right)^{-5/2} \left(\frac{\Omega_b h^2}{0.0224}\right)^{-2/5}$$

Energy stored in wave:

$$Q = \frac{3}{4}\rho_\gamma c_s^2 \left\langle \delta_\gamma^2(\mathbf{x}) \right\rangle$$

with

$$\left\langle \delta_\gamma^2(\mathbf{x}) \right\rangle = \int \frac{d^3k}{(2\pi)^3} P_\gamma(k)$$

with (Chluba et al (2011)):

$$P_\gamma(k) = \Delta_\gamma^2(k) P_\gamma^i(k) \qquad \Delta_\gamma(k) \approx 3\cos(kr_s) e^{-(k/k_D)^2}$$

$$P_\gamma^i = 1.45 \frac{2\pi^2 A_\zeta}{k^3} \left(\frac{k}{k_0}\right)^{n_s - 1 + \frac{1}{2}\ln(k/k_0)\alpha} \qquad \alpha = \frac{dn_s}{d\ln k}$$

(k_0=0.002 Mpc^{-1})

In case of modified gravity with k-dependent sound speed: (CvdB & Sculthorpe (2012))

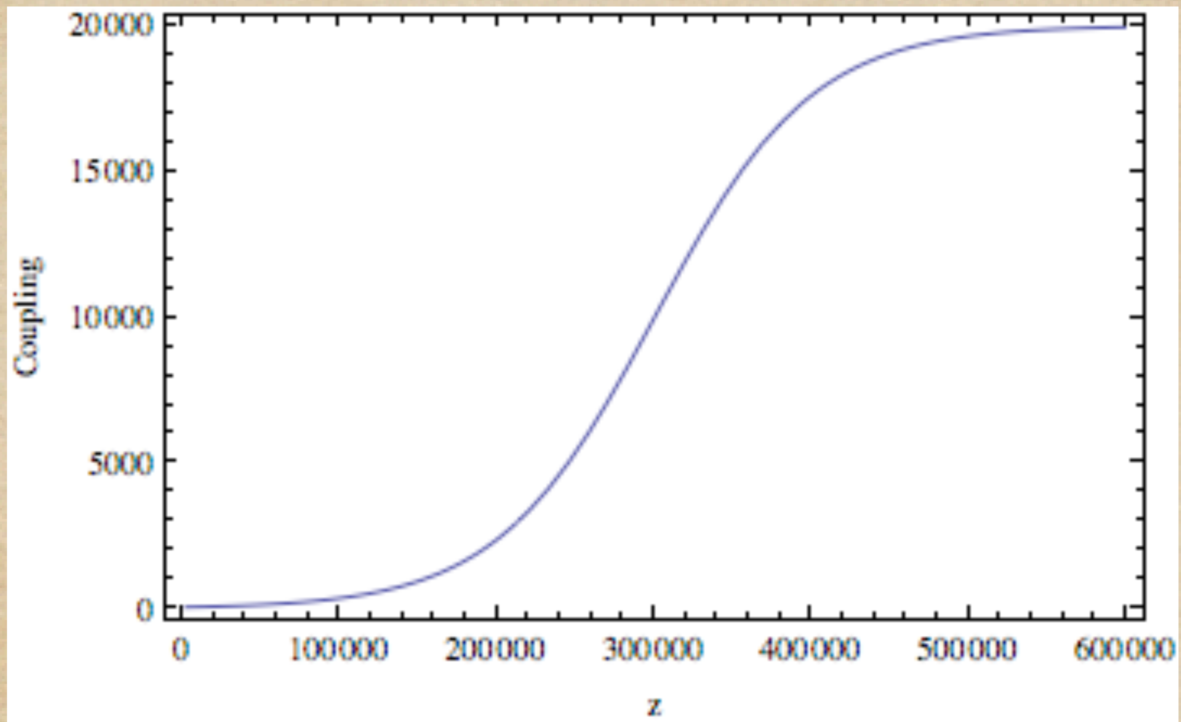
$$Q = \frac{3}{4}\rho_\gamma \int \frac{d^3k}{(2\pi)^3}\tilde{c}_s^2(k)P_\gamma(k)$$

