Sally Clark case

A mother is being prosecuted for the murder of her two children, found dead (two years apart) in their cots at the age of a few weeks. Her defence is that they died of unexplained "cot death". An expert witness for the prosecution (a noted professor of pediatrics) says that "the chance of one child dying of cot death is 1 in 8543, so that if you multiply these together, you get the chance of both dying by cot death being 1 in 73 million." The prosecutor adds "This is extremely small, making it very unlikely that the babies died naturally, and so strongly suggests murder." You are in the jury at this trial. How do you interpret this evidence?

Sally Clark is the mother of the two babies
D = data: two baby boys both about 10 weeks old found dead in their cots (two years apart)
SIDS = sudden infant death syndrome ("cot death")
C = hypothesis that SIDS is the cause of both deaths
M = hypothesis that murder is the cause of both deaths
Sally is arrested and charged with double murder after the second child is found dead
Sir Roy Meadow = expert witness (professor of pediatrics) at trial

Meadow testifies:

$$P(\text{one cot death}) = \frac{1}{8543} \tag{1}$$

$$P(\text{two cot deaths}) = P(\text{one cot death})^2 = \frac{1}{73 \times 10^6} \quad \text{and therefore} \tag{2}$$

$$P(C) = \frac{1}{73 \times 10^6} \tag{3}$$

Flaws:

1. The cause of SIDS is unknown, but there are likely to be environmental or genetic factors. This makes the two events non-independent, significantly lowering P(two cot deaths).
2. P(two cot deaths) will be (and was) widely interpreted as the probability of Sally Clark's innocence. Because the chance that both babies died of cot death appears extremely small, it therefore appears to be extremely unlikely that Sally Clark is innocent. But this number is not that at all: whatever its value, it is conceptually the probability of getting two cot deaths, and alone tells us nothing about the probability of guilt. This misinterpretation is the *prosecutor's fallacy*.
3. P(one cot death) is actually lower in this case, because both babies were boys. Based on past evidence, boys are more likely to die of cot death, with $P(\text{one cot death}) = 1/1300$.
4. However unlikely it is that both children died of SIDS, we must consider the probability of any alternative hypothesis. (There must be alternatives or you wouldn't hold a trial.) They might be even less likely.

Solution:
We need to calculate the *relative* probabilities of the two competing hypothesis, i.e.

$$\frac{P(C|D)}{P(M|D)} = \frac{P(D|C)}{P(D|M)} \frac{P(C)}{P(M)} \tag{4}$$

which follows from taking the ratio of Bayes' theorem ($P(D)$ cancels). The data on the nature of the deaths does not allow us to distinguish between the causes of death, so $P(D|C)/P(D|M) = 1$. Therefore

$$\frac{P(C|D)}{P(M|D)} = \frac{P(C)}{P(M)} \,. \tag{5}$$

Progress can only be made if we have some idea of $P(M)$. The fact is that double murder is also extremely unlikely. As a press release by the Royal Statistical Society on this case nicely summarized: "What matters is the relative likelihood of the deaths under each explanation, not just how unlikely they are under one explanation." Other research, not presented in the trial, suggests that in the causes of double infant deaths, only about a third are due to murder. Another estimate is that $P(C)/P(M)$ is 0.1–0.5, not nearly small enough to convict. (A probability of 0.1 or more is not "beyond reasonable doubt", in my opinion.) Note that even if M were favoured, it still would not establish that Sally Clark was the murderer.

The fact is that there is a lot of uncertainty about the causes and probability of SIDS, and a lot was made of that. In the trial, the figure of 1 in 73 million received far too much attention (the witness was an expert, right?), and almost certainly persuaded the jury. The fact is that the figure is almost irrelevant. Furthermore, $P(M)$ was never even considered in the trial. Sally was convincted in 1999 by a 10-2 majority, but released on her (second) appeal four years later following an outcry from statisticians[1] and lawyers. Meadow was discredited and struck off the medical register.

Take-home messages:

1. Think about what the presented probabilities really mean: avoid the prosecutor's fallacy
2. Always consider the alternative hypothesis: look at the odds ratio

[1] `http://www.rss.org.uk/site/cms/contentCategoryView.asp?category=264`