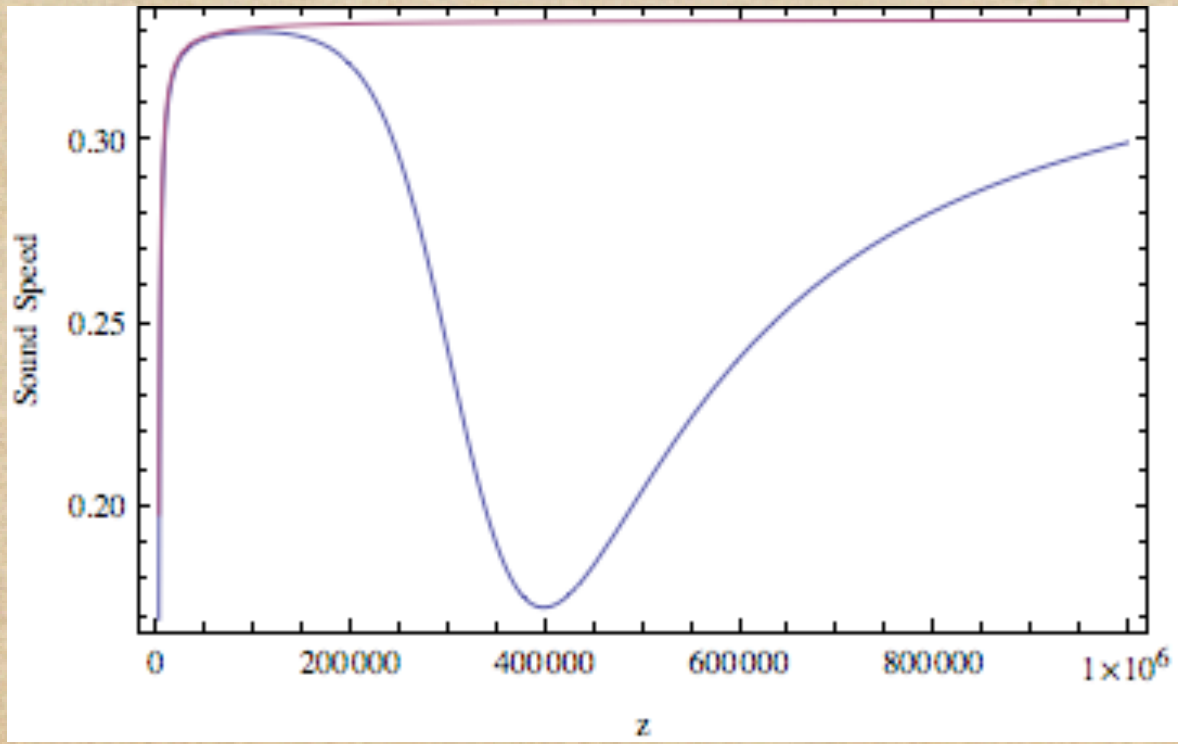
Modifications of transfer functions are in general necessary too, but we consider

$$m^2 > H^2$$

and modifications of gravity suppressed on large scales. Details will be model dependent.

Example case:

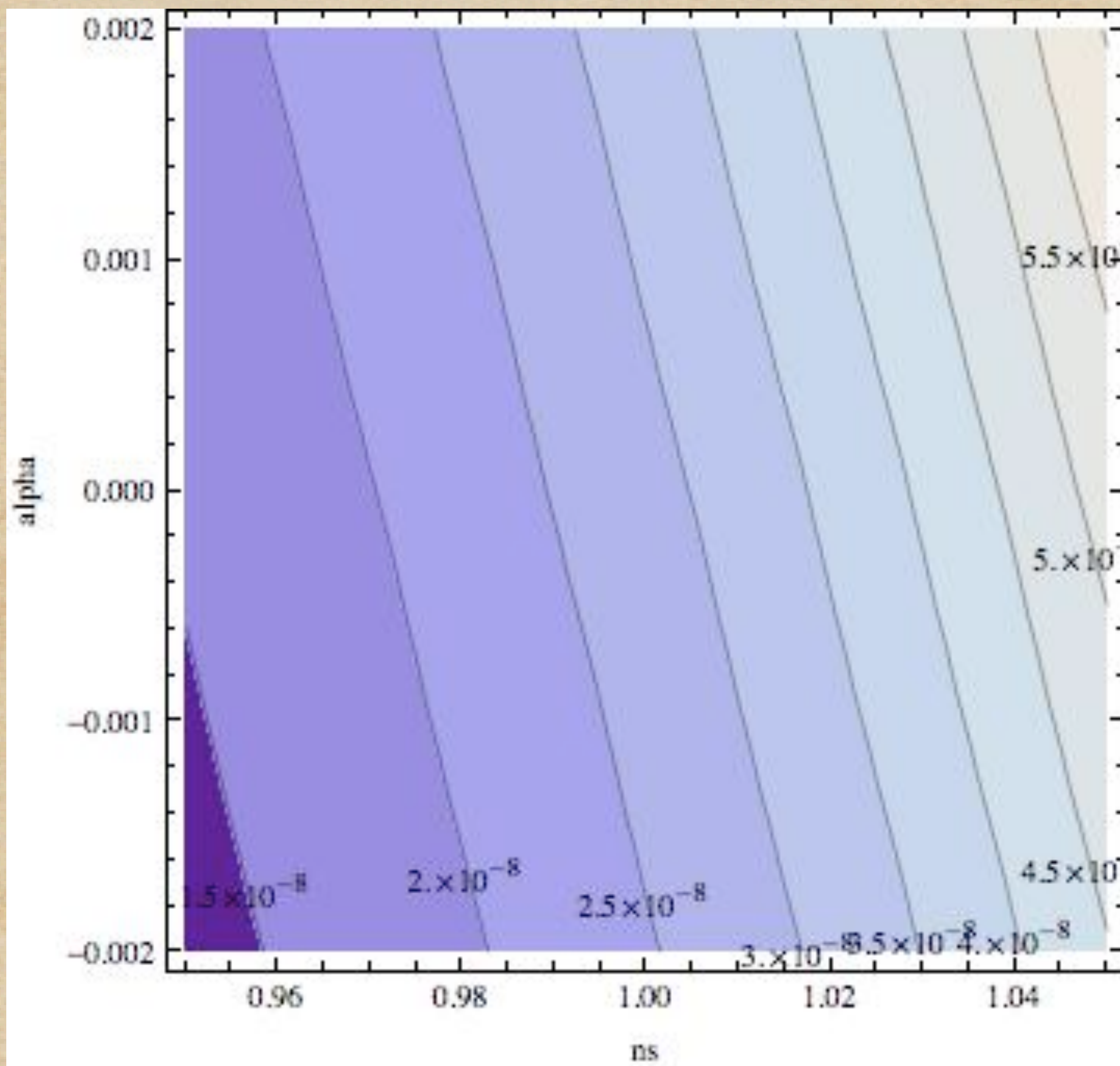




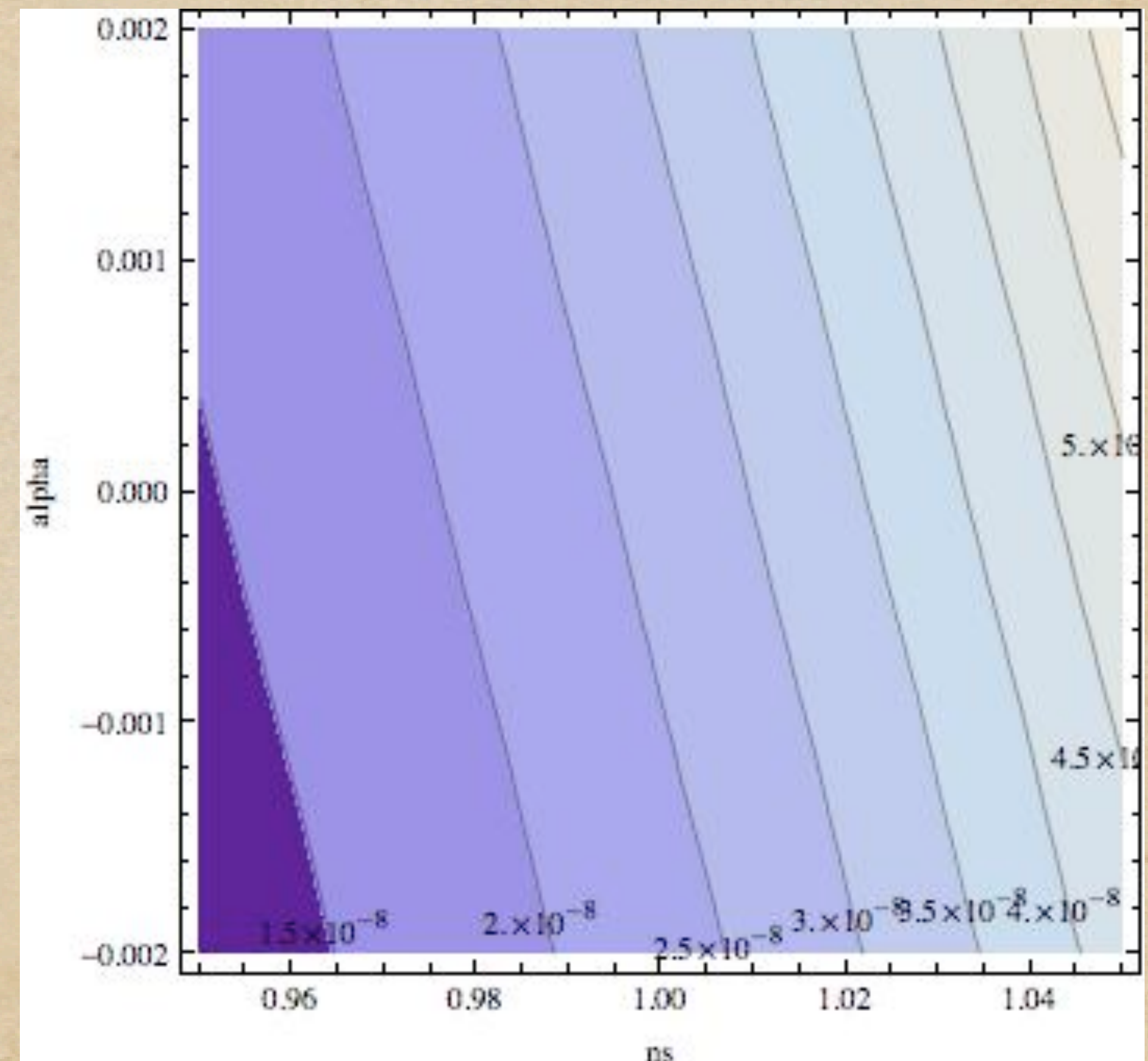
$$m(z) = 150H(z)$$

c_s^2 remains positive!

Standard GR case:



Modified Gravity:



As expected, signal is smaller in modified gravity case. For strongly coupled models, signal can be as large as other processes (Chluba et al (2012)).

Summary:

- Speed of sound of baryon-photon fluid is modified in modified gravity theories in which baryon couple to extra degrees of freedom
- Possible window to look for mod. grav. in the very early universe
- Need a moderate large coupling ($\gtrsim 10^3$) to significantly modify sound speed and a not too large effective mass; smaller couplings not constrainted by these considerations.
- Sound horizon at decoupling can deviate only very little from ΛCDM, but CMB distortions can be affected by mod. grav. effects.
- Signal reduced (smaller sound speed), but spectral distortions earliest possible direct test of mod. grav. effects.
- As usual: need to understand other contributions